Analytical and simulation-based approaches for quantifying multi-year risk in non-life insurance

Marc Linde | University of Ulm | October 01, 2019

Analytical and simulation-based approaches for quantifying multi-year risk in non-life insurance

Content	▪ Summary of our research and publications from 2013 – 2019 – Definition of Non-Life Insurance Risk in a multi-year view – Quantification of multi-year risk in (extended) univariate reserving models (ILR, CL models) – Quantification of multi-year risk in (extended) multivariate reserving models – with particular focus on simulation-based approaches („Stochastic Re-Reserving“)
Relevant for	▪ Practitioners in P&C Insurance companies – Risk Management – Actuarial Function – Capital Management ▪ Researchers in the field of stochastic reserving and stochastic modelling of non-life insurance risks
Application Fields	▪ Multi-year projections of Non-Life Underwriting Risks within the ORSA ▪ Assessment of Reserve uncertainty in a multi-year view ▪ Suitability assessment for standard formula assumptions and parameters ▪ Empirical estimation of correlation parameters ▪ Justification of Undertaking Specific Parameters (USPs)

Agenda

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Background

Definition of Non-Life Insurance Risk in a multi-year view

Quantification of multi-year Non-Life Insurance Risk in univariate reserving models

Quantification of multi-year Non-Life Insurance Risk in multivariate reserving models

Summary and Application Fields

Multi-Year Non-Life Insurance Risk - Background

- **Non-Life Insurance Risk** is typically composed of Reserve Risk and Premium Risk
  - **Reserve Risk** relates to claims that have already occurred in the past
  - **Premium Risk** relates to claims that will occur in the future

- Reserve Risk and Premium Risk have been typically modelled in an **ultimo view**
  - i.e. uncertainty about future claims occurrence and development is quantified up to final settlement

- Within the regulatory framework of Solvency II a **one-year view** has been introduced

  i.e. uncertainty about future claims occurrence and development needs to be quantified for one future calendar year

- For strategic decision making a **multi-year horizon** is required

  In the context of Solvency II, insurance companies are also prescribed to perform an *Own Risk and Solvency Assessment* (ORSA) as part of the risk management

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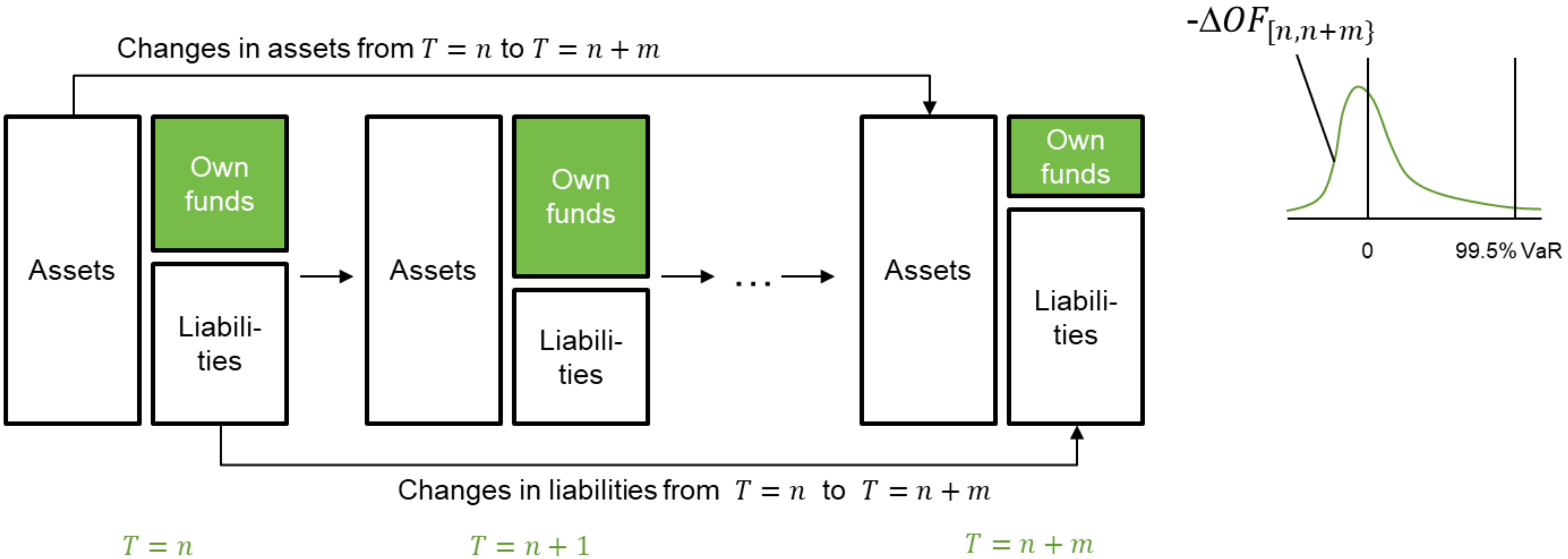
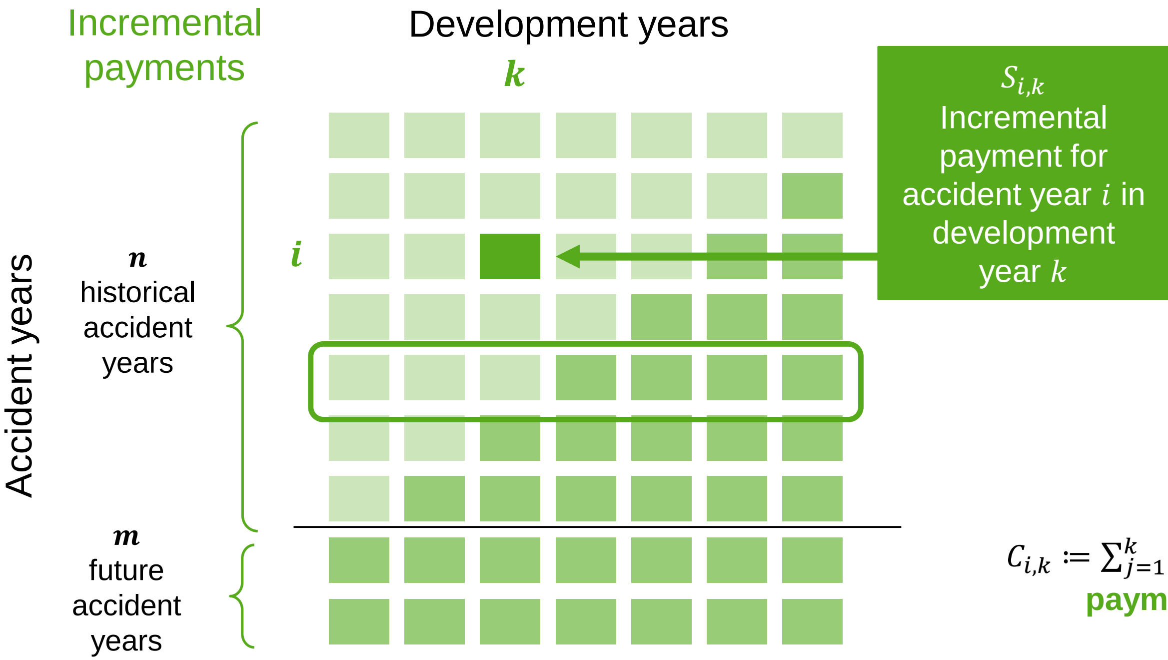
Framework for multi-year non-life insurance risk (1/7)

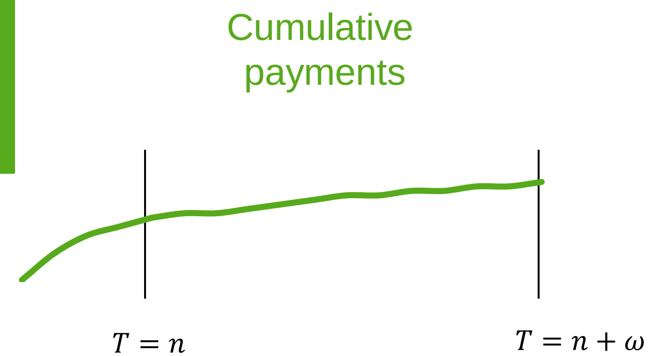
- Quantification of risk under Solvency II (i.e. calculation of the one-year SCR) is based on an economic balance sheet approach, which can be extended from a one-year perspective to an arbitrary time horizon of m future calendar years:



- Liabilities for Non-Life Insurance companies mainly consist of technical provisions which have to split into *outstanding claims provisions* and *premium provision* (for simplification reasons the latter one will be ignored in the following).
- *The market value of technical provisions* has to be calculated as *best-estimate* plus a *risk margin,* where the best estimate is derived from all relevant future cash-flows (premiums, expenses and claims payments).

Framework for multi-year non-life insurance risk (2/7)

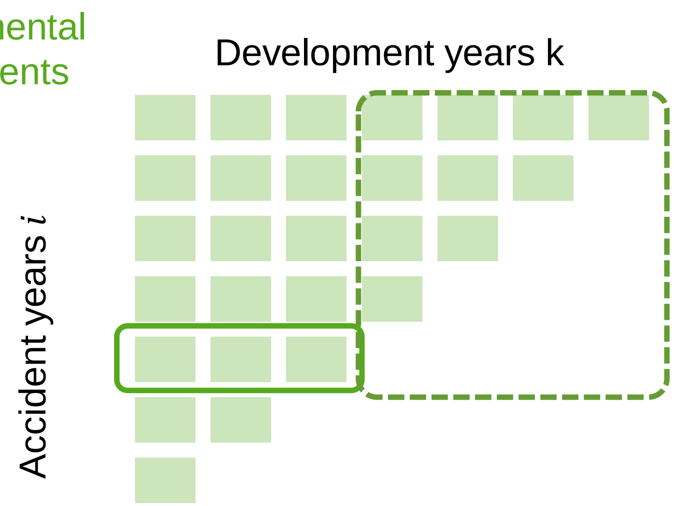


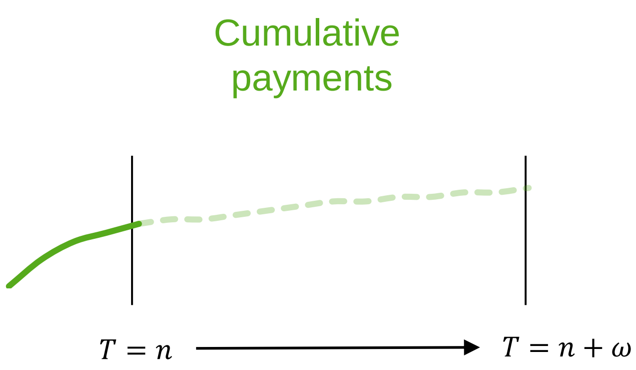


$C_{i,k} := \sum_{j=1}^{k} S_{i,j}$ represents the corresponding **cumulative payments** until development period $1 \leq k \leq \omega$

- The **claims triangle** $\mathcal{D}_n := \{S_{i,k}\}_{i+k \leq n+1}$ contains the **observed claims payments** up to $T = n$
- The payments $\{S_{i,k}\}_{n+1<i+k}$ represent **future payments** which are unknown at $T = n$ - thus we define:
  - $R_i^{(n)} := \sum_{k=n-i+2}^{\omega} S_{i,k}$ $\stackrel{\text{def}}{=}$ Cumulated future payments for **claims of a single accident year** $1 \leq i \leq n+m$
  - $U_i := C_{i,n-i+1} + R_i^{(n)}$ $\stackrel{\text{def}}{=}$ *Ultimate claim amount for a single accident year* $1 \leq i \leq n+m$
  - $R_{PY}^{(n)} := \sum_{i=1}^{n} R_i^{(n)}$ $\stackrel{\text{def}}{=}$ Cumulated future payments for **(outstanding) claims of prior accident years** $\{1, \dots, n\}$
  - $R^{(n)} := \sum_{i=1}^{n+m} R_i^{(n)}$ $\stackrel{\text{def}}{=}$ Cumulated future payments for **claims of all accident years** $\{1, \dots, n+m\}$

Framework for multi-year non-life insurance risk (3/7)



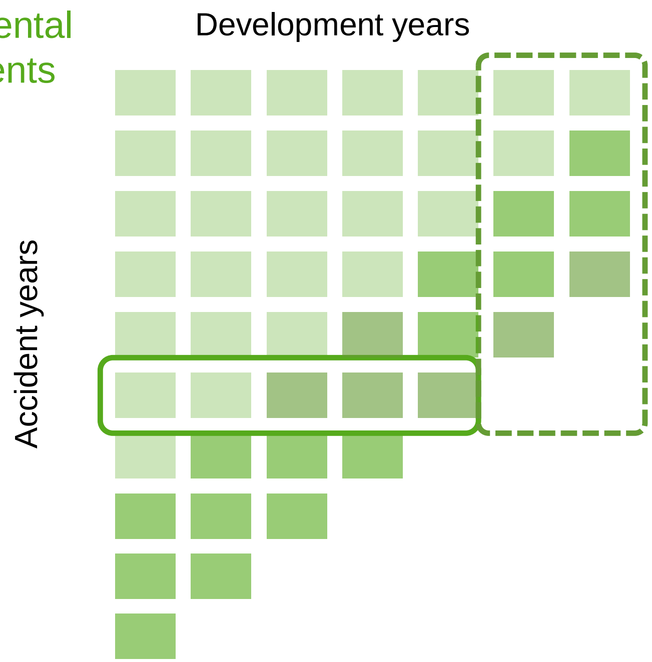


Application of an *actuarial reserving method* $\mathcal{T}_n$ (e.g. generalized chain-ladder methods, additive loss reserving method,…) yields *estimators* $\hat{S}_{i,k}^{(n)} := \hat{\mathbb{E}}[S_{i,k}|\mathcal{D}_n]$ for the *expected future payments* $\{\mathbb{E}[S_{i,k}|\mathcal{D}_n]\}_{n+1<i+k}$ as at $T = n$ – thus we define:

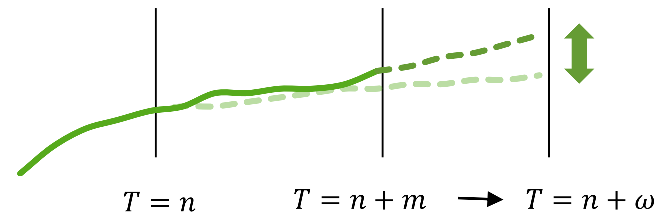
- $\hat{R}_i^{(n)} := \sum_{k=n-i+2}^{\omega} \hat{S}_{i,k}^{(n)}$ $\overset{\text{def}}{=}$ *Best-estimate* of cumulated **future claims payments for a single accident year** $1 \leq i \leq n+m$
- $\hat{U}_i^{(n)} := C_{i,n-i+1} + \hat{R}_i^{(n)}$ $\overset{\text{def}}{=}$ *Best-estimate* of the *ultimate claim amount for a single accident year* $1 \leq i \leq n+m$
- $\hat{R}_{PY}^{(n)} := \sum_{i=1}^{n} \hat{R}_i^{(n)}$ $\overset{\text{def}}{=}$ *Best-estimate reserve* for **outstanding claims of prior accident years** $\{1, \dots, n\}$
- $\hat{R}^{(n)} := \sum_{i=1}^{n+m} \hat{R}_i^{(n)}$ $\overset{\text{def}}{=}$ *Best-estimate* of cumulated *future claims payments for all prior and future accident years*

Framework for multi-year non-life insurance risk (4/7)

Incremental payments



Cumulative payments

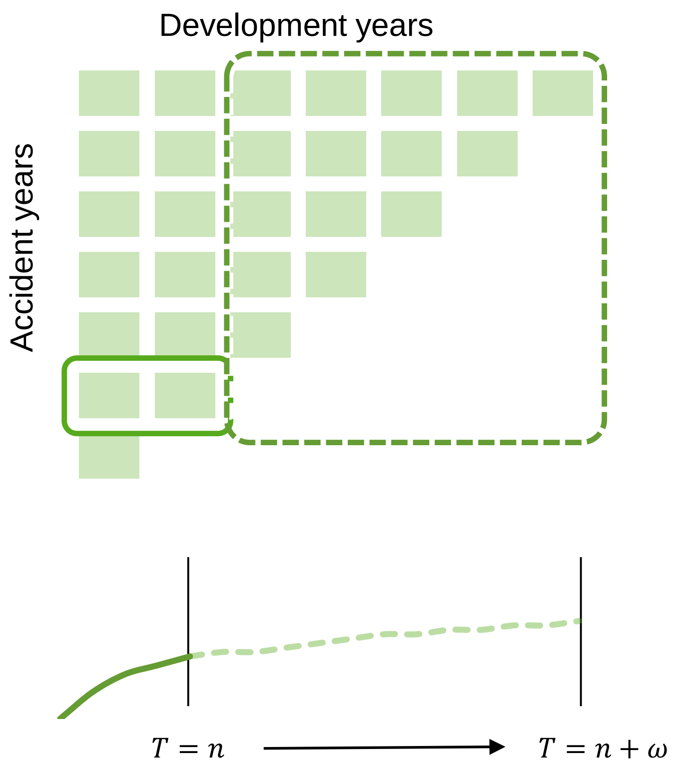


Application of *actuarial reserving method* $\mathcal{T}_{n+m}$ yields *estimators* $\hat{S}_{i,k}^{(n+m)} := \hat{\mathbb{E}}[S_{i,k} | \mathcal{D}_{n+m}]$ for the *expected future payments* $\{\mathbb{E}[S_{i,k} | \mathcal{D}_{n+m}]\}_{n+m+1<i+k}$ as at $T = n + m$:

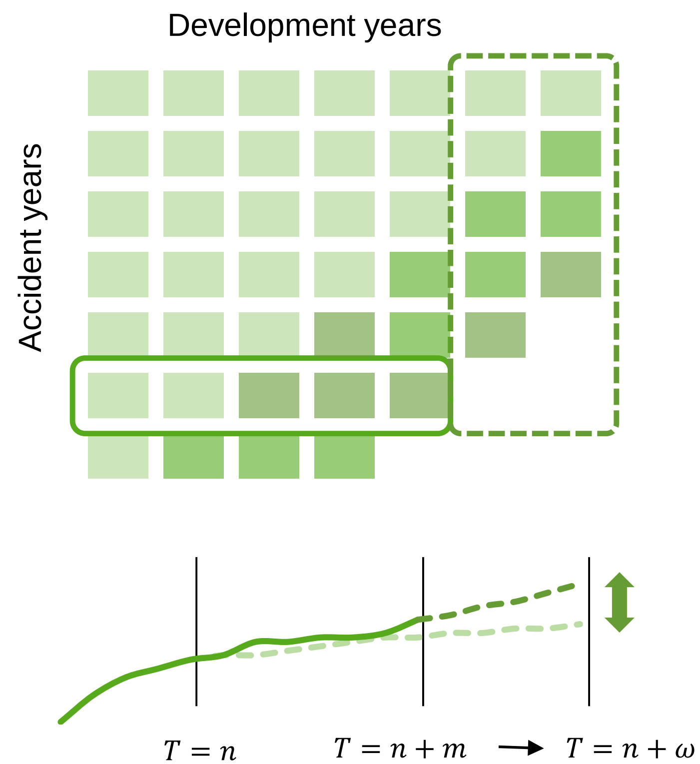
- $\hat{R}_i^{(n+m)} := \sum_{k=n-i+m+2}^{\omega} \hat{S}_{i,k}^{(n+m)}$ $\stackrel{\text{def}}{=}$ *Best-estimate* of cumulated ***future claims payments** for a single accident year* i
- $\hat{U}_i^{(n+m)} := C_{i,n-i+m+1} + \hat{R}_i^{(n+m)}$ $\stackrel{\text{def}}{=}$ *Best-estimate* of the *ultimate claim amount for a single accident year* $1 \leq i \leq n + m$
- $\hat{R}_{PY}^{(n+m)} := \sum_{i=1}^{n} \hat{R}_i^{(n+m)}$ $\stackrel{\text{def}}{=}$ *Best-estimate reserve* for *outstanding claims of prior accident years* $\{1, \ldots, n\}$
- $\hat{R}^{(n+m)} := \sum_{i=1}^{n+m} \hat{R}_i^{(n+m)}$ $\stackrel{\text{def}}{=}$ *Best-estimate* of cumulated *future claims payments for all prior and future accident years*

Framework for multi-year non-life insurance risk (5/7)

(Initial) Valuation as per $T = n$



(Re-)Valuation as per $T = n + \mathrm{m}$



Incremental payments

Cumulative payments

Multi-year claims development result for single accident year i

$$\widehat{\mathrm{CDR}}_i^{(n \to n+m)} := \widehat{U}_i^{(n)} - \widehat{U}_i^{(n+m)}$$

Framework for multi-year non-life insurance risk (6/7)

Reserve Risk – Claims from prior accident years $\{1, \dots, n\}$

$$\widehat{\mathrm{CDR}}_{PY}^{(n\to n+m)} := \sum_{i=1}^{n} \widehat{\mathrm{CDR}}_{i}^{(n\to n+m)}$$

$\widehat{\mathrm{CDR}}_{PY}^{(n\to n+m)} < 0 \ (> 0)$ is equal to a cumulated economic loss (economic profit) over the next m calendar years, hence leads to a reduction / an increase of the basic own funds leaving all other equal.

Premium Risk – Claims from future accident years $\{n+1, \dots, n+m\}$

$$\widehat{\mathrm{CDR}}_{NY}^{(n\to n+m)} := \sum_{i=n+1}^{n+m} \widehat{\mathrm{CDR}}_{i}^{(n\to n+m)}$$

The *cumulated estimated m-year underwriting result* $\sum_{i=n+1}^{n+m} \widehat{\mathrm{UW}}_i^{(n+m)}$ for accident years $n+1, \dots, n+m$ can be expressed in terms of the *claims development result for future accident years* $\widehat{\mathrm{CDR}}_{NY}^{(n\to n+m)}$:

$$\sum_{i=n+1}^{n+m} \widehat{\mathrm{UW}}_i^{(n+m)} = \sum_{i=n+1}^{n+m} P_i - E_i - \hat{U}_i^{(n+m)} = \widehat{\mathrm{CDR}}_{NY}^{(n\to n+m)} + \left(\sum_{i=n+1}^{n+m} \widehat{\mathrm{UW}}_i^{(n)} \right)$$

Hereby P_i denotes the *future earned premium* for accident year $n+1 \le i \le n+m$, E_i the corresponding *expenses*. Hence $\sum_{i=n+1}^{n+m} \widehat{\mathrm{UW}}_i^{(n+m)} < 0 \ (> 0)$ means a cumulated economic loss (economic profit) over the next m years.

Framework for multi-year non-life insurance risk (7/7)

Non-Life Insurance Risk – claims from prior and future accident years

$$\widehat{\mathrm{CDR}}^{(n\to n+m)} := \widehat{\mathrm{CDR}}_{PY}^{(n\to n+m)} + \widehat{\mathrm{CDR}}_{NY}^{(n\to n+m)}$$

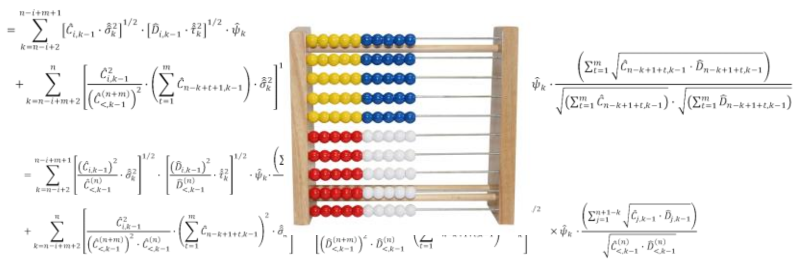
$$\sum_{i=n+1}^{n+m} \widehat{\mathrm{UW}}_i^{(n+m)} + \widehat{\mathrm{CDR}}_{PY}^{(n\to n+m)} = \sum_{i=n+1}^{n+m} \widehat{\mathrm{UW}}_i^{(n)} + \widehat{\mathrm{CDR}}^{(n\to n+m)},$$

- We have seen that the m-year reserve risk, m-year premium risk and hence m-year non-life insurance risk can be expressed in terms of ***volatility of the claims development result*** over the next m calendar years.
- In general, different (and separate) models for Premium Risk and Reserve Risk are applied, explicit correlation assumptions necessary to combine both risks.
- Next milestone: Define integrated modelling framework!

Agenda

Quantification of Non-Life Insurance Risk – General Overview

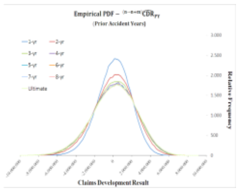
Analytical Approaches

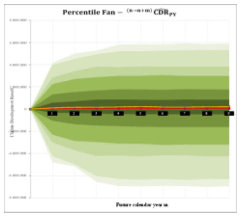


- Estimators for mean squared error of prediction
- Only provides moments up to 2nd order (= standard deviation of the predictive distribution)
- Closed-form analytical formulae
- Fast computation
- Available for most common reserving models
- one-year and ultimo view

Closed-form estimators for the mean squared error of prediction (msep) for $\widehat{\mathrm{CDR}}^{(n\to n+m)}$ in distribution-free stochastic reserving models

Bootstrap Approaches





- Resampling, simulation-based
- Leads to a full predictive distribution (provides also higher moments and risk measures like VaR and TVaR)
- More time-consuming, subject to simulation error
- Applicable to most common reserving models

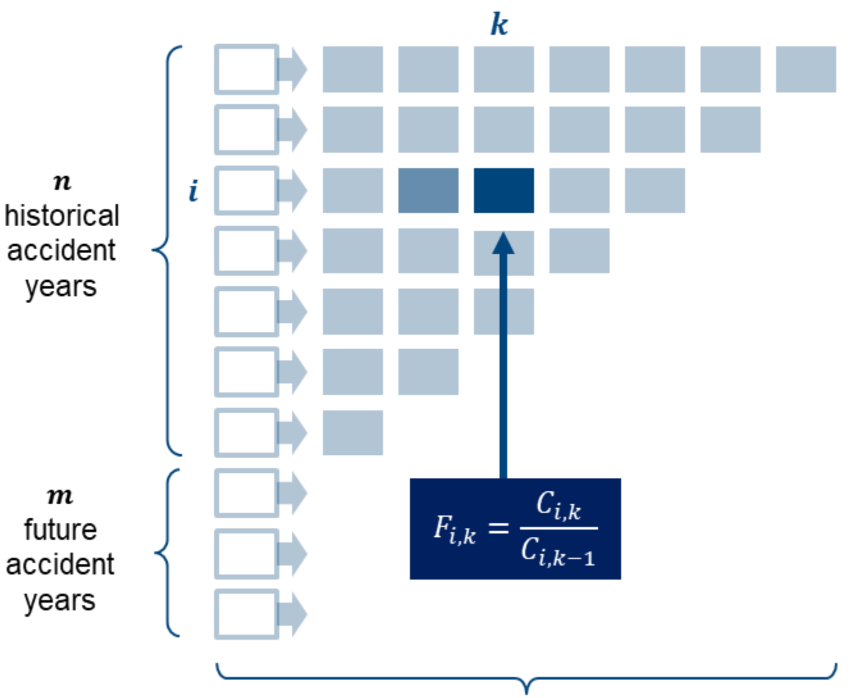
Stochastic m-year re-reserving (“actuary in a box”): to estimate full predictive distribution of $\widehat{\mathrm{CDR}}^{(n\to n+m)}$ generalizing the one-year bootstrap by Ohlsson and Lauzeningks (2009)

Quantification of Non-Life Insurance Risk - univariate reserving models

- The **reserving methods** used for best-estimate prediction determine the **stochastic model**.
- Focus on two of the most common reserving methods:
  - **C**hain-**L**adder method (CL) $\Longrightarrow$ Mack chain-ladder model
  - **I**ncremental **L**oss **R**atio method (ILR) $\Longrightarrow$ Additive loss reserving model
- Estimate parameters:
  - CL: Chain-ladder factors, Mack volatility parameters
  - ILR: Incremental loss ratios, volatility parameters
- Extend the current models to allow for future accident years (Premium Risk) as well: for the chain ladder model, we introduce a volume model for the first development year.

Results for univariate reserving models – (Extended) Chain Ladder Model

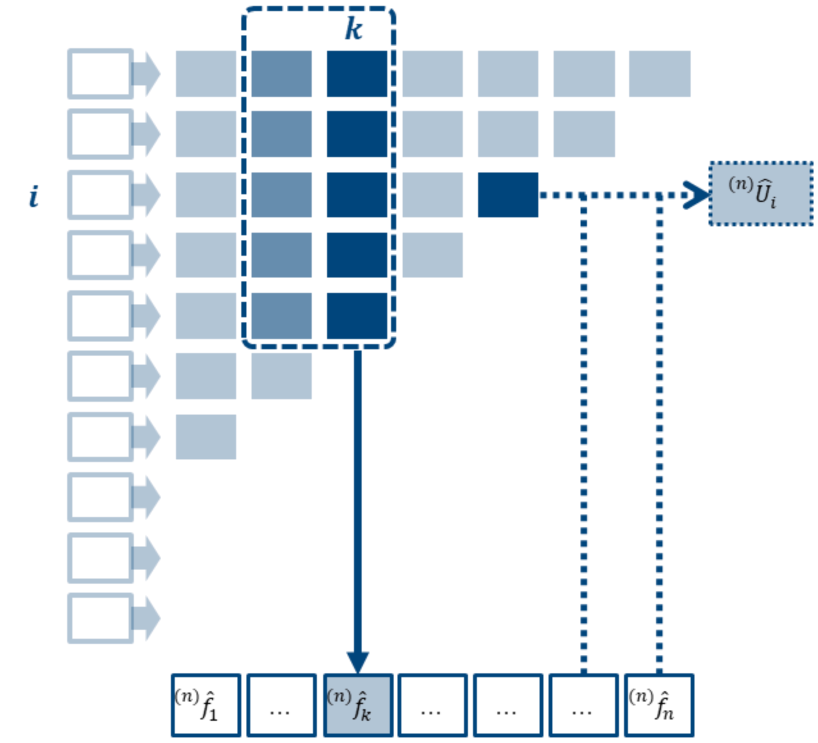
Model Definition



Extended Mack chain-ladder model

- Independence of accident years
- $\mathbb{E}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = f_k$
- $\mathbb{V}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = \sigma_k^2 / C_{i,k-1}$

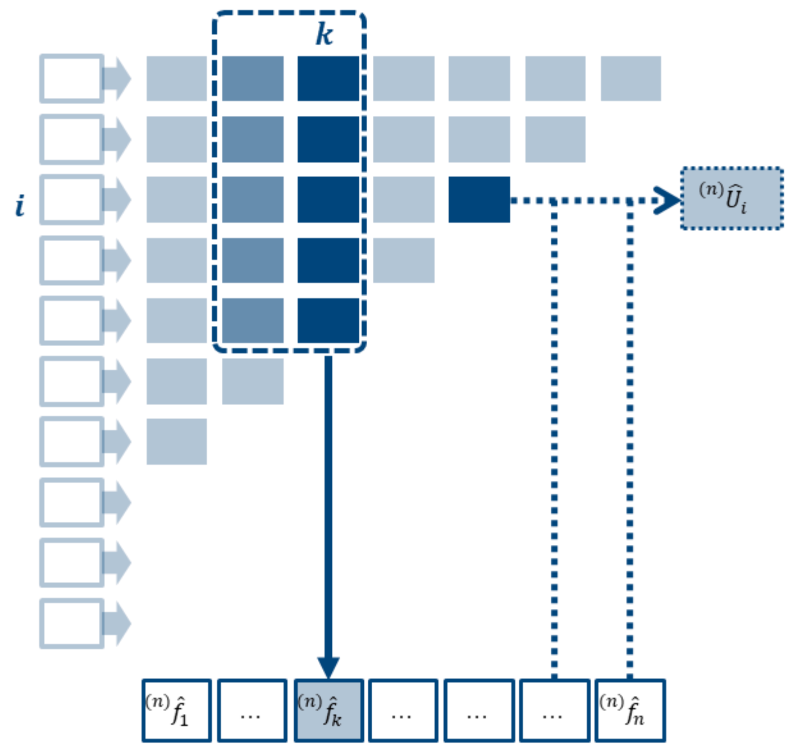
(Initial) Valuation as per $T = n$



- $\hat{f}_k^{(n)} = \sum_{i=1}^{n-k+1} C_{i,k} \; / \; \sum_{i=1}^{n-k+1} C_{i,k-1}$
- $\widehat{U}_i^{(n)} = C_{i,n-i+1} \cdot \left(\prod_{k=n-i+2}^{n} \hat{f}_k^{(n)} \right)$

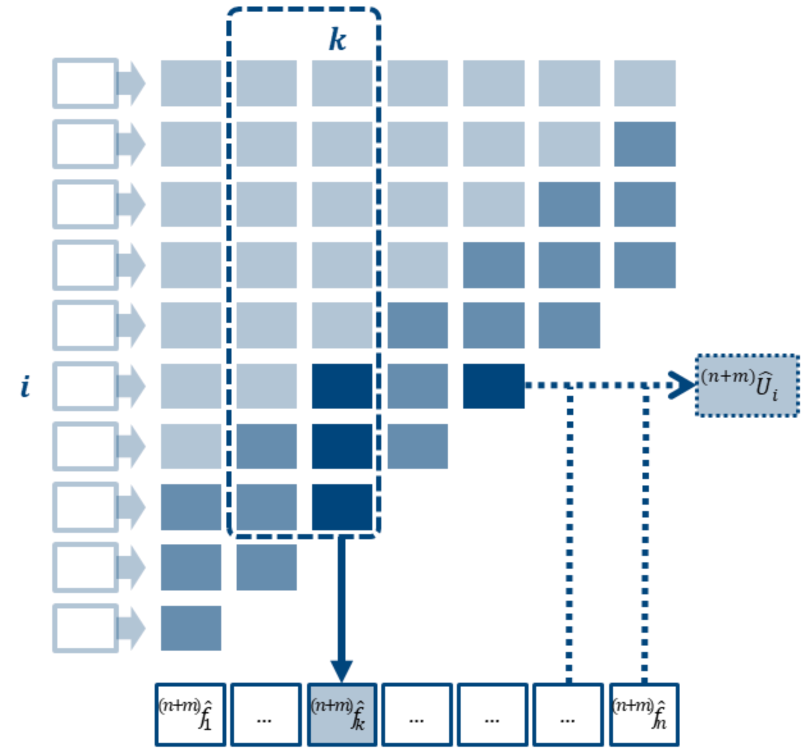
Results for univariate reserving models – (Extended) Chain Ladder Model

(Initial) Valuation as per $T = n$



- $\hat{f}_k^{(n)} = \sum_{i=1}^{n-k+1} C_{i,k} \;/\; \sum_{i=1}^{n-k+1} C_{i,k-1}$
- $\widehat{U}_i^{(n)} = C_{i,n-i+1} \cdot \left(\prod_{k=n-i+2}^{n} \hat{f}_k^{(n)}\right)$

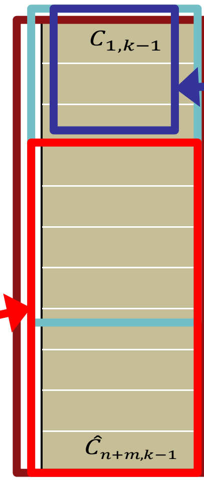
(Re-)Valuation as per $T = n + \mathrm{m}$



- $\hat{f}_k^{(n+\mathbf{m})} = \sum_{i=1}^{n-k+\mathbf{m}+1} C_{i,k} \;/\; \sum_{i=1}^{n-k+\mathbf{m}+1} C_{i,k-1}$
- $\widehat{U}_i^{(n+\mathbf{m})} = C_{i,n-i+\mathbf{m}+1} \cdot \left(\prod_{k=n-i+\mathbf{m}+2}^{n} \hat{f}_k^{(n)}\right)$

Results for univariate reserving models – (Extended) Chain Ladder Model

The **mean squared errors of prediction** for the various multi-year claims development results can be written in a **closed analytical form** (see [Diers, Hahn and Linde 2016]) - formulas were derived by means of a First-Order Taylor approximation.



$$\hat{C}^{(n)}_{<,k-1} := \sum_{i=1}^{n+1-k} \hat{C}_{i,k-1}$$

$$\hat{C}^{(n+m)}_{<,k-1} := \sum_{i=1}^{n+m+1-k} \hat{C}_{i,k-1}$$

$$C^{(n)}_{\geq,k-1} := \hat{C}^{(n)}_{+,k-1} - \hat{C}^{(n)}_{<,k-1}$$

$$\hat{C}^{(n+m)}_{+,k-1} := \sum_{i=1}^{n+m} \hat{C}_{i,k-1}$$

Single accident year $i \in \{1, \dots, n\}$

$$\widehat{\text{msep}}_{\text{CDR}_i^{(n\to n+m)}|\mathcal{D}_n}(0) = \sum_{k=n+2-i}^{n+m+1-i} \hat{C}_{i,k-1} \cdot \left(1 + \frac{\hat{C}_{i,k-1}}{\hat{C}^{(n)}_{<,k-1}}\right) \hat{\hat{\sigma}}_k^2 + \sum_{k=n+m+2-i}^{n} \frac{\hat{C}^2_{i,k-1}}{C^{(n+m)}_{<,k-1}\, C^{(n)}_{<,k-1}} \cdot \left(\sum_{t=1}^{m} C_{n+1+t-k,k-1}\right) \hat{\hat{\sigma}}_k^2$$

Prior years $PY := \{1, \dots, n\}$

$$\widehat{\text{msep}}_{\text{CDR}_{PY}^{(n\to n+m)}|\mathcal{D}_n}(0) = \sum_{k=2}^{m} \frac{\hat{C}^{(n)}_{+,k-1}\, \hat{C}^{(n)}_{\geq,k-1}}{\hat{C}^{(n)}_{<,k-1}} \hat{\hat{\sigma}}_k^2 + \sum_{k=m+1}^{n} \frac{\left(\hat{C}^{(n)}_{+,k-1}\right)^2}{\hat{C}^{(n+m)}_{<,k-1}\, \hat{C}^{(n)}_{<,k-1}} \cdot \left(\sum_{t=1}^{m} \hat{C}_{n-k+1+t,k-1}\right) \hat{\hat{\sigma}}_k^2$$

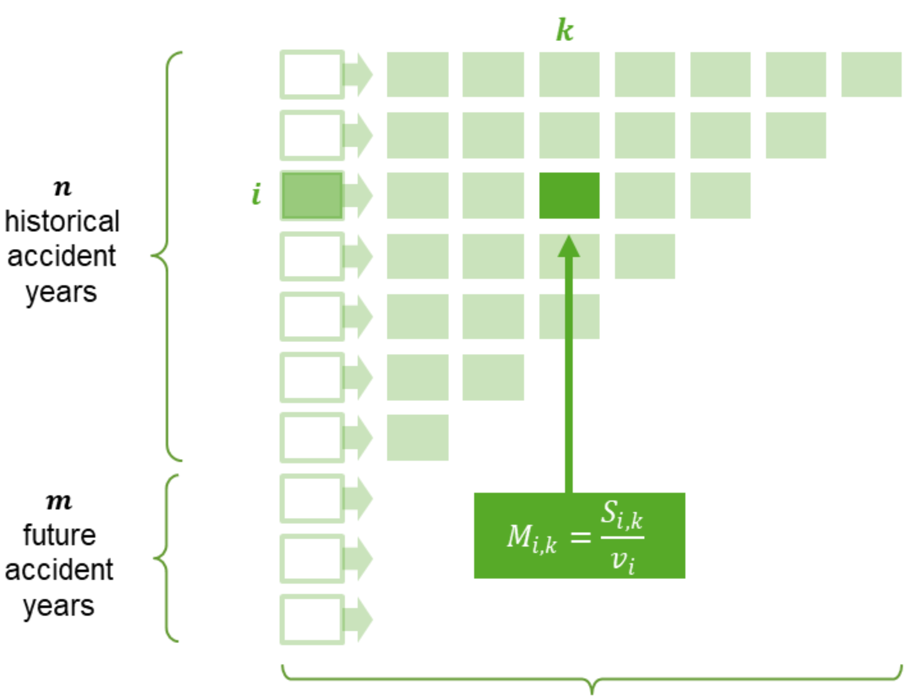
Prior and future accident years

$$\widehat{\text{msep}}_{\text{CDR}^{(n\to n+m)}|\mathcal{D}_n}(0) = \sum_{k=1}^{n} \frac{\left(\hat{C}^{(n+m)}_{+,k-1}\right)^2}{\hat{C}^{(n+m)}_{<,k-1}\, \hat{C}^{(n)}_{<,k-1}} \cdot \left(\sum_{t=1}^{m} \hat{C}_{n-k+1+t,k-1}\right) \hat{\hat{\sigma}}_k^2$$

$$\hat{\sigma}_k^2 := \frac{1}{n-k} \cdot \sum_{i=1}^{n-k+1} C_{i,k-1} \cdot \left(\frac{C_{i,k}}{C_{i,k-1}} - \hat{f}_k^{(n)}\right)^2, \hat{\sigma}_n^2 := \min\{\hat{\sigma}^2_{n-3}, \hat{\sigma}^2_{n-2}, \hat{\sigma}^2_{n-1}\} \qquad \hat{\hat{\sigma}}_k^2 := \hat{\sigma}_k^2 \cdot \prod_{l=k+1}^{n} \left(\hat{f}_l^{(n)}\right)^2$$

Results for univariate reserving models – (Extended) Additive Model

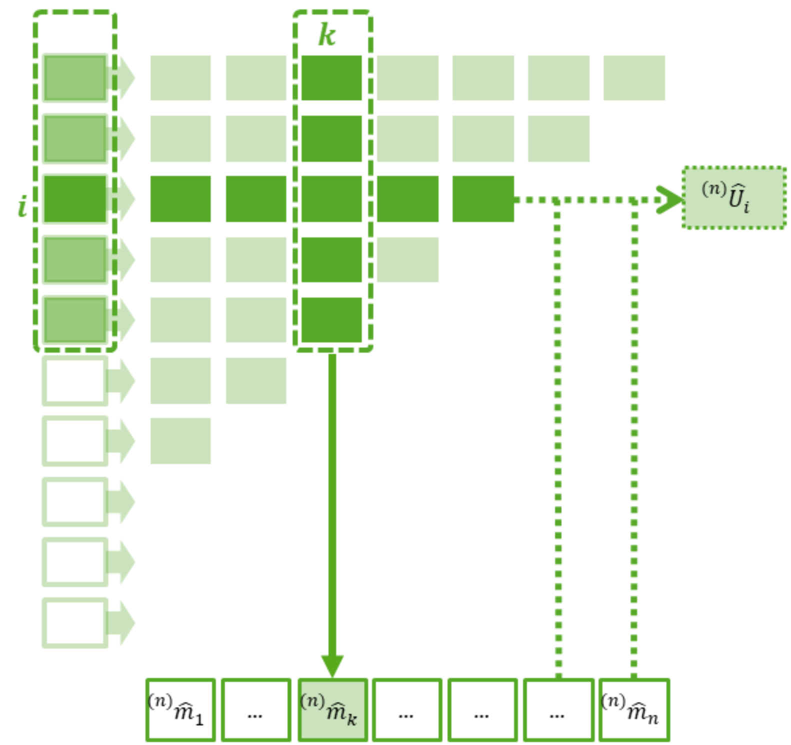
Model Definition



Additive reserving model

- Independence of incremental payments
- $\mathbb{E}[M_{i,k}] = m_k$
- $\mathbb{V}[M_{i,k}] = s_k^2/v_i$

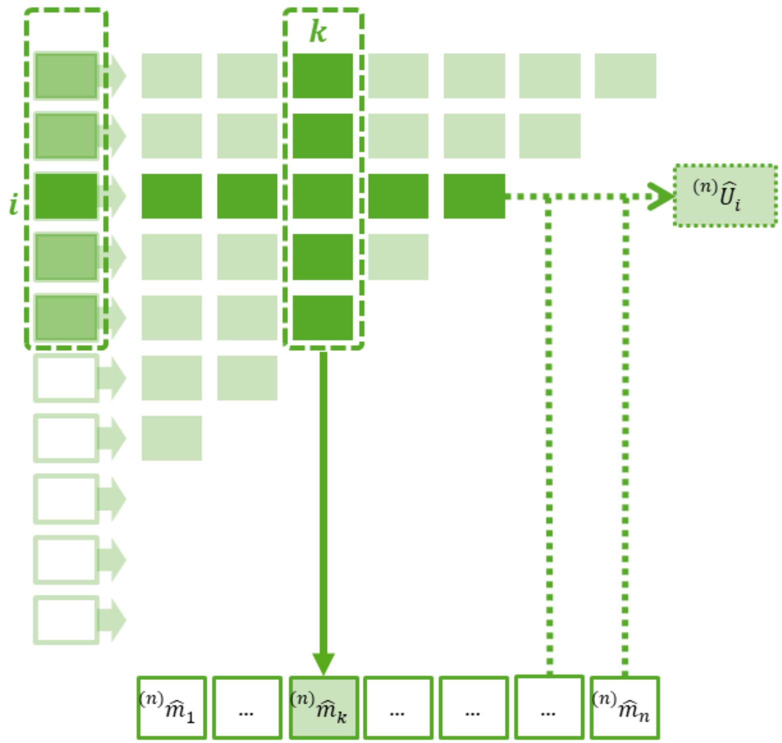
(Initial) Valuation as per $T = n$



- $\widehat{m}_k^{(n)} := \sum_{i=1}^{n-k+1} S_{i,k} \;/\; \sum_{i=1}^{n-k+1} v_i$
- $\widehat{R}_i^{(n)} = v_i \cdot \left(\sum_{k=n-i+2}^{n} \widehat{m}_k^{(n)}\right)$

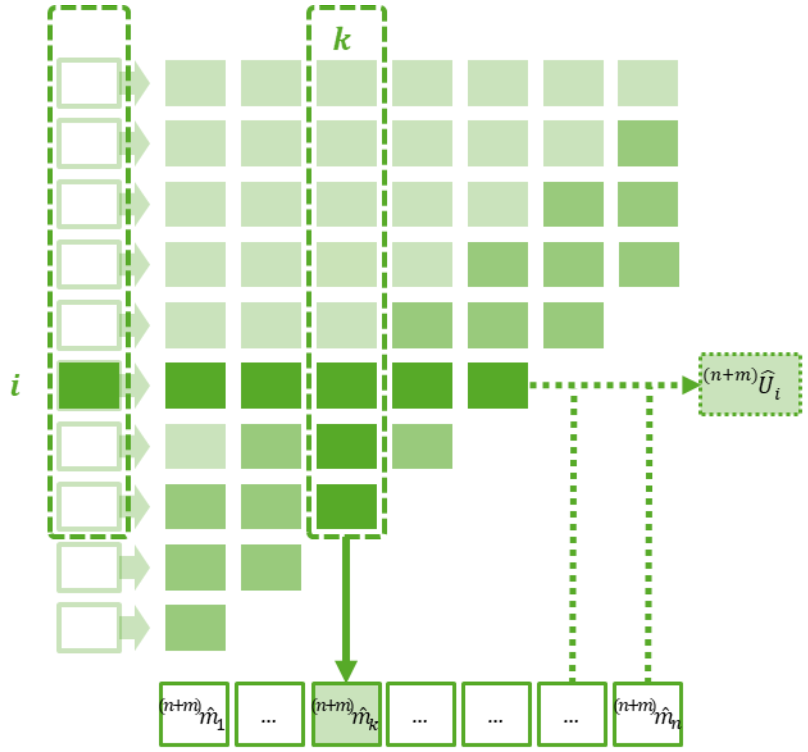
Results for univariate reserving models – (Extended) Additive Model

(Initial) Valuation as per $T = n$



- $\hat{m}_k^{(n)} := \sum_{i=1}^{n-k+1} S_{i,k} \ / \ \sum_{i=1}^{n-k+1} v_i$
- $\hat{R}_i^{(n)} = v_i \cdot \left(\sum_{k=n-i+2}^{n} \hat{m}_k^{(n)} \right)$

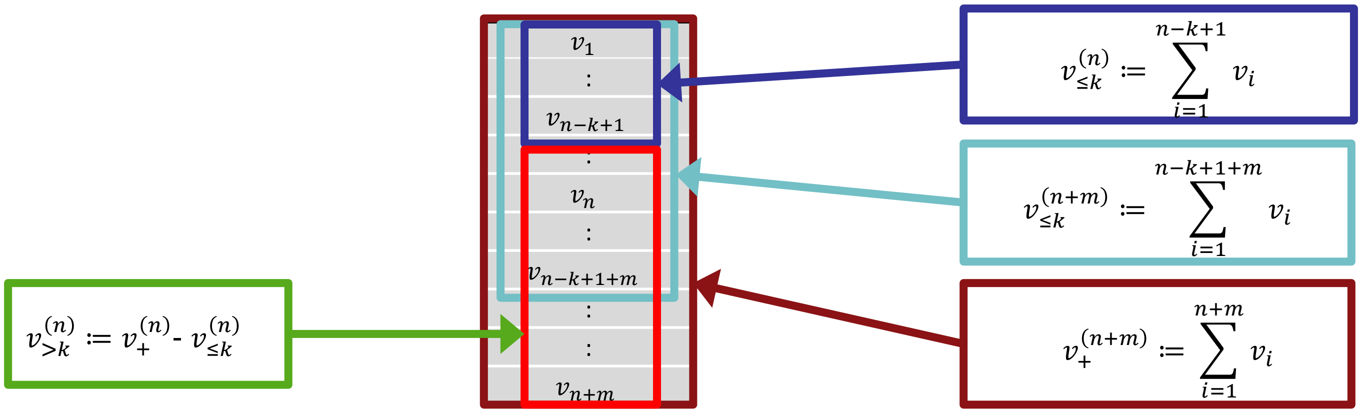
(Re-)Valuation as per $T = n + \mathrm{m}$



- $\hat{m}_k^{(n+\boldsymbol{m})} := \sum_{i=1}^{n-k+\boldsymbol{m}+1} S_{i,k} \ / \ \sum_{i=1}^{n-k+\boldsymbol{m}+1} v_i$
- $\hat{R}_i^{(n+\boldsymbol{m})} = v_i \cdot \left(\sum_{k=n-i+\boldsymbol{m}+2}^{n} \hat{m}_k^{(n+\boldsymbol{m})} \right)$

Results for univariate reserving models – (Extended) Additive Model

Also for the additive reserving model, the **mean squared errors of prediction** for the multi-year claims development results can be written in a **closed analytical form** (see [Diers and Linde 2013]):



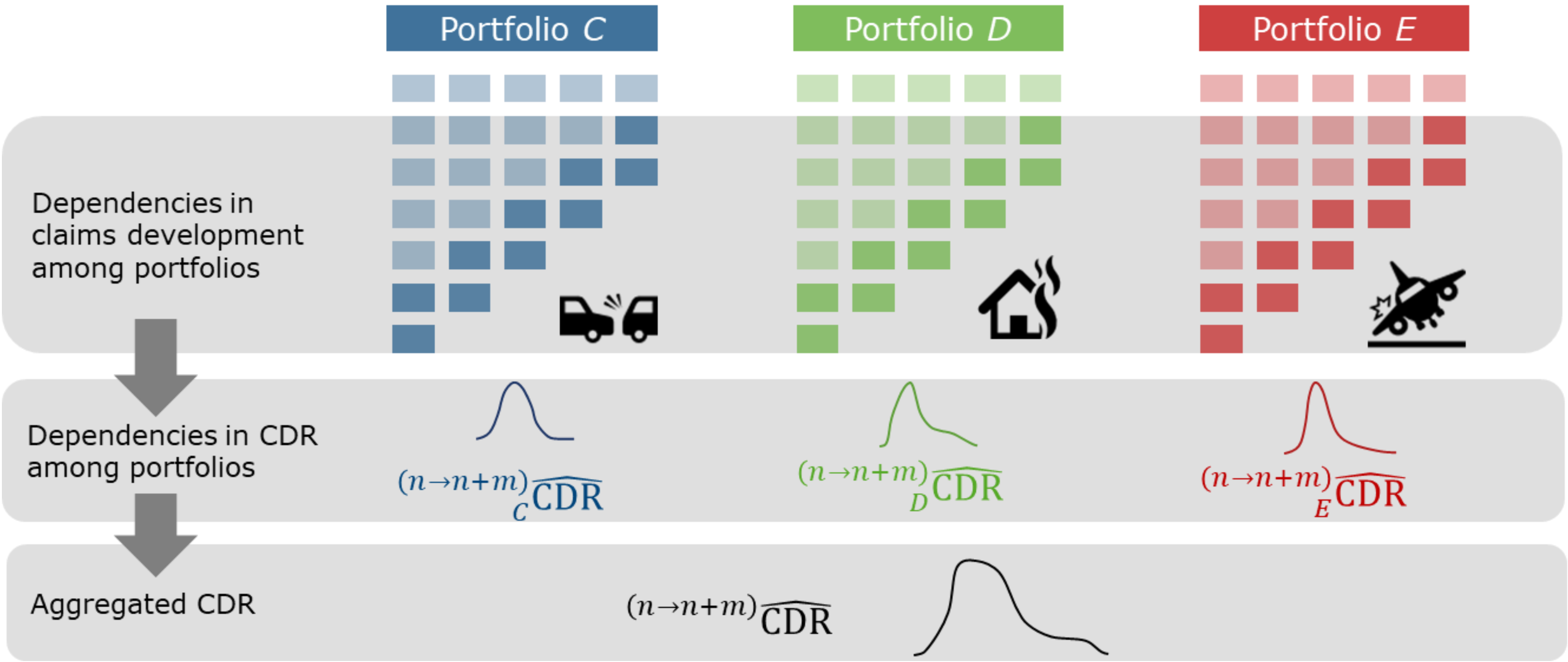
Single accident year $i \in \{1,\dots,n\}$	$\widehat{\mathrm{msep}}_{\widehat{\mathrm{CDR}}_i^{(n\to n+m)}\mid\mathcal{D}_n}(0) = \sum_{k=n-i+2}^{n-i+1+m} v_i \cdot \left(1+\frac{v_i}{v_{\le k}^{(n)}}\right)\hat{s}_k^2 + \sum_{k=n-i+2+m}^{n} \frac{v_i^2}{v_{\le k}^{(n+m)}\, v_{\le k}^{(n)}} \cdot \left(\sum_{t=1}^{m} v_{n-k+1+t}\right)\hat{s}_k^2$
Prior years $PY := \{1,\dots,n\}$	$\widehat{\mathrm{msep}}_{\widehat{\mathrm{CDR}}_{PY}^{(n\to n+m)}\mid\mathcal{D}_n}(0) = \sum_{k=2}^{m} \frac{v_+^{(n)}\, v_{>k}^{(n)}}{v_{\le k}^{(n)}}\hat{s}_k^2 + \sum_{k=m+1}^{n} \frac{\left(v_+^{(n)}\right)^2}{v_{\le k}^{(n+m)}\, v_{\le k}^{(n)}} \cdot \left(\sum_{t=1}^{m} v_{n-k+1+t}\right)\hat{s}_k^2$
Prior and future accident years	$\widehat{\mathrm{msep}}_{\widehat{\mathrm{CDR}}^{(n\to n+m)}\mid\mathcal{D}_n}(0) = \sum_{k=1}^{n} \frac{\left(v_+^{(n+m)}\right)^2}{v_{\le k}^{(n+m)}\, v_{\le k}^{(n)}} \cdot \left(\sum_{t=1}^{m} v_{n-k+1+t}\right)\hat{s}_k^2$

$$\hat{s}_k^2 = \frac{1}{n-k} \cdot \sum_{i=1}^{n-k+1} v_i \cdot \left(\frac{S_{i,k}}{v_i} - \widehat{m}_k^{(n)}\right)^2, \hat{s}_n^2 := \min\{\hat{s}_1^2,\dots,\hat{s}_{n-1}^2\}$$

Agenda

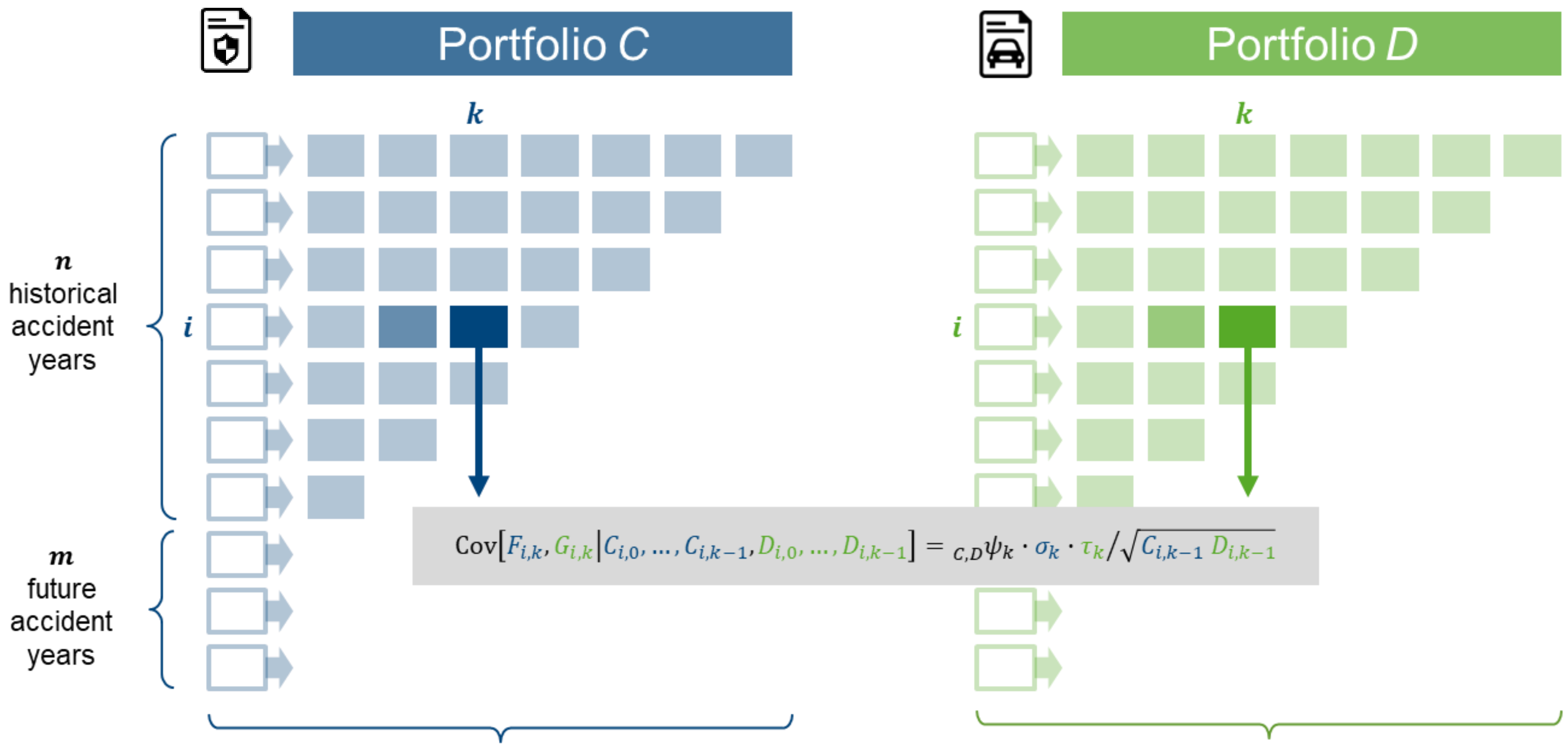
Multi-year claims development result for multiple portfolios

- Application of actuarial reserving methods for best estimate valuation needs to be performed on the level of homogeneous risk groups (at least SII Minimum lines of business)
- Segmentation into several lines of business and further division into portfolios
- Dependencies between portfolios become obvious in the aggregation process to the aggregated claims development result on company level.



Multi-year non-life insurance risk in the multivariate Chain Ladder Model – Model framework and results (1/7)

Bivariate chain-ladder model



$$\mathrm{Cov}\left[F_{i,k}, G_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}, D_{i,0}, \dots, D_{i,k-1}\right] = {}_{C,D}\psi_k \cdot \sigma_k \cdot \tau_k / \sqrt{C_{i,k-1}\, D_{i,k-1}}$$

Marginal Mack chain-ladder model

- Independence of accident years
- $\mathbb{E}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = f_k$
- $\mathbb{V}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = \sigma_k^2 / C_{i,k-1}$

Marginal Mack chain-ladder model

- Independence of accident years
- $\mathbb{E}\left[G_{i,k} \middle| D_{i,0}, \dots, D_{i,k-1}\right] = g_k$
- $\mathbb{V}\left[G_{i,k} \middle| D_{i,0}, \dots, D_{i,k-1}\right] = \tau_k^2 / D_{i,k-1}$

Multi-year non-life insurance risk in the multivariate Chain Ladder Model – Model framework and results (2/7)

- The original Braun model is also known as bivariate chain ladder model and provides an extension of the classical chain ladder type-models to the case of two portfolios allowing for dependencies in their claims development processes.
- Braun introduces a correlation between single development factors of different portfolios at the same stage of development
- As single development factors are correlated according to the assumptions of the Braun model, also chain ladder factors (estimated as per $T = n$ und $T = n + m$) are correlated.
- Thus, also Claims Development Results ${}^{C}\widehat{\mathrm{CDR}}^{(n \to n+m)}$ and ${}^{D}\widehat{\mathrm{CDR}}^{(n \to n+m)}$ are correlated, hence in general the mean squared error of predictions (msep) per single portfolio can not be simply summed up to the msep on aggregated level.

Multi-year non-life insurance risk in the multivariate Chain Ladder Model – Model framework and results (3/7)

$$\widehat{\mathrm{msep}}_{{}^{C+D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}|Data}(0) = \widehat{\mathrm{msep}}_{{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}|Data}(0) + \widehat{\mathrm{msep}}_{{}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}|Data}(0) + 2\cdot\widehat{\mathrm{Cov}}\left[{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}, {}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} \mid Data\right]$$

The prediction covariance $\widehat{\mathrm{Cov}}\left[{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}, {}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} \mid \mathrm{Data}\right]$ will be further divided into a *process error and an estimation error component*

$$\widehat{Cov}\left[{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}, {}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} \mid Data\right] = {}_{\mathrm{process}}\widehat{Cov}\left[{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}, {}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} \mid Data\right] + {}_{\mathrm{estimation}}\widehat{Cov}\left[{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}, {}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} \mid Data\right]$$

$${}_{\mathrm{process}}\widehat{Cov}\left[{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}, {}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} \mid Data\right] = \sum_{k=n-i+2}^{n-i+m+1}\left[\hat{C}_{i,k-1}\cdot\hat{\hat{\sigma}}_k^2\right]^{1/2}\cdot\left[\widehat{D}_{i,k-1}\cdot\hat{\hat{\tau}}_k^2\right]^{1/2}\cdot\hat{\psi}_k$$
$$+\sum_{k=n-i+m+2}^{n}\left[\frac{\hat{C}_{i,k-1}^2}{\left(\hat{C}_{<,k-1}^{(n+m)}\right)^2}\cdot\left(\sum_{t=1}^{m}\hat{C}_{n-k+t+1,k-1}\right)\cdot\hat{\hat{\sigma}}_k^2\right]^{1/2}\cdot\left[\frac{\widehat{D}_{i,k-1}^2}{\left(\widehat{D}_{<,k-1}^{(n+m)}\right)^2}\cdot\left(\sum_{t=1}^{m}\widehat{D}_{n-k+t+1,k-1}\right)\cdot\hat{\hat{\tau}}_k^2\right]^{1/2}\times\hat{\psi}_k\cdot\frac{\left(\sum_{t=1}^{m}\sqrt{\hat{C}_{n-k+1+t,k-1}\cdot\widehat{D}_{n-k+1+t,k-1}}\right)}{\sqrt{\left(\sum_{t=1}^{m}\hat{C}_{n-k+1+t,k-1}\right)}\cdot\sqrt{\left(\sum_{t=1}^{m}\widehat{D}_{n-k+1+t,k-1}\right)}}$$

$${}_{\mathrm{estimation}}\widehat{Cov}\left[{}^{C}\widehat{\mathrm{CDR}}_i^{(n\to n+m)}, {}^{D}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} \mid Data\right] = \sum_{k=n-i+2}^{n-i+m+1}\left[\frac{\left(\hat{C}_{i,k-1}\right)^2}{\hat{C}_{<,k-1}^{(n)}}\cdot\hat{\hat{\sigma}}_k^2\right]^{1/2}\cdot\left[\frac{\left(\widehat{D}_{i,k-1}\right)^2}{\widehat{D}_{<,k-1}^{(n)}}\cdot\hat{\hat{\tau}}_k^2\right]^{1/2}\cdot\hat{\psi}_k\cdot\frac{\left(\sum_{j=1}^{n+1-k}\sqrt{\hat{C}_{j,k-1}\cdot\widehat{D}_{j,k-1}}\right)}{\sqrt{\hat{C}_{<,k-1}^{(n)}\cdot D_{<,k-1}^{(n)}}}$$
$$+\sum_{k=n-i+m+2}^{n}\left[\frac{\hat{C}_{i,k-1}^2}{\left(\hat{C}_{<,k-1}^{(n+m)}\right)^2\cdot\hat{C}_{<,k-1}^{(n)}}\cdot\left(\sum_{t=1}^{m}\hat{C}_{n-k+1+t,k-1}\right)^2\cdot\hat{\hat{\sigma}}_k^2\right]^{1/2}\cdot\left[\frac{\widehat{D}_{i,k-1}^2}{\left(\widehat{D}_{<,k-1}^{(n+m)}\right)^2\cdot\widehat{D}_{<,k-1}^{(n)}}\cdot\left(\sum_{t=1}^{m}\widehat{D}_{n-k+1+t,k-1}\right)^2\cdot\hat{\hat{\tau}}_k^2\right]^{1/2}\times\hat{\psi}_k\cdot\frac{\left(\sum_{j=1}^{n+1-k}\sqrt{\hat{C}_{j,k-1}\cdot\widehat{D}_{j,k-1}}\right)}{\sqrt{\hat{C}_{<,k-1}^{(n)}\cdot\widehat{D}_{<,k-1}^{(n)}}}$$

Multi-year non-life insurance risk in the multivariate Chain Ladder Model – Model framework and results (4/7)

Bivariate chain-ladder model

General Third Party Liability (GTPL)

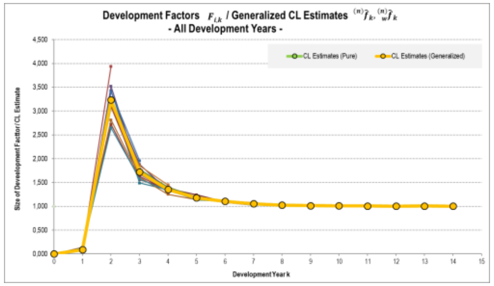
Volume Measures ${}^{C}v_i$

Accident year	Volume
1	510.301
2	632.897
3	658.133
4	723.456
5	709.312
6	845.673
7	904.378
8	1.156.778
9	1.214.569
10	1.397.123
11	1.832.676
12	2.156.781
13	2.559.345
14	2.456.991
15	2.616.438
16	2.775.885
17	2.935.332
18	3.094.779
19	3.254.226

Triangle Data: Cumulative Claims Payments $C_{i,k}$

Development Years

1	2	3	4	5	6	7	8	9	10	11	12	13	14
59.966	163.152	254.512	349.524	433.265	475.778	513.660	520.309	527.978	539.039	537.301	540.873	547.696	549.589
49.685	153.344	272.936	383.349	458.791	503.358	532.615	551.437	555.792	556.671	560.844	563.571	562.795	
51.914	170.048	319.204	425.029	503.999	544.769	559.475	577.425	588.342	590.985	601.296	602.710		
84.937	273.183	407.318	547.288	621.738	687.139	736.304	757.440	758.036	782.084	784.632			
98.921	278.329	448.530	561.691	641.332	721.696	742.110	752.434	768.638	768.373				
71.708	245.587	416.882	560.958	654.652	726.813	768.358	793.603	811.100					
92.350	285.507	466.214	620.030	741.226	827.979	873.526	896.728						
95.731	313.144	553.702	755.978	857.859	962.825	1.022.241							
97.518	343.218	575.441	769.017	934.103	1.019.303								
173.686	459.416	722.336	955.335	1.141.750									
139.821	436.958	809.926	1.174.196										
154.965	528.080	1.032.684											
196.124	772.971												
204.325													



Motor Third Party Liability (MTPL)

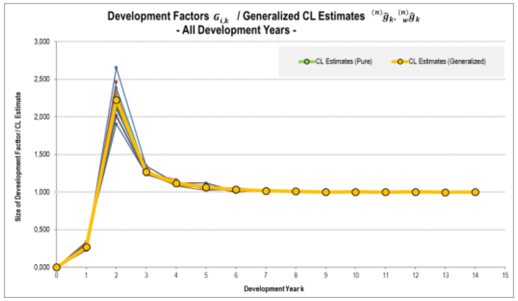
Volume Measures ${}^{D}v_i$

Accident year	Volume
1	413.213
2	537.988
3	589.145
4	523.419
5	501.498
6	598.345
7	608.376
8	698.993
9	704.129
10	903.557
11	947.326
12	1.134.129
13	1.538.916
14	1.487.234
15	1.564.735
16	1.642.236
17	1.719.737
18	1.797.238
19	1.874.739

Triangle Data: Cumulative Claims Payments $D_{i,k}$

Development Years

1	2	3	4	5	6	7	8	9	10	11	12	13	14
114.423	247.961	312.982	344.340	371.479	371.102	380.991	385.468	385.152	392.260	391.225	391.328	391.537	391.428
152.296	305.175	376.613	418.299	440.308	465.623	473.584	478.427	478.314	479.907	480.755	485.138	483.974	
144.325	307.244	413.609	464.041	519.265	527.216	535.450	536.859	538.920	539.589	539.765	540.742		
145.904	307.636	387.094	433.736	463.120	478.931	482.529	488.056	485.572	486.034	485.016			
170.333	341.501	434.102	470.329	482.201	500.961	504.141	507.679	508.627	507.752				
189.643	361.123	446.857	508.083	526.562	540.118	547.641	549.605	549.693					
179.022	396.224	497.304	553.487	581.849	611.640	622.884	635.452						
205.908	416.047	520.444	565.721	600.609	630.802	648.365							
210.951	426.429	525.047	587.893	640.328	663.152								
213.426	509.222	649.433	731.692	790.901									
249.508	580.010	722.136	844.159										
258.425	686.012	915.109											
368.762	909.066												
394.997													

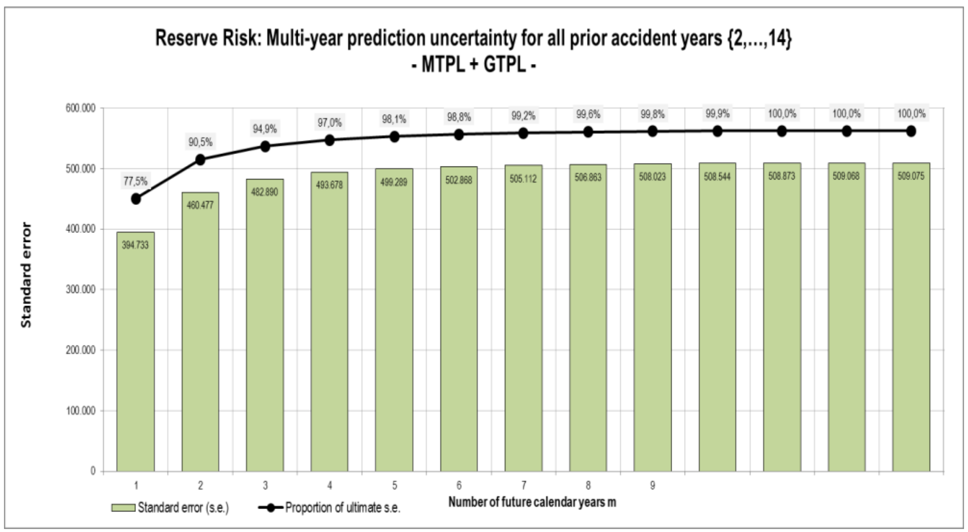


Multi-year non-life insurance risk in the multivariate Chain Ladder Model – Model framework and results (5/7)

Bivariate chain-ladder model

Analytical results for Multi-Year Reserve Risk:

- standard errors for the *multi-year claims development result of all prior accident years* per portfolio
- corresponding standard errors on aggregated level
- as well as the implied correlation between both portfolios



Number of future calendar years m	m-year Reserve Risk - GTPL -	m-year Reserve Risk - MTPL -	m-year Reserve Risk - GTPL + MTPL -	m-year Reserve Risk Corr GTPL <> MTPL
1	330.991	134.242	394.733	*31,78%*
2	387.867	147.534	460.477	*34,80%*
3	405.193	154.906	482.890	*35,85%*
4	413.613	159.099	493.678	*35,96%*
5	418.363	160.847	499.289	*35,96%*
6	421.655	161.601	502.868	*35,93%*
7	423.718	162.099	505.112	*35,91%*
8	425.297	162.442	506.863	*35,93%*
9	426.342	162.690	508.023	*35,94%*
10	426.811	162.790	508.544	*35,94%*
11	427.103	162.858	508.873	*35,95%*
12	427.281	162.870	509.068	*35,96%*
13	427.289	162.872	509.075	*35,96%*

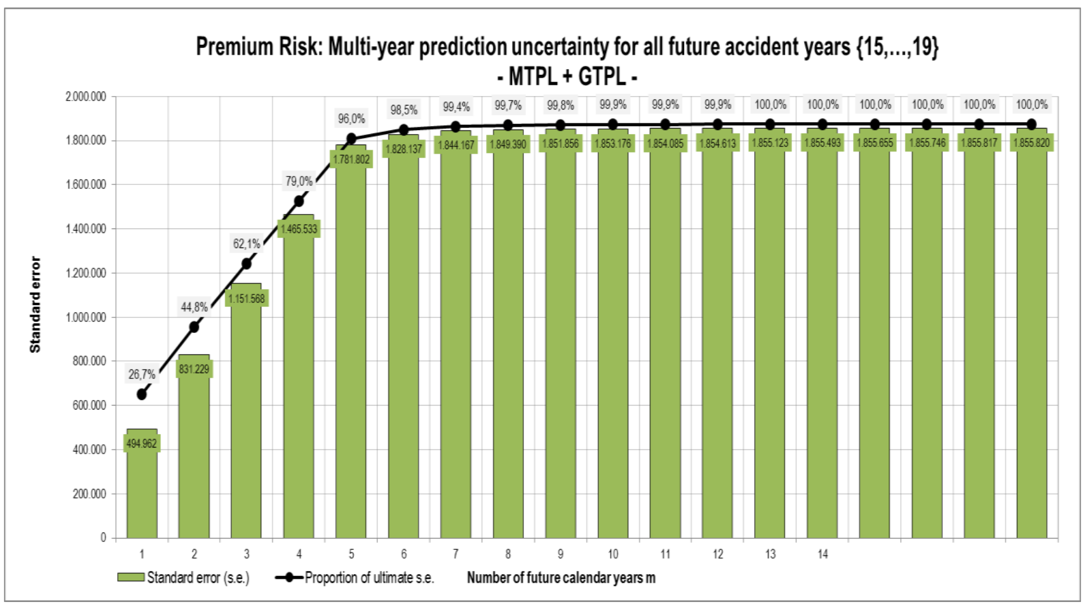
- Figures from [Braun 2004] for the ultimo view on univariate level are reproduced
- Implied correlation is time-dependent
- Results for future one-year risks can be found in [Linde 2016]

Multi-year non-life insurance risk in the multivariate Chain Ladder Model – Model framework and results (6/7)

Bivariate chain-ladder model

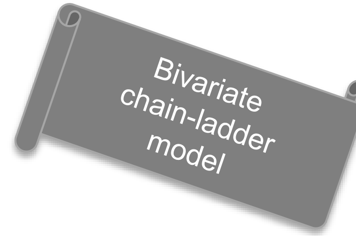
Analytical results for Multi-Year Premium Risk:

- standard errors for the *multi-year claims development result of all future accident years* per portfolio
- corresponding standard errors on aggregated level
- as well as the implied correlation between both portfolios



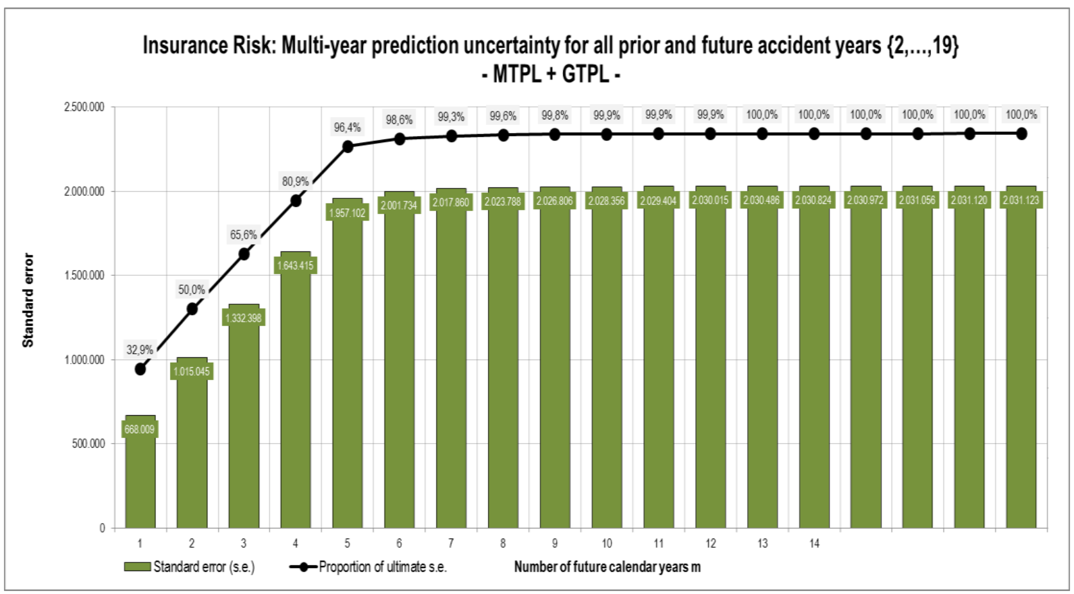
Number of future calendar years m	m-year Premium Risk - GTPL -	m-year Premium Risk - MTPL -	m-year Premium Risk - GTPL + MTPL -	m-year Premium Risk Corr GTPL <> MTPL
1	430.569	136.098	494.962	*35,05%*
2	717.025	248.232	831.229	*32,36%*
3	993.278	345.141	1.151.568	*32,14%*
4	1.263.978	438.589	1.465.533	*32,27%*
5	1.536.805	532.343	1.781.802	*32,37%*
6	1.574.308	551.354	1.828.137	*32,24%*
7	1.587.933	554.764	1.844.167	*32,45%*
8	1.591.707	556.750	1.849.390	*32,54%*
9	1.593.471	557.934	1.851.856	*32,56%*
10	1.594.513	558.459	1.853.176	*32,56%*
11	1.595.324	558.668	1.854.085	*32,56%*
12	1.595.792	558.827	1.854.613	*32,56%*
13	1.596.236	558.934	1.855.123	*32,57%*
14	1.596.561	559.018	1.855.493	*32,57%*
15	1.596.704	559.049	1.855.655	*32,57%*
16	1.596.782	559.072	1.855.746	*32,57%*
17	1.596.846	559.076	1.855.817	*32,57%*
18	1.596.849	559.076	1.855.820	*32,57%*

Multi-year non-life insurance risk in the multivariate Chain Ladder Model – Model framework and results (7/7)



Analytical results for Multi-Year Non-Life Insurance Risk:

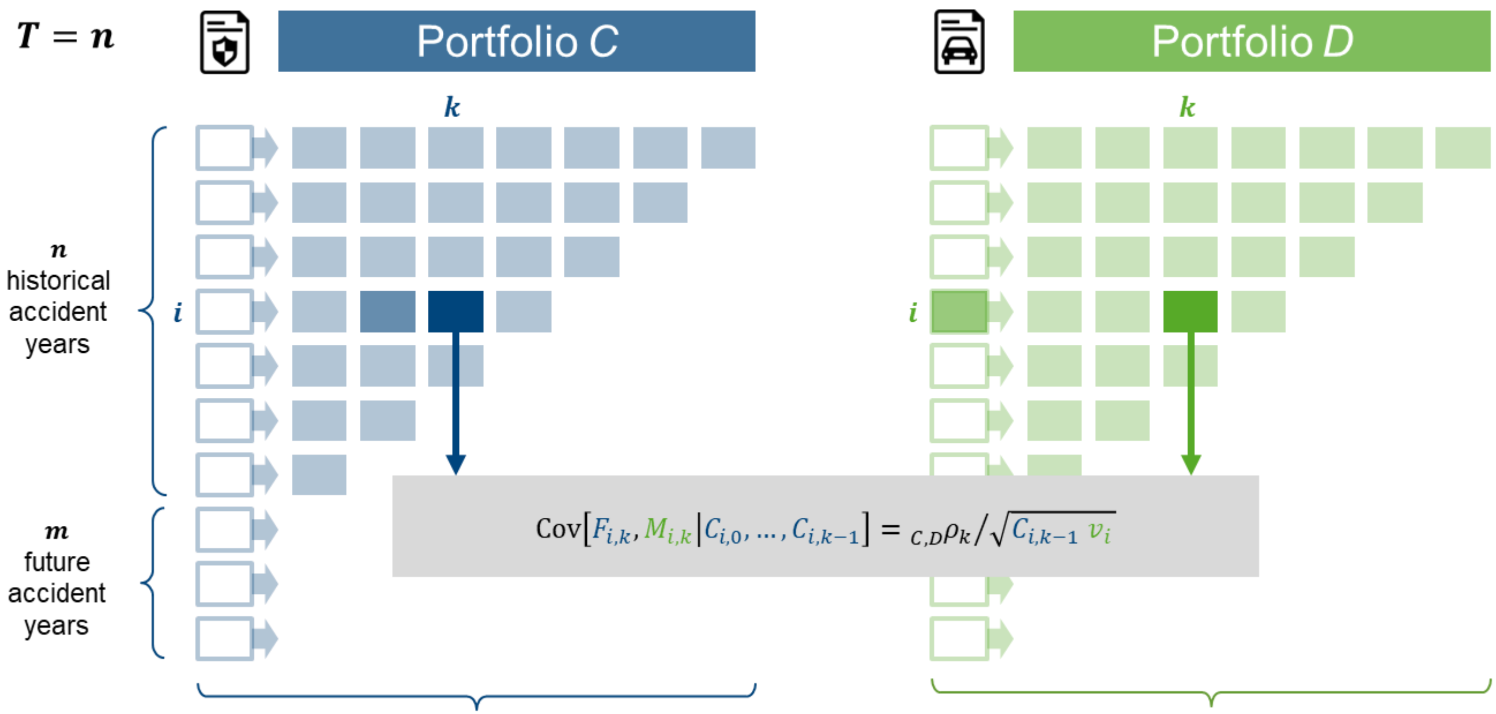
- standard errors for the *multi-year claims development result of all prior and future accident years* per portfolio
- corresponding standard errors on aggregated level
- as well as the implied correlation between both portfolios



Number of future calendar years m	m-year Insurance Risk - GTPL -	m-year Insurance Risk - MTPL -	m-year Insurance Risk - GTPL + MTPL -	m-year Insurance Risk Corr GTPL <> MTPL
1	572.038	203.925	668.009	33,19%
2	869.441	309.622	1.015.045	33,16%
3	1.142.263	405.284	1.332.398	33,08%
4	1.410.245	498.100	1.643.415	33,02%
5	1.681.102	590.792	1.957.102	32,98%
6	1.717.591	608.485	2.001.734	32,85%
7	1.731.425	611.911	2.017.860	33,01%
8	1.735.911	613.942	2.023.788	33,09%
9	1.738.208	615.182	2.026.806	33,11%
10	1.739.467	615.726	2.028.356	33,12%
11	1.740.401	615.961	2.029.404	33,12%
12	1.740.947	616.113	2.030.015	33,12%
13	1.741.359	616.211	2.030.486	33,12%
14	1.741.657	616.287	2.030.824	33,12%
15	1.741.787	616.315	2.030.972	33,12%
16	1.741.859	616.336	2.031.056	33,13%
17	1.741.918	616.340	2.031.120	33,13%
18	1.741.920	616.340	2.031.123	33,13%

Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (1/8)

Model Calibration | Bootstrap Process | Simulation of Future Payments | Stochastic Re-reserving | Portfolio Aggregation



Marginal Mack chain-ladder model

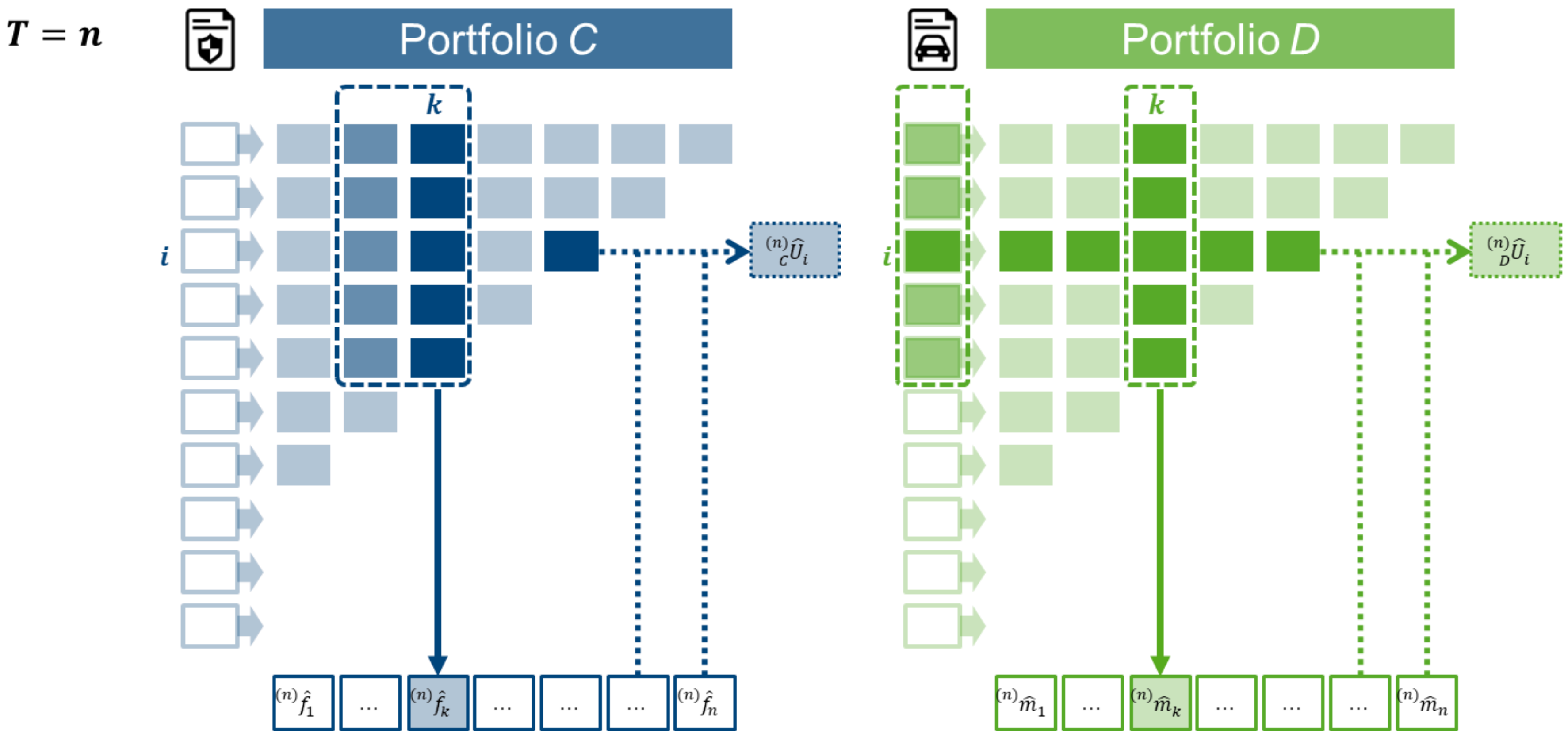
- Independence of accident years
- $\mathbb{E}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = f_k$
- $\mathbb{V}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = \sigma_k^2 / C_{i,k-1}$

Marginal additive reserving model

- Independence of incremental payments
- $\mathbb{E}\left[M_{i,k}\right] = m_k$
- $\mathbb{V}\left[M_{i,k}\right] = s_k^2 / v_i$

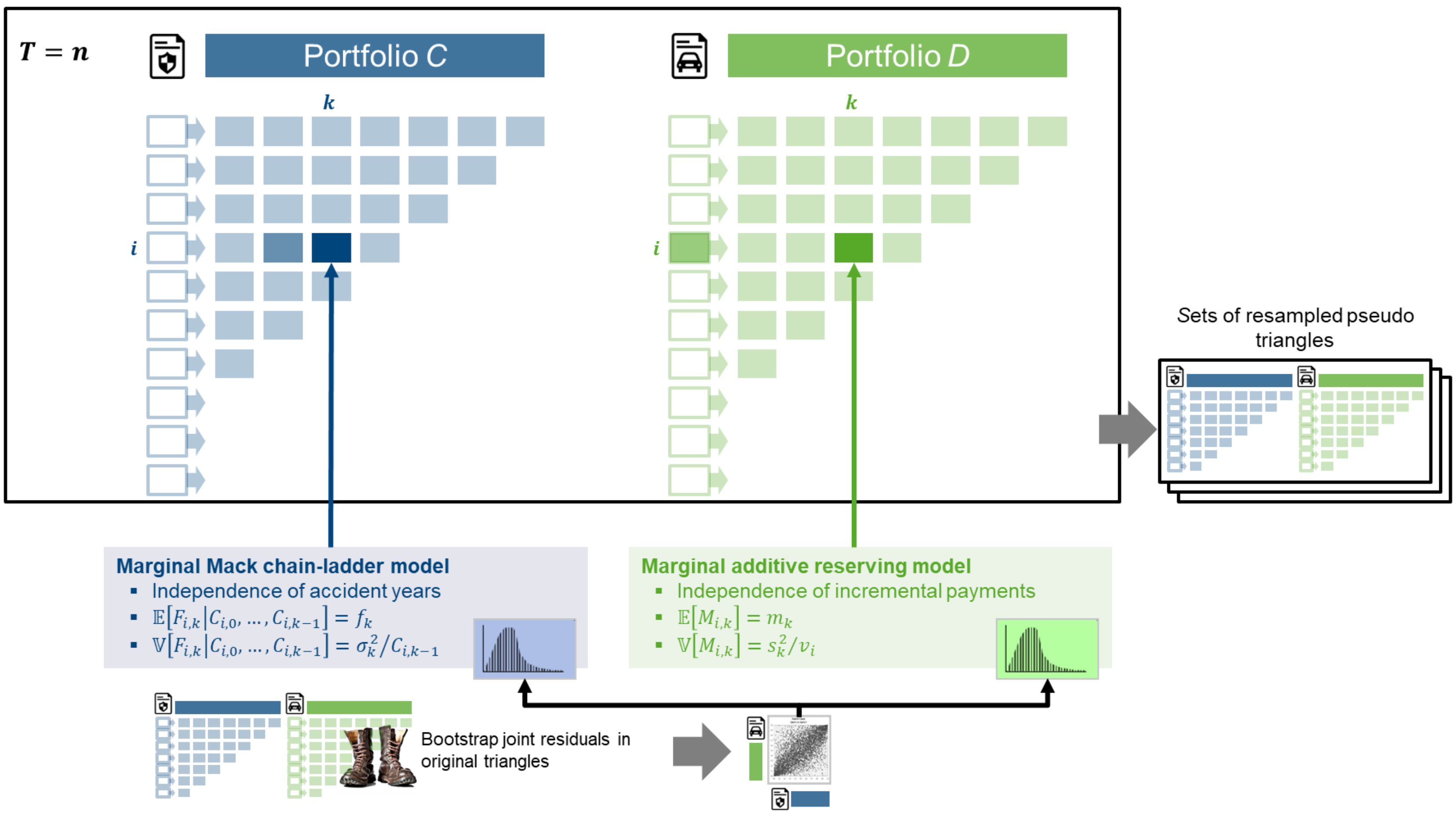
Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (2/8)

Model Calibration | Bootstrap Process | Simulation of Future Payments | Stochastic Re-reserving | Portfolio Aggregation



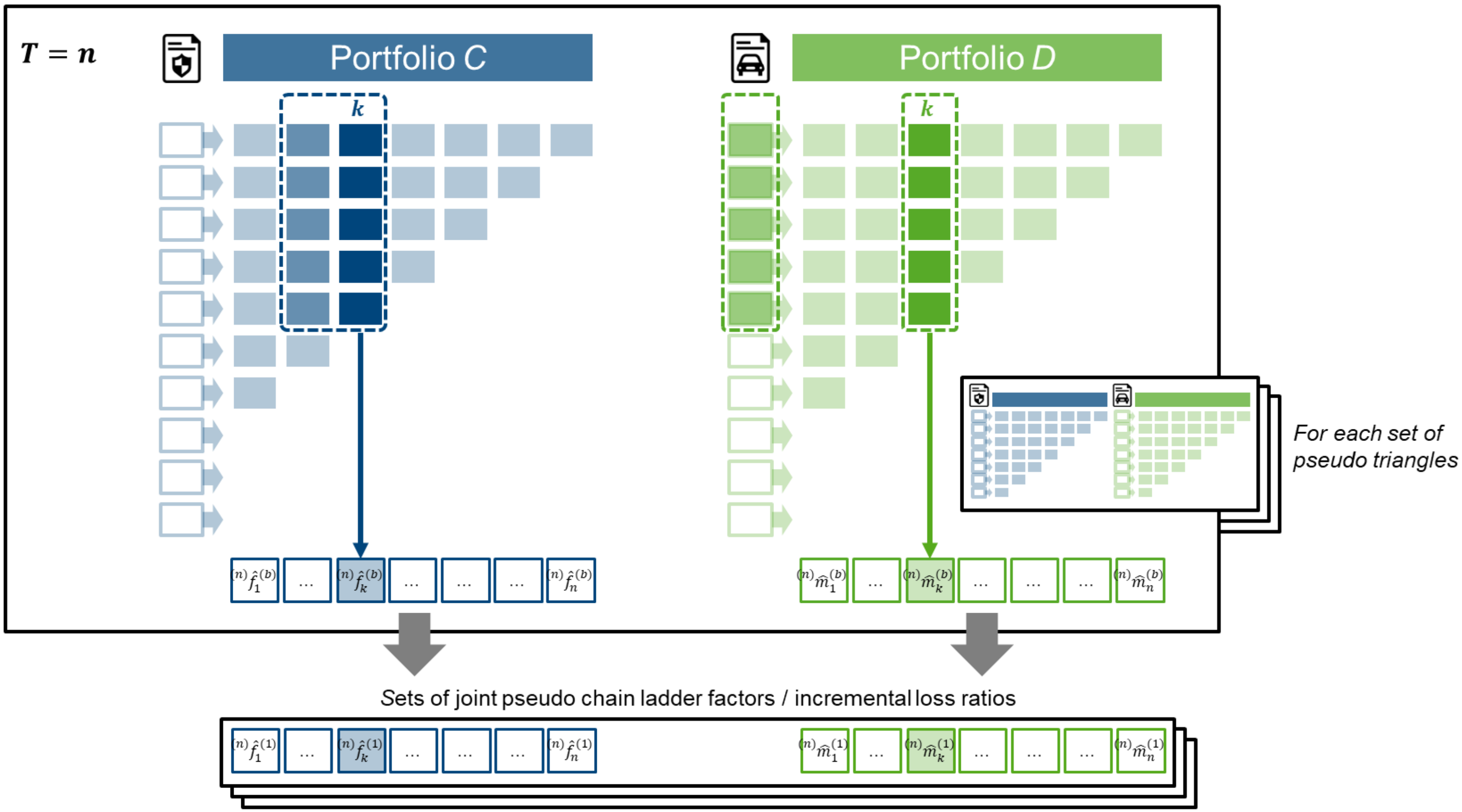
Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (3/8)

Model Calibration | **Bootstrap Process** | Simulation of Future Payments | Stochastic Re-reserving | Portfolio Aggregation



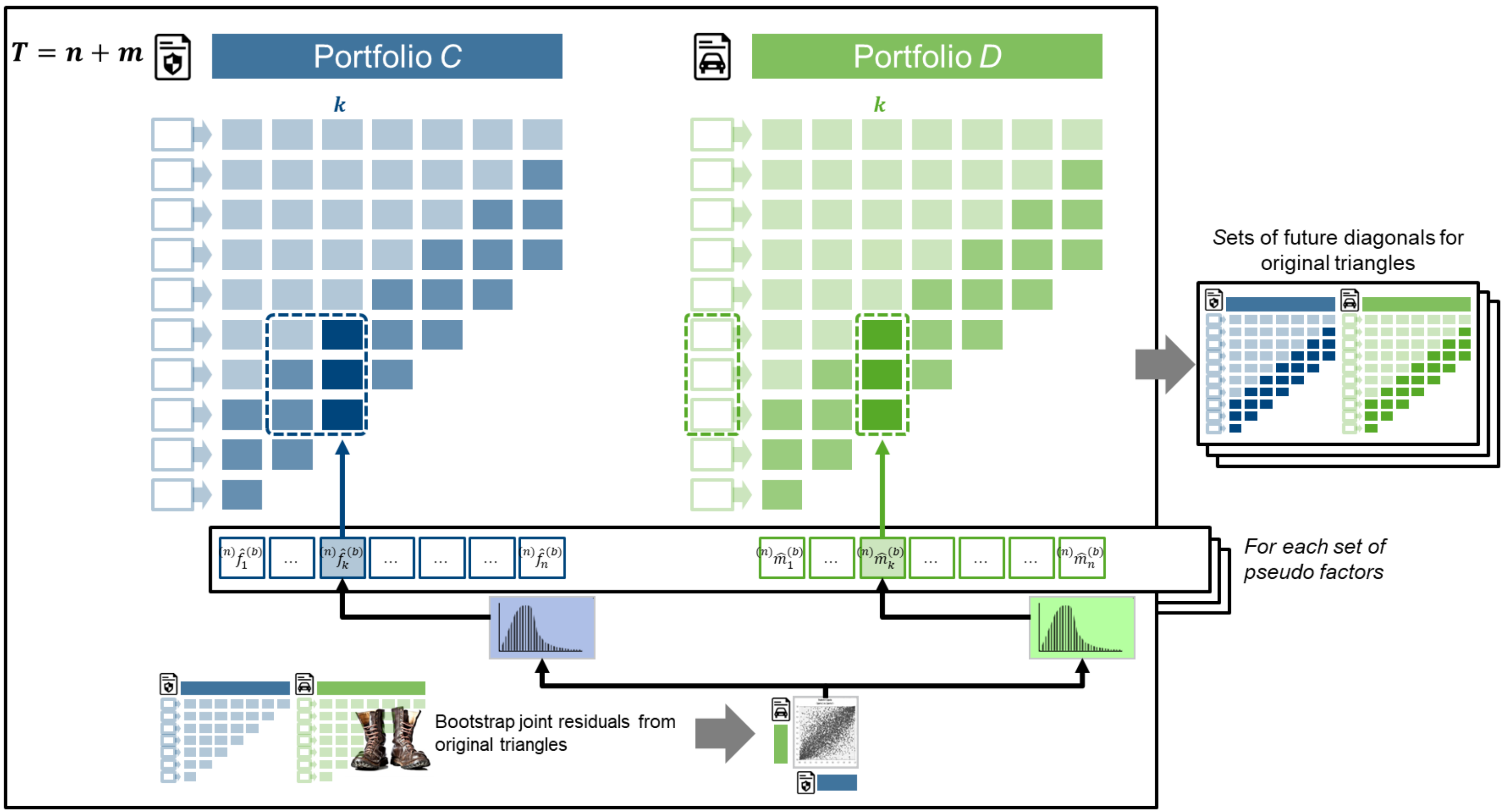
Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (4/8)

Model Calibration | **Bootstrap Process** | Simulation of Future Payments | Stochastic Re-reserving | Portfolio Aggregation



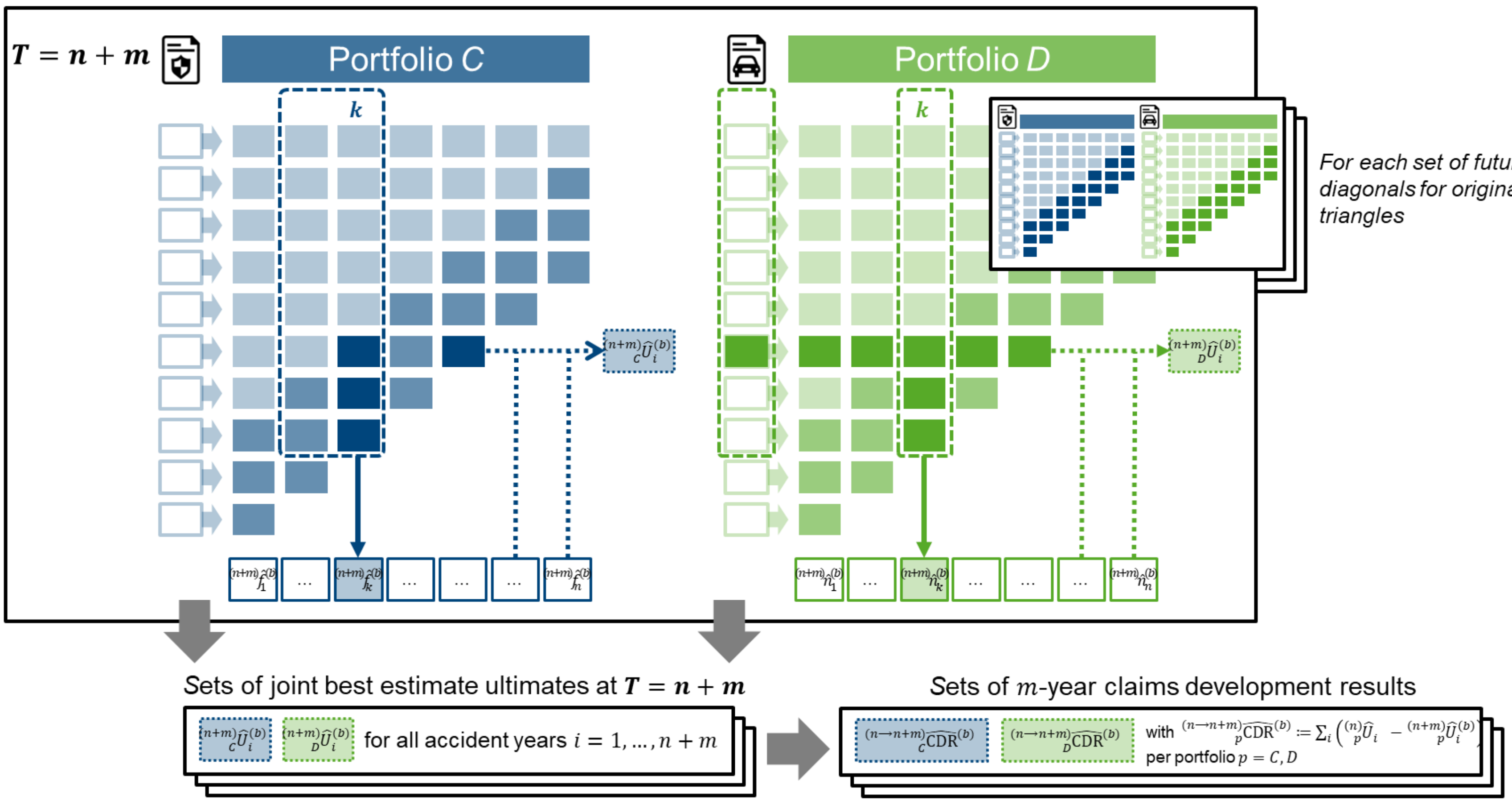
Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (5/8)

Model Calibration | Bootstrap Process | **Simulation of Future Payments** | Stochastic Re-reserving | Portfolio Aggregation



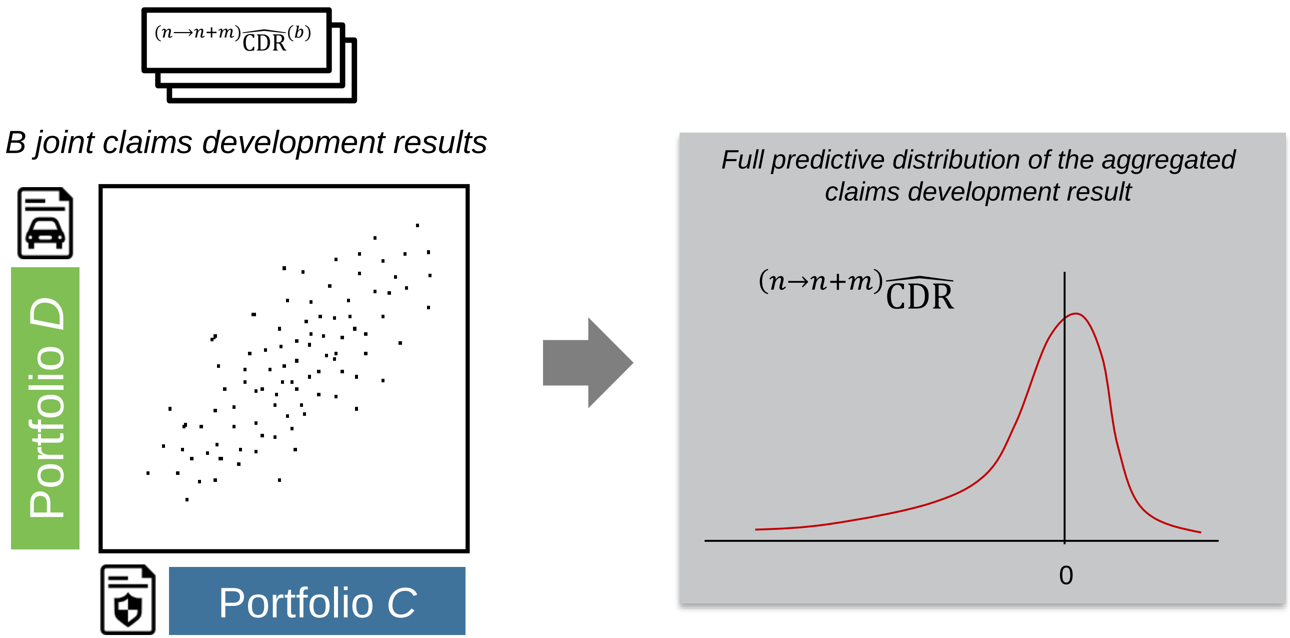
Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (6/8)

Model Calibration | Bootstrap Process | Simulation of Future Payments | **Stochastic Re-reserving** | Portfolio Aggregation

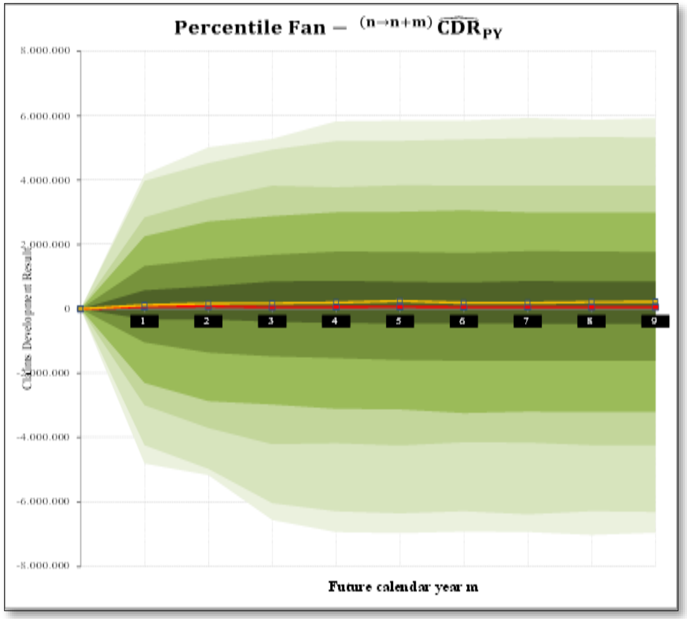


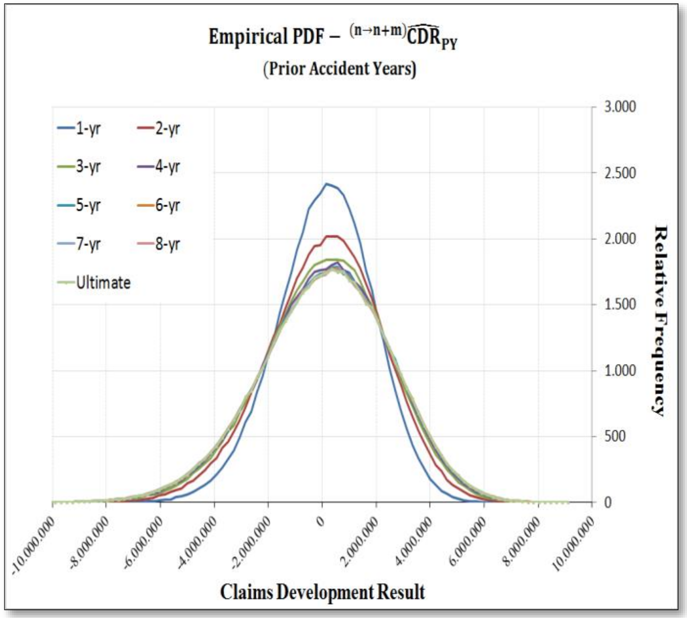
Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (7/8)

Model Calibration | Bootstrap Process | Simulation of Future Payments | Stochastic Re-reserving | **Portfolio Aggregation**



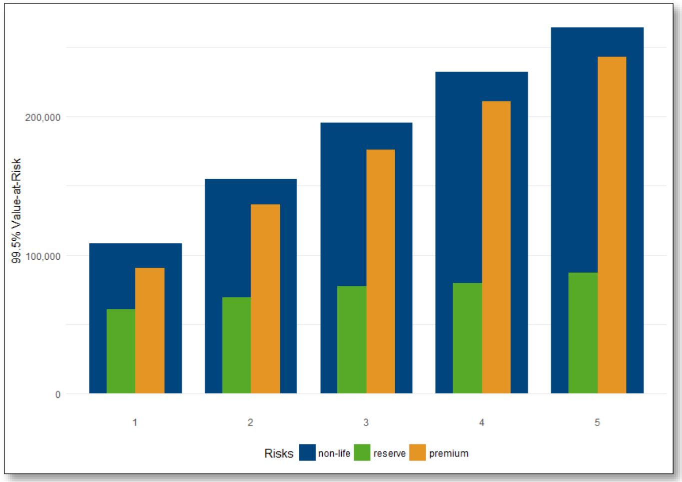
Quantification of multi-year non-life insurance risk for dependent portfolios with Bootstrap approach (8/8)





Bootstrap approach allows to

- estimate multi-year overall solvency needs,
- derive future one-year capital requirements (rolling forward definition of reserve and premium risks),
- calculate risk margins and safety loadings,
- perform sensitivity analyses.



Agenda

Background

Definition of Non-Life Insurance Risk in a multi-year view

Quantification of multi-year Non-Life Insurance Risk in univariate reserving models

Quantification of multi-year Non-Life Insurance Risk in multivariate reserving models

Summary and Application Fields

Summary and Application Fields (1/2)

Definition of multi-year claims development result: Diers et al. (2013)

Analytical approaches

- Based on distribution-free claims reserving models (additive reserving model and chain-ladder model) extended for future business
- closed-form estimators for mean squared error of prediction
  - Univariate models: Diers and Linde (2013), Diers et al. (2016)
    - Building the bridge between Mack (1993) and Merz and Wüthrich (2008) for chain-ladder model
  - Multivariate models: Linde (2016), Hahn (2017)

Simulation-based approaches

- Bootstrap and stochastic re-reserving ("actuary in the box")
- risk measures derived from full predictive distribution of the multi-year claims development result
  - Univariate models: Diers et al. (2013)
    - Generalization of England and Verrall (2006) and Ohlsson and Lauzeningks (2009)
  - Multivariate models: Hahn and Linde (2019)

Summary and Application Fields (2/2)

Practical Value of Bootstrap Approach

- The bootstrap approach allows to quantify premium and reserve risk
  - among lines of business (or more granular segments)
  - through various risk measures,
  - over a time horizon of multiple future accounting years.
- input for overall solvency needs in the ORSA process
- understanding of how dependencies between the occurrence and settlement of claims among different loss portfolios influence the aggregated risk capital
- suitability assessment of the standard formula
- indicator for possible benefits from undertaking-specific parameters
- derivation of risk capitals in future one-year view to compute a risk margin under a run-off scenario

Outlook on R Implementation

- chain-ladder and additive loss reserving models subject to development year correlations
  - combinations and generalized versions possible
- non-parametric and parametric bootstraps/simulations of historic and future triangles
  - different marginals and copulae for parametric approach
- various analyses based on full predictive distributions
  - risk by accident years, reserve and premium risks
  - flexible (e.g. pairwise or complete) portfolio aggregations
  - split into estimation and process error
  - m-year and (updated) future one-year view
  - extensive plotting functions (model diagnostics, results)
- closed-form analytical estimators for benchmarking

Thanks for your attention!

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