Product sales strategy and risk-return efficiency

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Abstract

What should be the sales volume of two different products in order to optimize the risk-return efficiency?

The approach based on Portfolio Theory

- ✓ Can determine the optimal investment ratio by considering the effect of diversification on risk between two products.
- ✓ Cannot be applied as is when the sales volume and profit are non-linear.

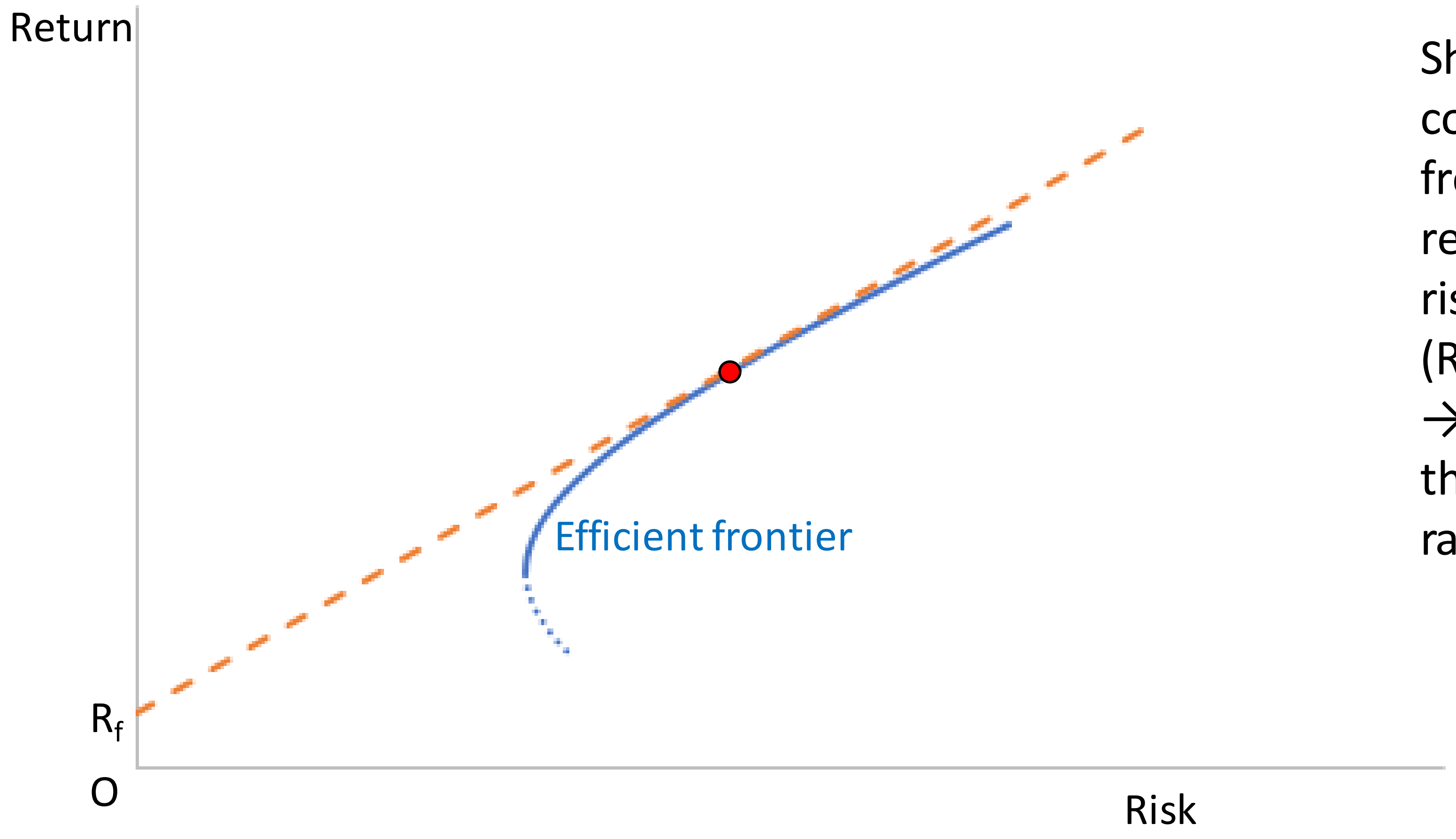
The approach based on profit maximization

- ✓ Can determine the optimal sales volume when sales volume and profit are non-linear.
- ✓ Cannot consider risk.

→Determine the optimal sales volume of two products by using both portfolio theory and profit maximization approach.

1. Portfolio Theory and Insurance Product Sales

Risk-return effeciency is optimized when Sharpe ratio is maximized.



Sharpe ratio: Slope of the line connecting a point on the efficient frontier and the point $(0,R_f)$ representing the investment in risk-free assets
(R_f: Return on risk-free assets)
→When the straight line touches the efficient frontier, the Sharpe ratio is maximized .

1. Portfolio Theory and Insurance Product Sales

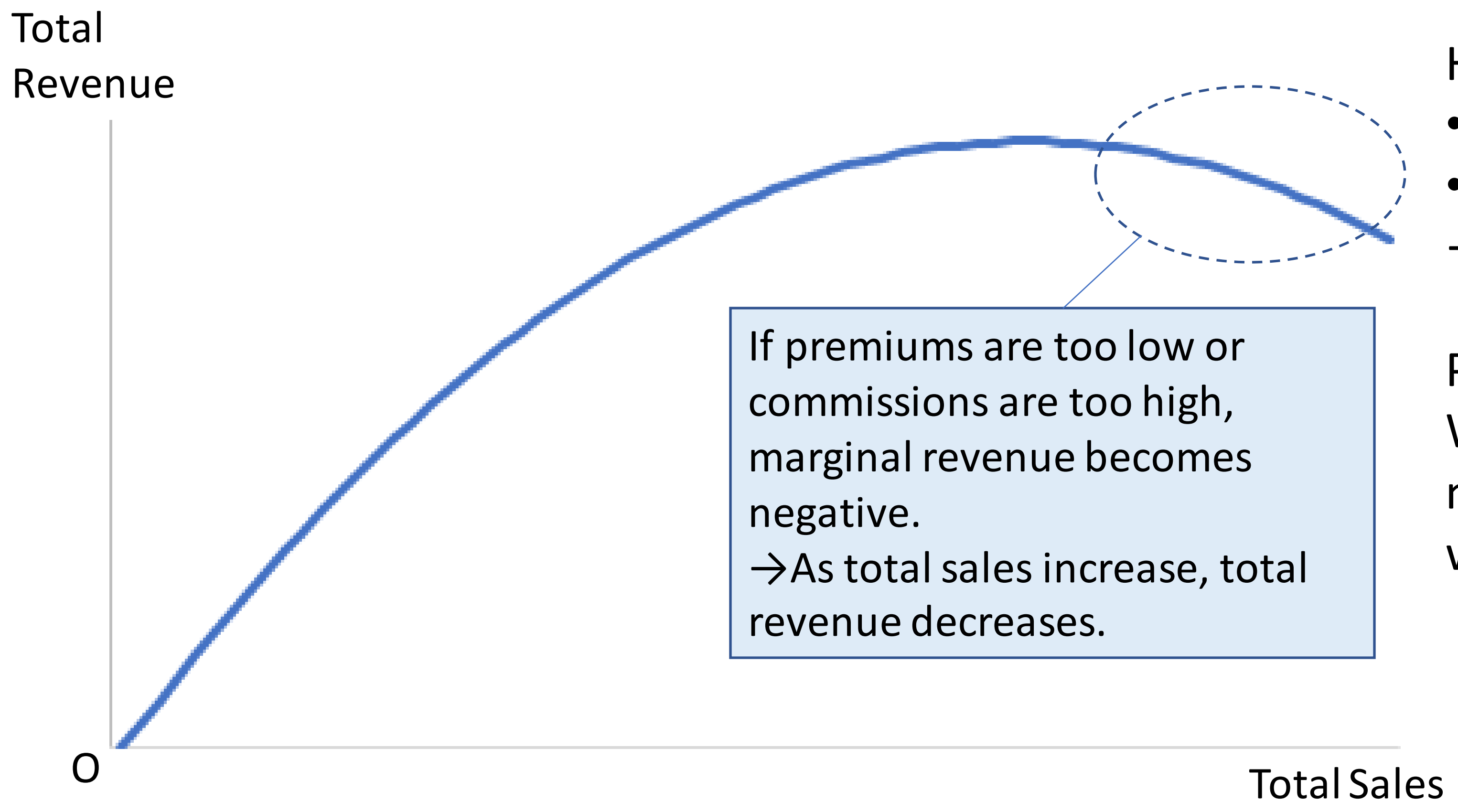
Investment Strategy vs. Sales Strategy

	Investment Strategy	Sales Strategy
Investment entity	Investor	Insurer
Initial investment	Purchase cost of financial instruments	Commissions paid to sales representatives/agents (sales commissions, training costs, etc.)
Risks borne by the investment entity (Different types of risks create a diversification effect.)	• Interest rate risk • Equity risk • Spread risk • Currency risk	• Mortality risk • Longevity risk • Disability/Morbidity risk • Lapse risk • Expenses risk
Return earned by the investment entity	• Interest and dividend income • Capital gains	• Estimated revenue • Return due to the accident rate being lower than estimated

→Insurers can use portfolio theory not only in Investment Strategy but also in Sales Strategy.

2. The Concept of Profit Maximization

In general, sales and revenue have a non-linear relationship.



How can we increase sales?

- Lower premiums
- Raise commissions, etc.

→Marginal revenue decreases.

Problem of profit maximization:
When sales and revenue have a non-linear relationship, how can we maximize the revenue?

3. Risk Limits and Profit Maximization

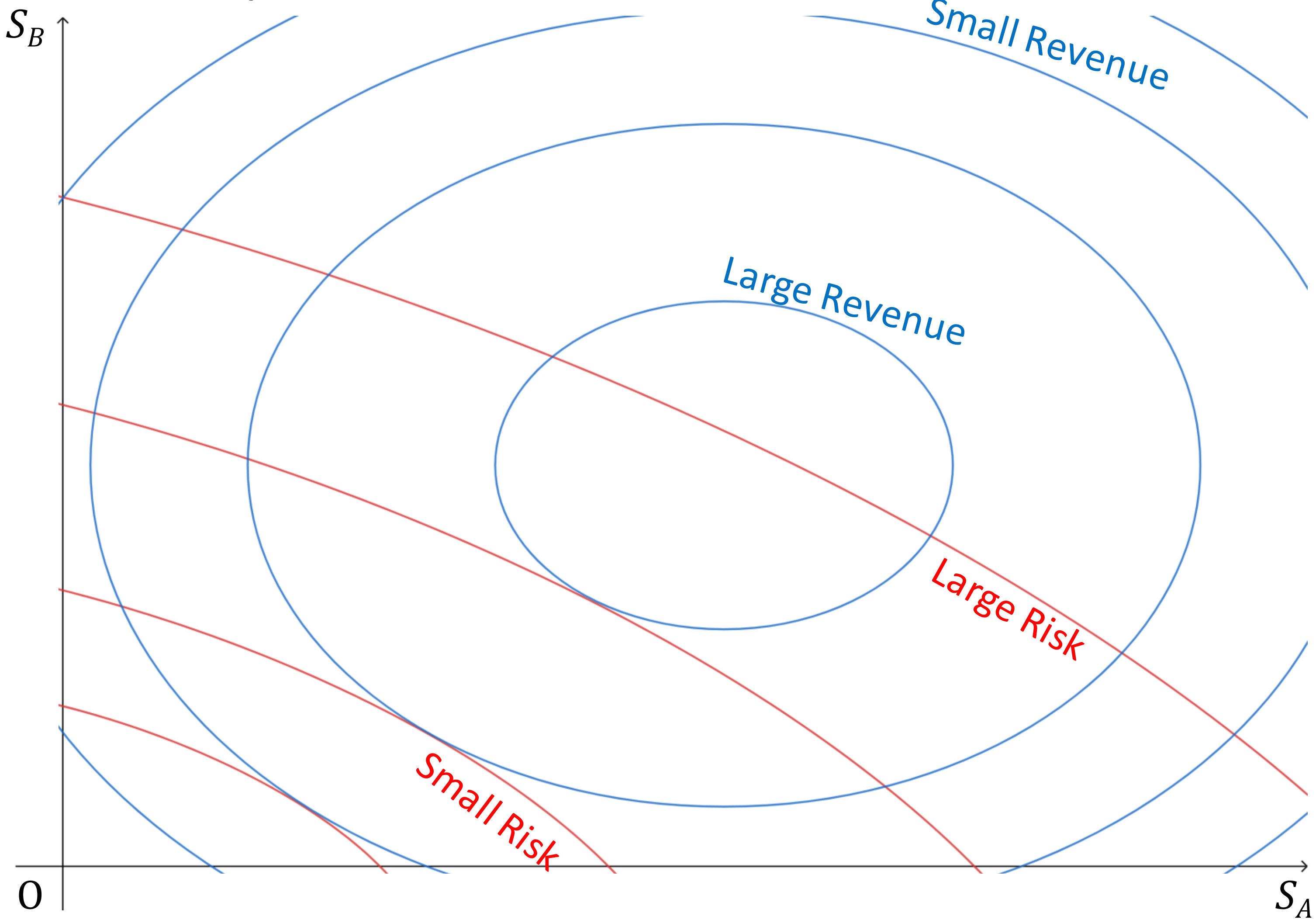
Consider profit maximization under a certain constrain.

	Product A	Product B
Sales	S_A	S_B
Revenue	$Q_A=f(S_A)$	$Q_B=g(S_B)$
Risk	$R_A=r_AS_A$	$R_B=r_BS_B$
Risk correlation coefficient	ρ_{AB} $(R(S_A,S_B)=\{(r_AS_A)^2+(r_BS_B)^2+2\rho_{AB}r_Ar_BS_AS_B\}^{0.5})$	

Identify (S_A,S_B) that maximizes the revenue $f(S_A)+g(S_B)$
under the constraint $R(S_A,S_B)\leqq L$, where L stands for the risk limit.

3. Risk Limits and Profit Maximization

Consider profit maximization under a certain constrain.



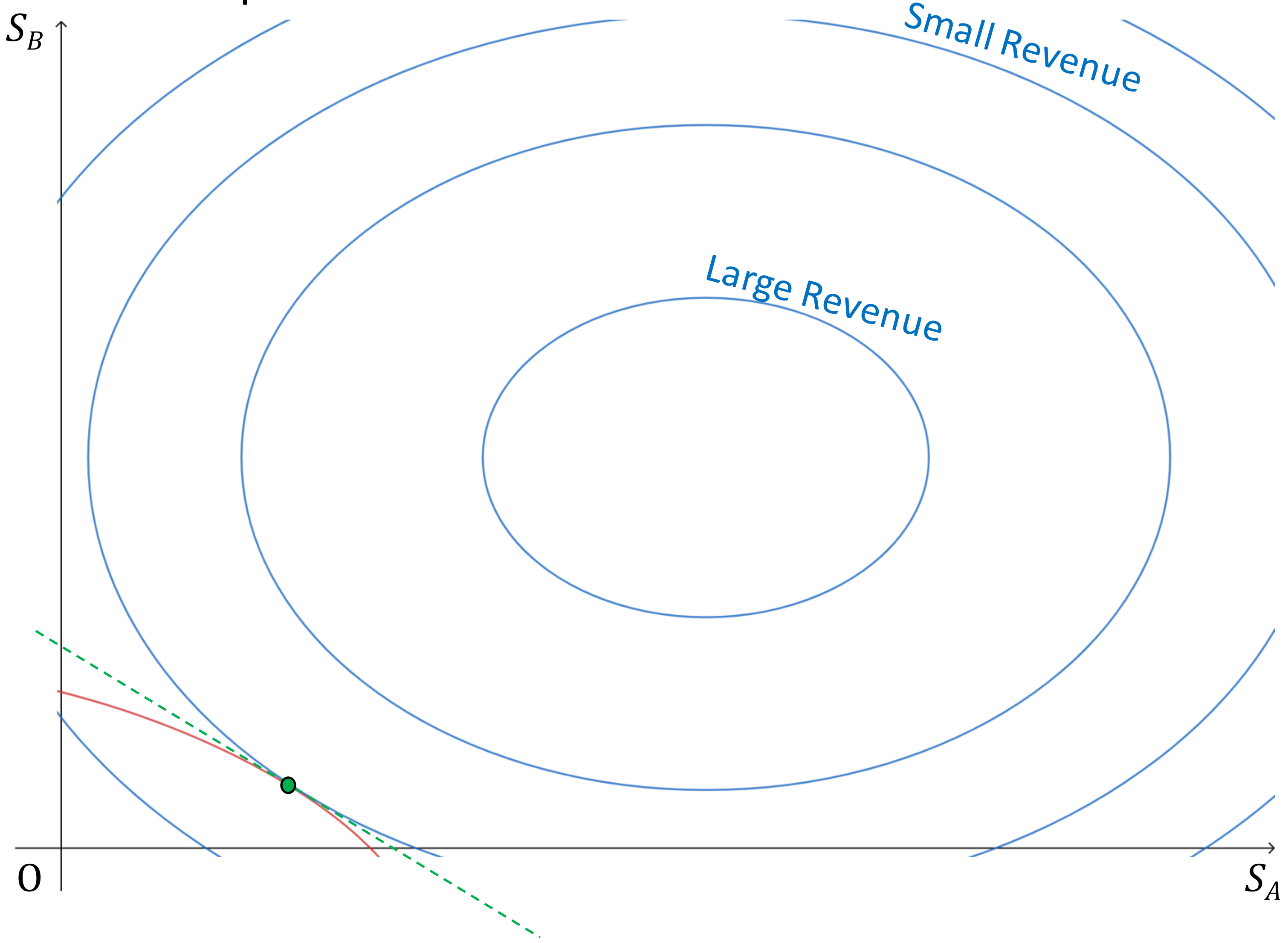
Horizontal axis: S_A
Vertical axis: S_B

Revenue: Contour line in blue
Risk: Contour line in red

Identify (S_A,S_B) that maximizes the revenue under certain risk limit.
→We can use the method of Lagrange multipliers.
(Let F=$f(S_A)+g(S_B)-\lambda(R-L)$ and solve the simultaneous equation $\partial F/\partial S_A=\partial F/\partial S_B=\partial F/\partial\lambda=0$.)

3. Risk Limits and Profit Maximization

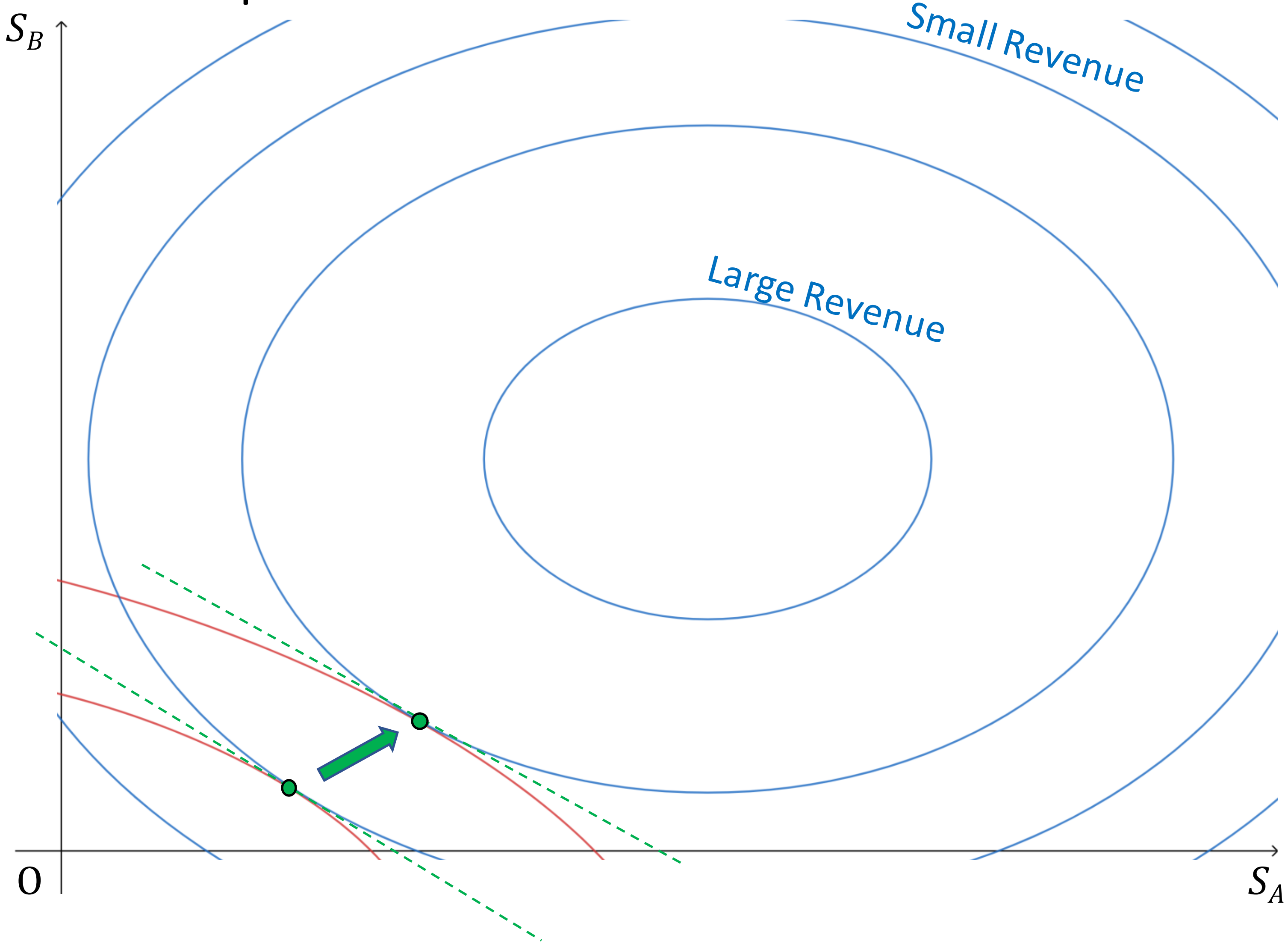
Consider profit maximization under a certain constrain.



The red line in the figure represents the contour line R=L. To maximize revenue under a constrain L, find the point where revenue (blue contour lines) and risk (red contour lines) meet.

3. Risk Limits and Profit Maximization

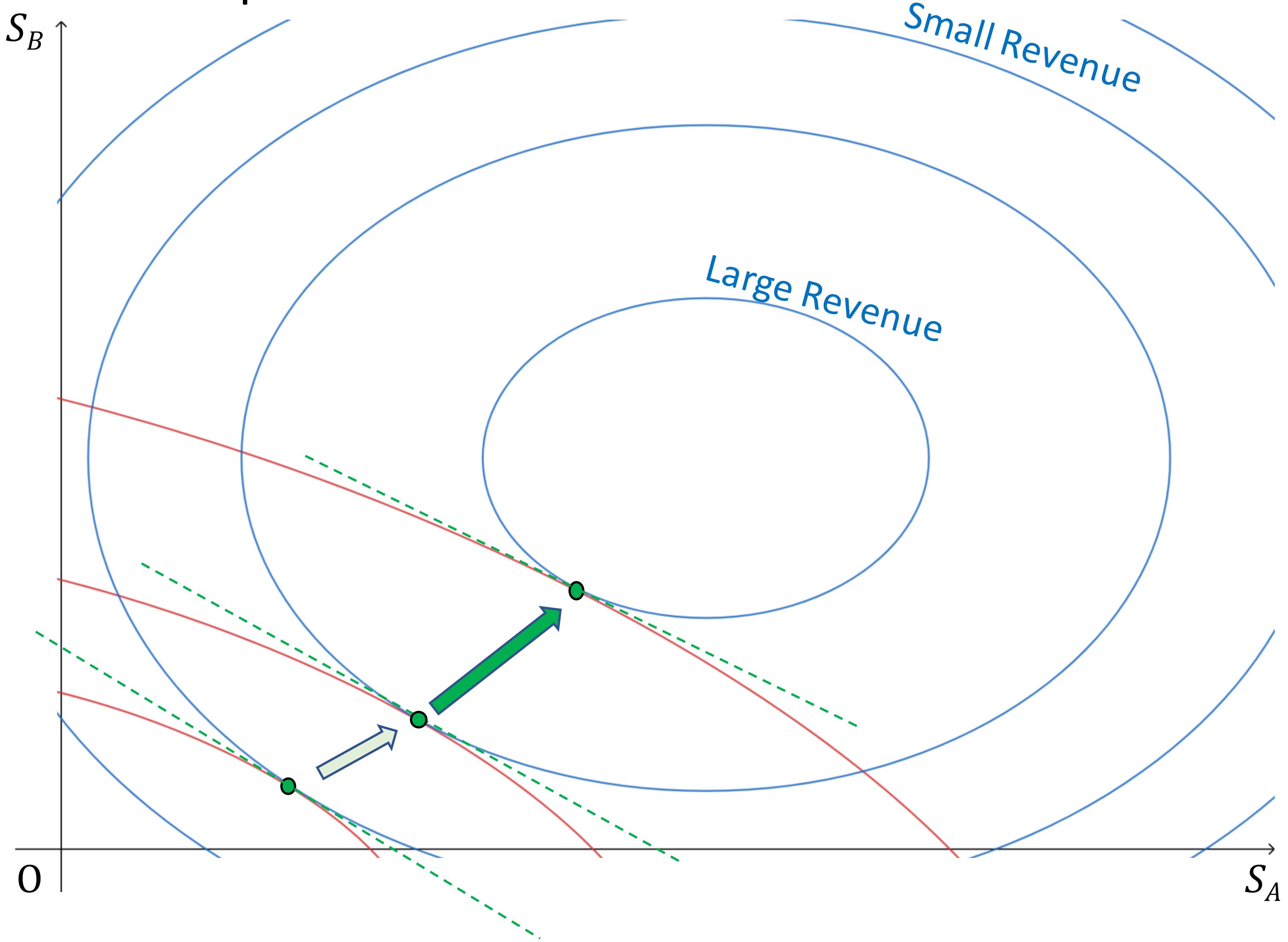
Consider profit maximization under a certain constrain.



As the risk limit L increases, the contour line R=L moves to the upper right.
Then, the point that maximizes revenue also moves to the upper right, where the red contour line meets a blue contour line.

3. Risk Limits and Profit Maximization

Consider profit maximization under a certain constrain.



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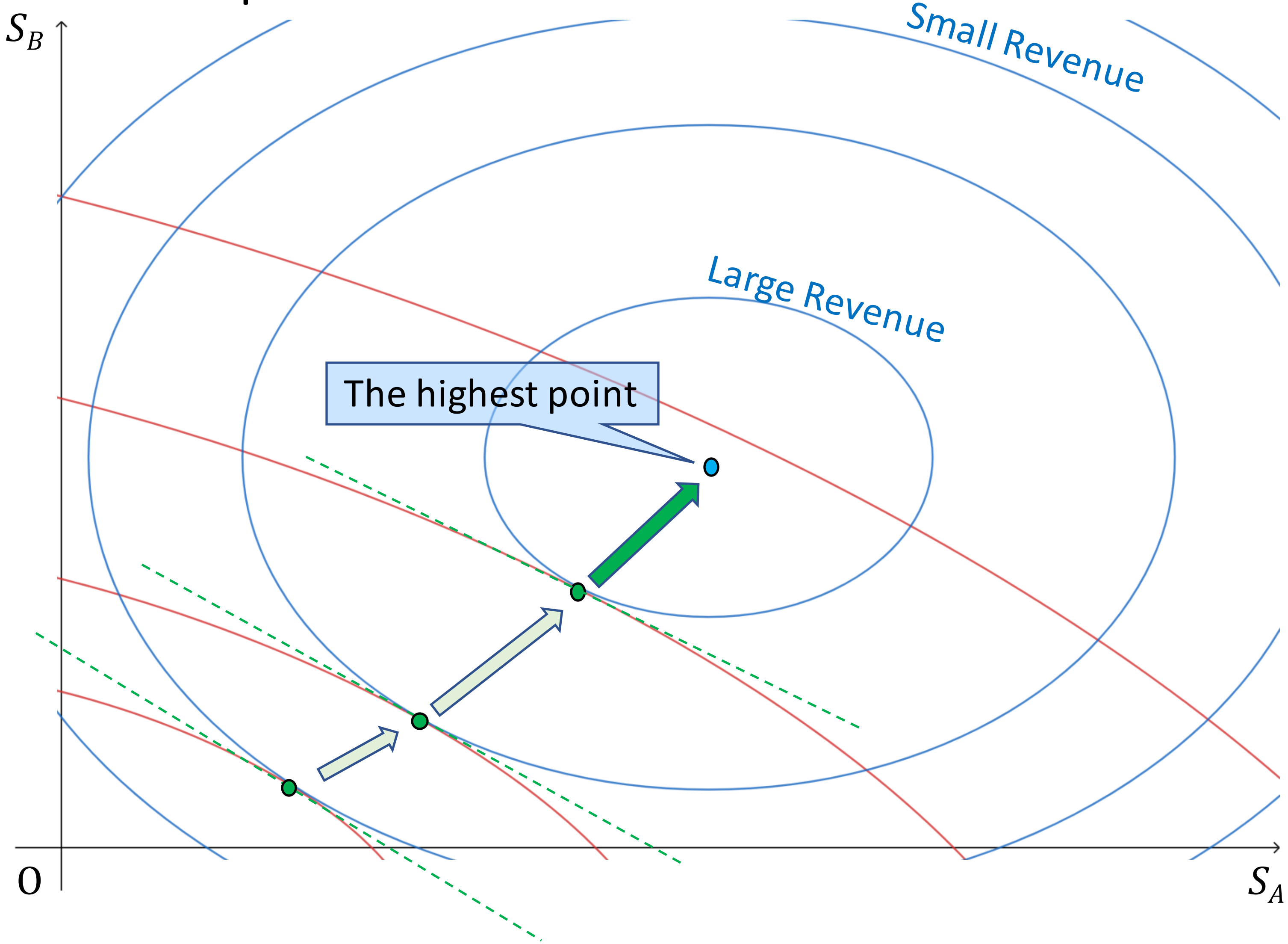
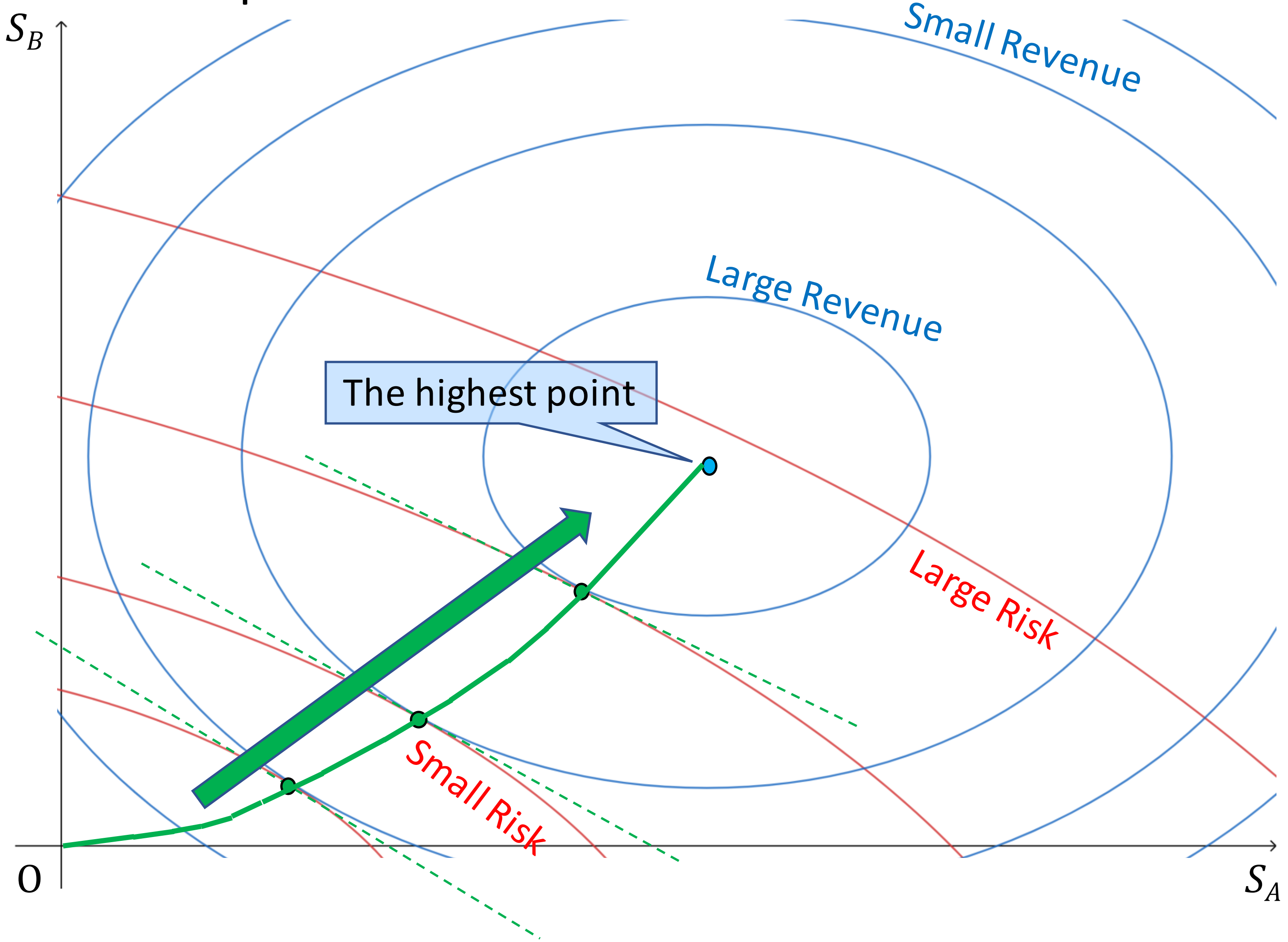


When the risk limit L is sufficiently large, the contour line R=L moves to the upper right above the highest point of the blue contour line.
Then the point that maximizes revenue also moves to the upper right but stops at the highest point.

3. Risk Limits and Profit Maximization

Consider profit maximization under a certain constrain.



Thus, when the risk limit L increases, the point that maximizes revenue moves from (0,0) to the highest point of the blue contour line.

4. Numerical Example

The sales, revenue, and risks of Product A and Product B are as follows:

	Product A	Product B
Sales	S_A	S_B
Revenue	$Q_A=f(S_A)=0.1S_A-0.001S_A^2$	$Q_B=g(S_B)=0.12S_B-0.002S_B^2$
Risk	$R_A=0.1S_A$	$R_B=0.2S_B$
Risk correlation coefficient	$\rho_{AB}=0.5$ $(R(S_A,S_B)=\{(0.1S_A)^2+(0.2S_B)^2+2*0.5*0.1*0.2*S_AS_B\}^{0.5})$	

Identify (S_A,S_B) that maximizes the revenue $f(S_A)+g(S_B)$
under the constraint $R(S_A,S_B) \leqq L$, where L stands for the risk limit.

4. Numerical Example

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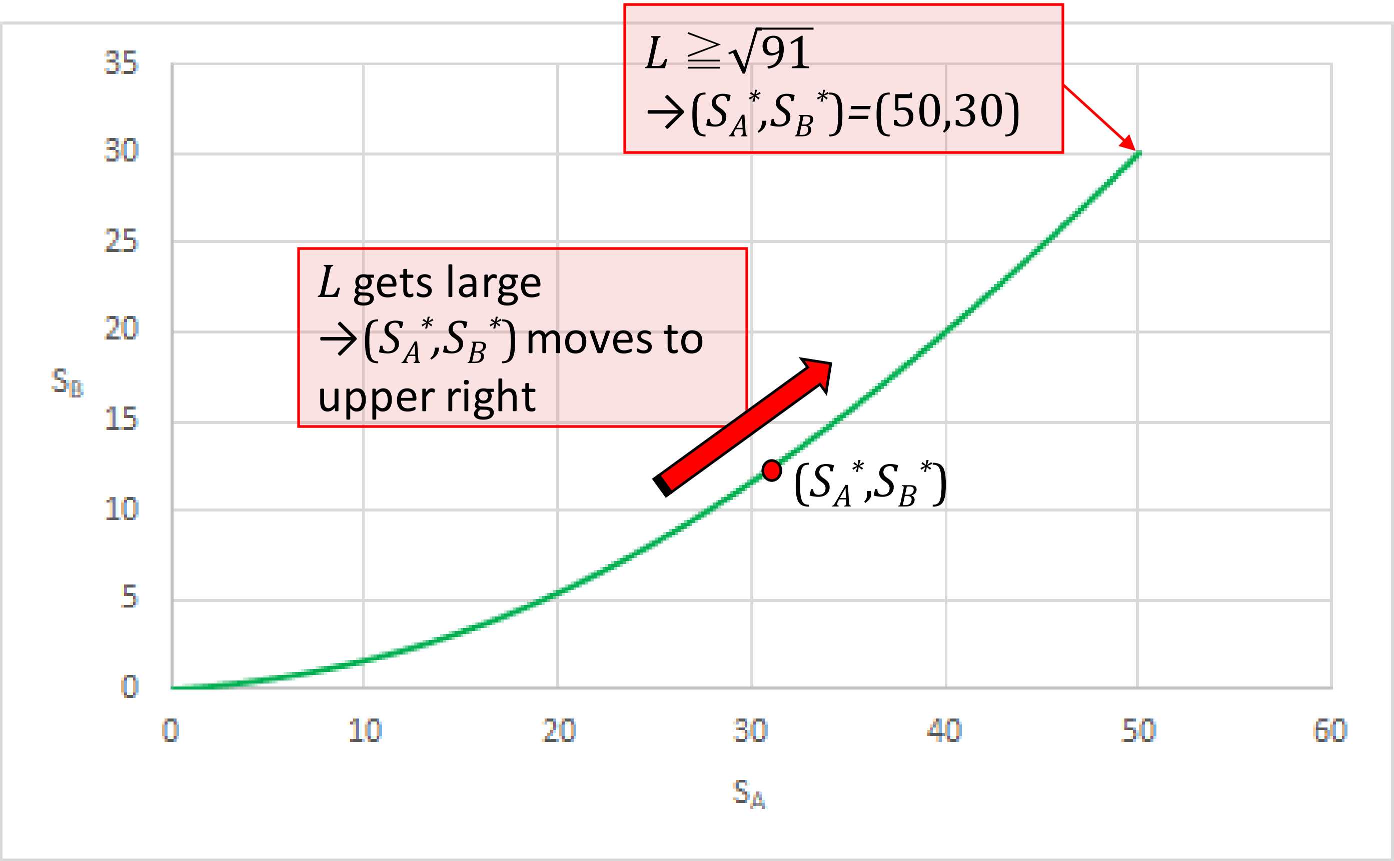
$Q_A=f(S_A)=-0.001(S_A-50)^2+2.5$, $Q_B=g(S_B)=-0.002(S_B-30)^2+1.8$, then:

- If we don't care the risk Limit L, $Q(S_A,S_B)(=Q_A+Q_B)$ is maximized at $(S_A,S_B)=(50,30)$. Then $R(50,30)=\sqrt{91}$.
- If the risk limit $L<\sqrt{91}$, we can use Lagrange multiplier to find the point that maximizes $Q(S_A,S_B)$, under the constraint $R=L$.
  $F=f(S_A)+g(S_B)-\lambda(R-L)$, then solve $\partial F/\partial S_A=\partial F/\partial S_B=\partial F/\partial\lambda=0$.

→The point at which revenue is maximized is denoted by (S_A^*,S_B^*). (Depends on L)

4. Numerical Example

When L varies, (S_A^*, S_B^*) draws the following curve.



- The larger L is, the more (S_A^*, S_B^*) moves to the upper right corner, and stops at $(50,30)$.
- When point (S_A^*, S_B^*) is in the lower right, increasing the ratio of S_A leads to higher risk-return efficiency.
- When point (S_A^*, S_B^*) is in the upper left, increasing the ratio of S_B leads to higher risk-return efficiency.

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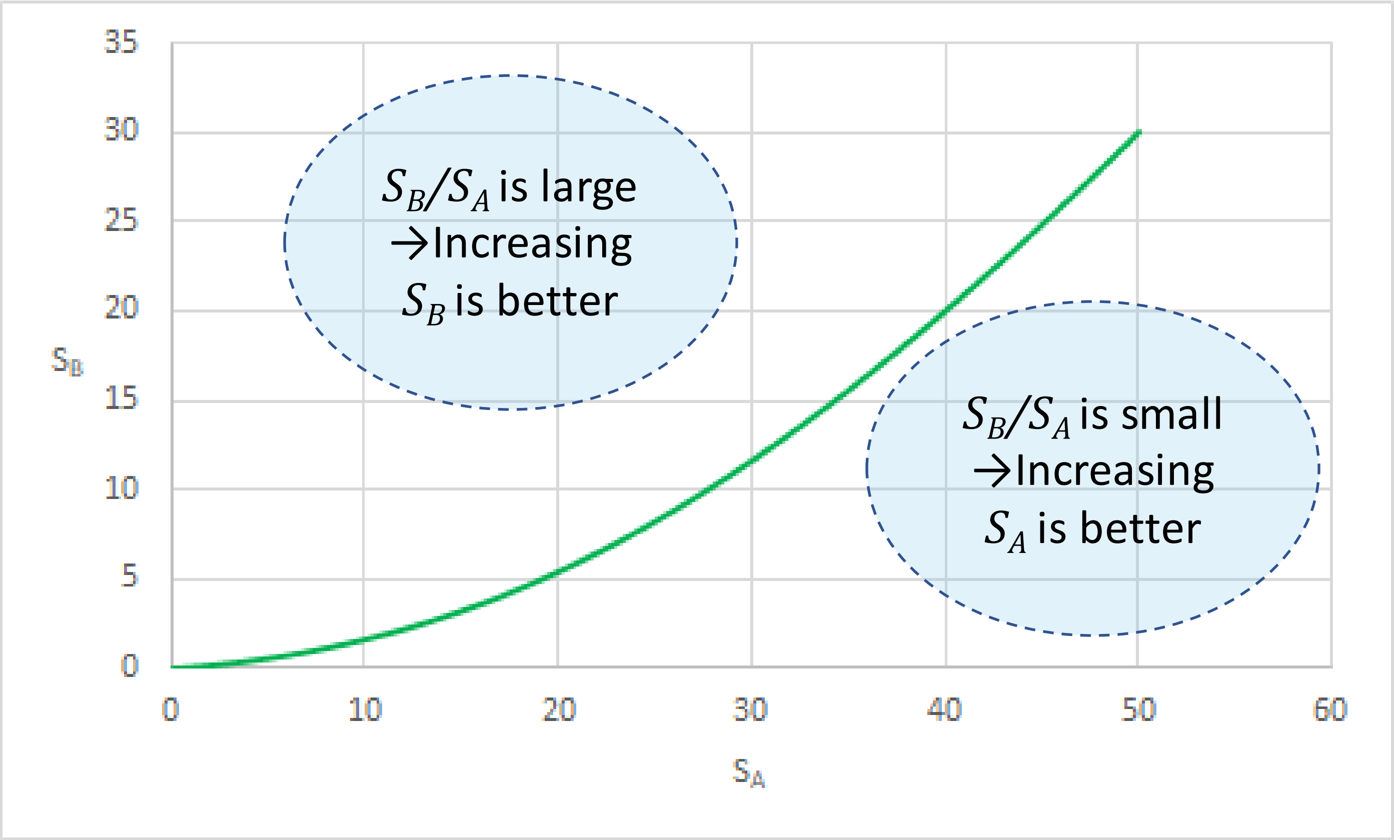

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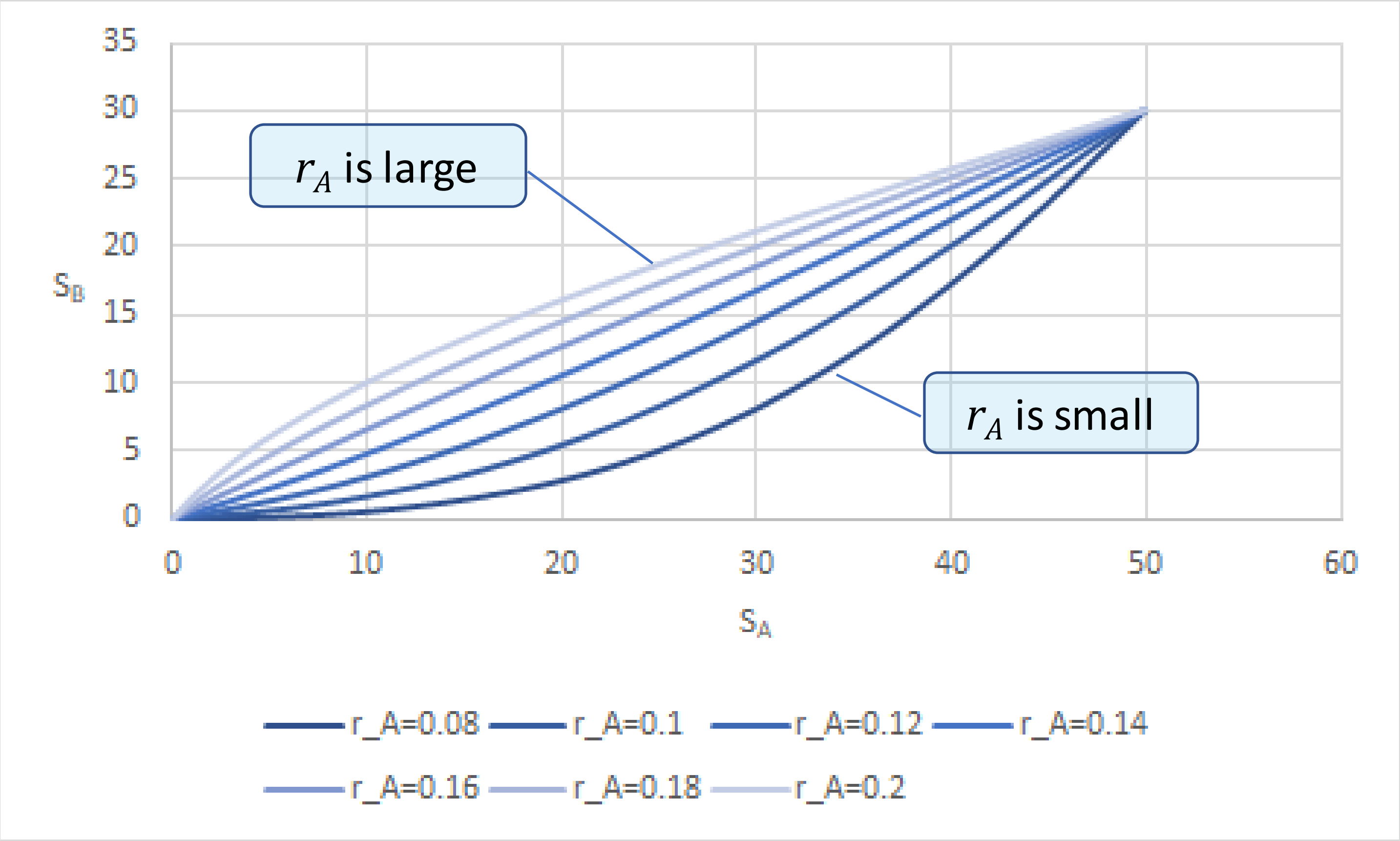
- When point (S_A^*, S_B^*) is in the upper left, increasing the ratio of S_B leads to higher risk-return efficiency.

4. Numerical Example

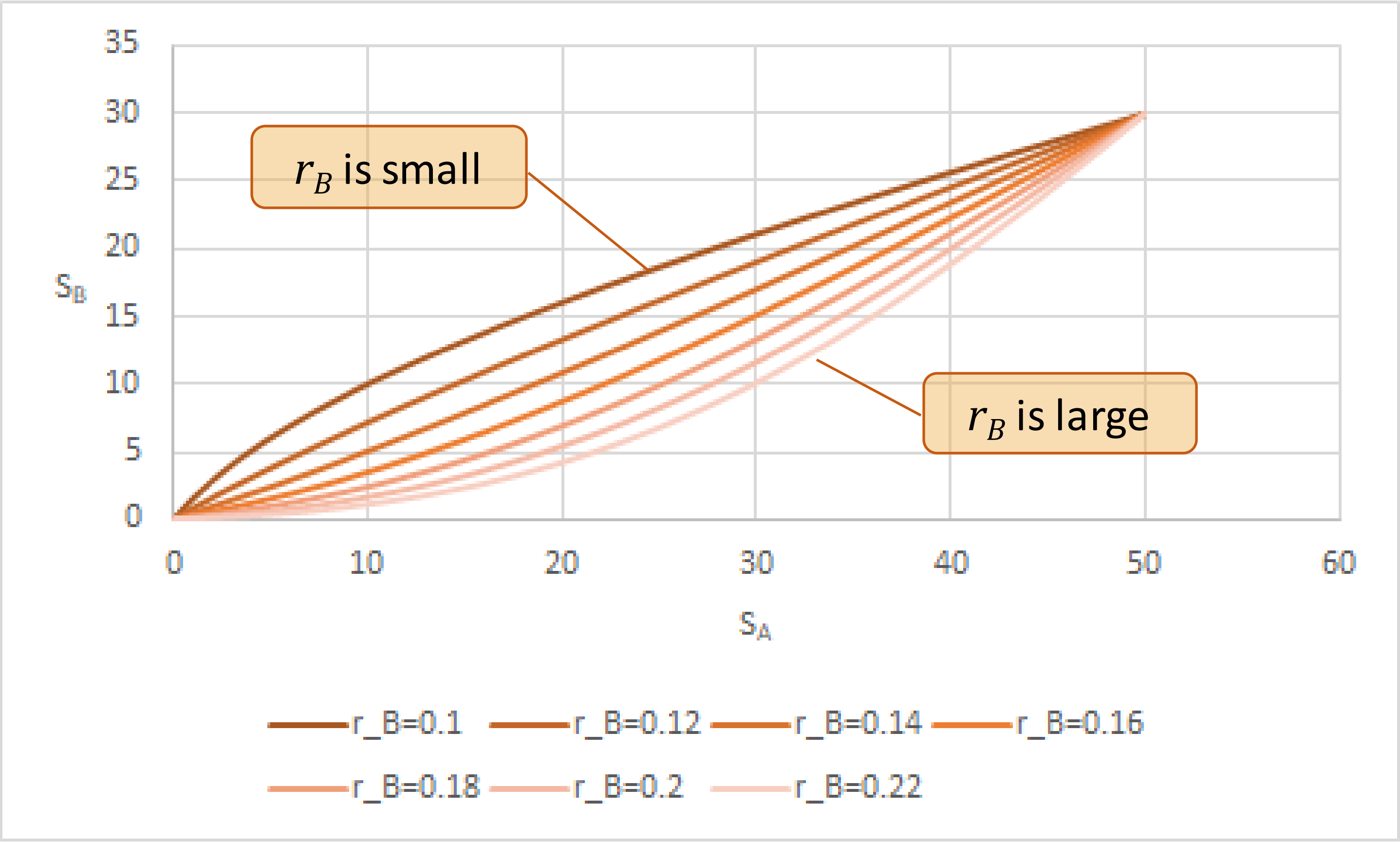
When r_A varies, the curve changes as below.



- r_A is small
  →The curve passes through the lower right corner (where S_B/S_A is small)
  →Increasing the ratio of S_A is better
- r_A is large
  →The curve passes through the upper left corner (where S_B/S_A is large)
  →Increasing the ratio of S_B is better

4. Numerical Example

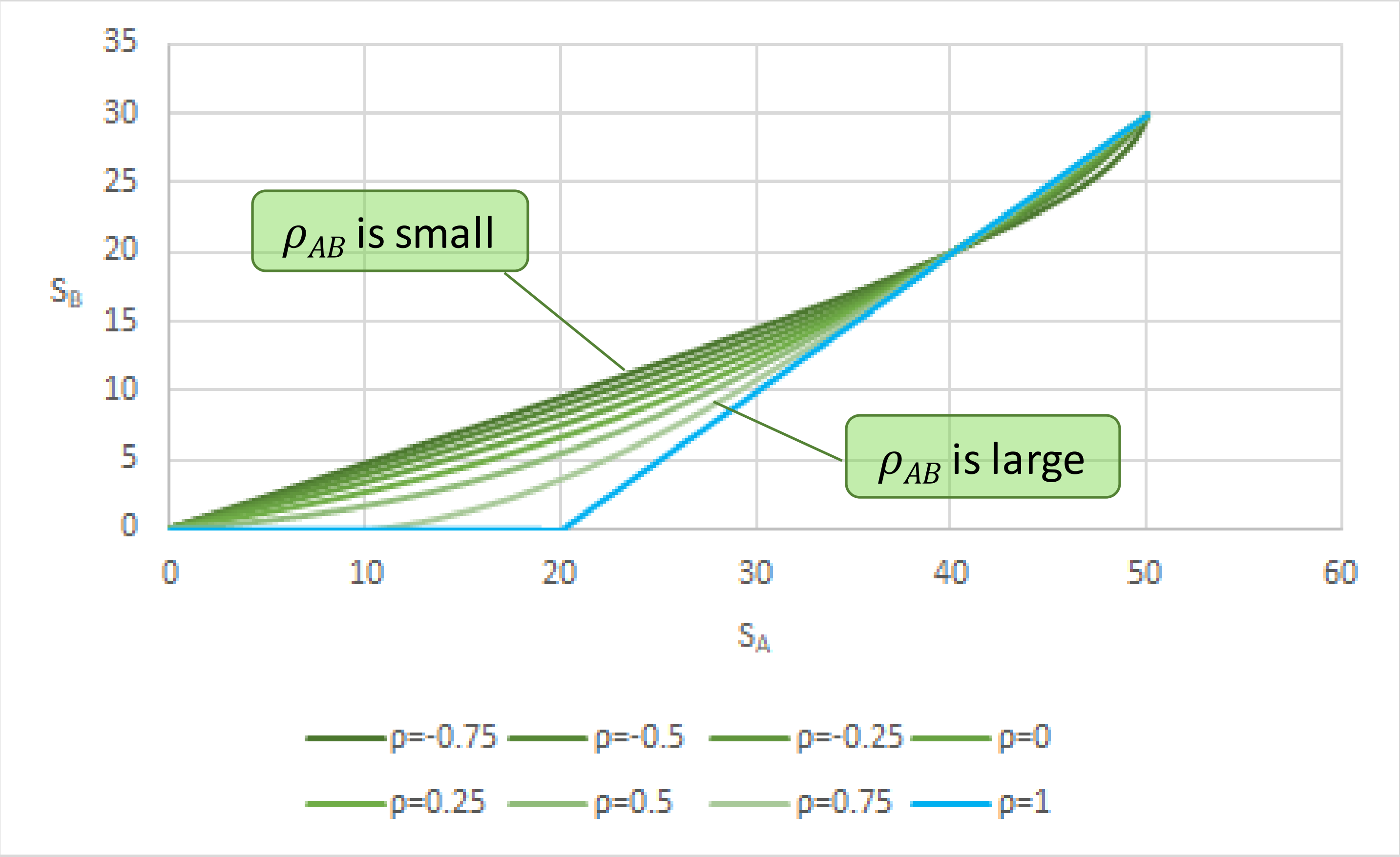
When r_B varies, the curve changes as below.



- r_B is small
  →The curve passes through the upper left corner (where S_B/S_A is large)
  →Increasing the ratio of S_B is better
- r_B is large
  →The curve passes through the lower right corner
  (where S_B/S_A is small)
  →Increasing the ratio of S_A is better

4. Numerical Example

When ρ_{AB} varies, the curve changes as below.



Where S_A<40,S_B<20

- ρ_{AB} is small
  →The curve passes through the upper left corner (where S_B/S_A is large)
  →Increasing the ratio of S_B is better
- ρ_{AB} is large
  →The curve passes through the lower right corner
  (where S_B/S_A is small)
  →Increasing the ratio of S_A is better

As ρ_{AB} approaches 1, the curve converges to the line show in blue.

5. Conclusion

- When sales and revenue have a non-linear relationship, optimizing risk-return efficiency becomes a complex problem.
- In order to optimize risk-return efficiency, insurance companies need to consider the following:
  - ✓How to evaluate the risk coefficients of each product and the correlation coefficients of risk between different products (problem of portfolio theory)
  - ✓How to formulate a non-linear relationship between sales and profit (problem of profit maximization)
  - ✓How to set the risk limit (problem of management policy)

  Each of these problems is also difficult and requires further consideration.

Thank you very much for your attention!

If you have any questions, please email me at wataru.hirose@fi.fukoku-life.co.jp.