Success and failure of the financial regulation on a surplus-driven financial company

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Outline:

- Motivation
- Model setup and optimization problems
- Solutions to the optimization problems
- Conclusion

Motivation

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- secure the stability of the financial system as a whole.

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- Insurance regulation: Solvency II
  - Three-pillar framework;
  - Pillar I: the quantitative requirements to ensure the solvency of the banks/insurance companies;

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- e.g., Value-at-Risk (VaR):
  - a measure of the maximum loss of a portfolio over a given horizon and a pre-determined confidence level;
- Average Value-at-Risk (aka, Expected Shortfall)
  - a measure of the average of all potential losses exceeding the VaR at a given confidence level;

A stream of literatures focus on the impact of these risk measures on the investment strategies of the financial institutions.

- [Basak and Shapiro, 2001]
  optimal asset allocation under the VaR/expected discounted shortfall constraint;
- [Cuoco et al., 2008]
  optimal asset allocation under multiple VaR constraints;
- [Chen et al., 2018]
  optimal asset allocation under VaR and minimum insurance;
- [Nguyen and Stadje, 2020]
  nonconcave optimal investment with VaR constraints;

Considering the surplus-driven characteristic of the financial company, we are interested in how these risk measures protect the liability holders of the financial institution.

Therefore, we explicitly distinguish the liability and surplus in the asset structure, and investigate the optimal investment problem under risk constraints.

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  - The company has limited liability;
- payoff of the equity holders: $\varphi_E(X_T) = X_T - \varphi_L(X_T) = \max(X_T - D_T, 0) = (X_T - D_T)^+$

Let $\rho(X_T)$ denote a risk measure of the terminal wealth, we formulate the following optimization problem:

$$\max_{X_T \in \mathcal{X}} \mathbb{E}[U((X_T - D_T)^+)] \tag{1}$$

$$\text{s.t.} \quad \mathbb{E}[X_T \xi_T] \leq x_0, \quad \rho(X_T) \quad \geq \quad \leq .$$

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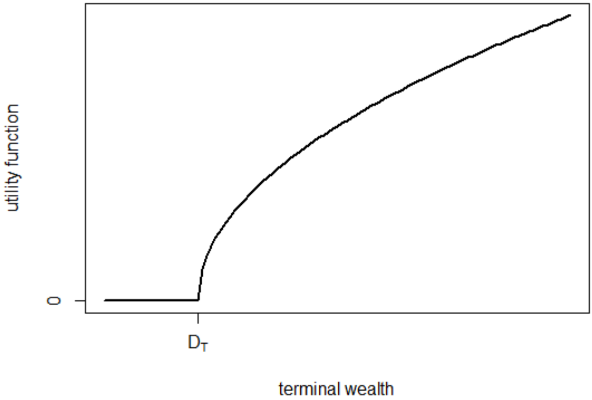
Remark:

- ξ_T is the state price density or stochastic discount factor;

$$\mathbb{E}^{\mathbb{Q}}[e^{-\int_0^T r(s)\mathrm{d}s} X_T] \le x_0 \iff \mathbb{E}^{\mathbb{P}}[\xi_T X_T] \le x_0.$$

- In a complete financial market, each state-contingent claim (X_T) can be replicated by a self-financing trading strategy;


utility function
0
D_T
terminal wealth

The formal definition of the risk constraint $\rho(X_T)$

quantile-based risk constraints:

- Value-at-Risk:

$$VaR_{X_T}(\alpha) := \sup\{x | \mathbb{P}(X_T < x) \leq \alpha\}. \tag{2}$$

As a constraint, $VaR_{X_T}(\alpha) \geq L$ is equivalent to $\mathbb{P}(X_T \geq L) \geq 1 - \alpha$.

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- average Value-at-Risk:

$$AVaR_{X_T}(\alpha) := \frac{1}{\alpha}\int_0^{\alpha} VaR_{X_T}(\beta)\mathrm{d}\beta \geq L'. \tag{3}$$

Shortfall-based risk constraints:

- Expected Shortfall under the physical measure $\mathbb{P}$

$$ES_{\mathbb{P}} := \mathbb{E}[(L - X_T)^+] \leq \epsilon_{\mathbb{P}}. \tag{4}$$

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- Expected Shortfall under the risk-neutral measure $\mathbb{Q}$

$$ES_{\mathbb{Q}} := \mathbb{E}[\xi_T (L - X_T)^+] \leq \epsilon_{\mathbb{Q}}. \tag{5}$$

$$(\mathbb{E}^{Q}[e^{-\int_0^T r(s)\mathrm{d}s}(L - X_T)^+] \equiv \mathbb{E}^{\mathbb{P}}[\xi_T (L - X_T)^+])$$

Essentially, we formulate four optimization problems:

$$\max_{X_T \in \mathcal{X}} \mathbb{E}[U((X_T - D_T)^+)]$$

$$\text{s.t.} \quad \mathbb{E}[X_T \xi_T] \le x_0, \quad \rho(X_T) \in \{\text{VaR}, \text{AVaR}, ES_{\mathbb{P}}, ES_{\mathbb{Q}}\}.$$

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In comparison, the benchmark problem is

$$\max_{X_T \in \mathcal{X}} \mathbb{E}[U((X_T - D_T)^+)]$$

$$\mathbb{E}[X_T \xi_T] \le x_0.$$

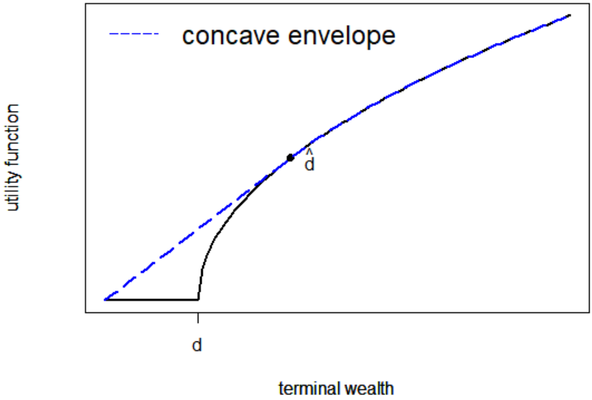
Solutions to the optimization problems

The benchmark solution:

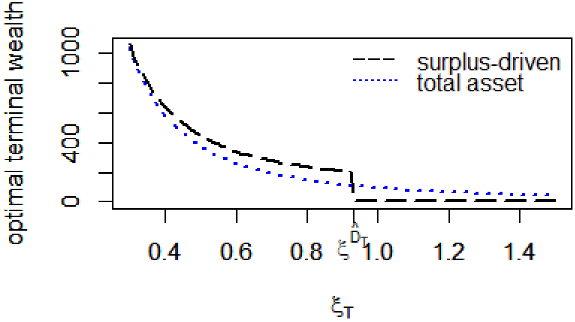
$$X_T^B = (I(\lambda_B \xi_T) + D_T)\mathbb{1}_{\xi_T < \xi^{\widehat{D}_T}}, \tag{6}$$

where $\widehat{D}_T$ is the tangent point with respect to D_T,
$\xi^{\widehat{D}_T} = U'(\widehat{D}_T - D_T)/\lambda_B$, and λ_B is obtained by solving $\mathbb{E}[X_T^B \xi_T] = x_0$.

Concave envelope: the smallest concave function that dominates the original non-concave function;



The benchmark wealth will either be above the tangent point $\widehat{D}_T$ or jump to zero.



$ES_{\mathbb{Q}}$ constraint:

a) If $L \leq D_T$, the optimal solution to is:

$$X^{ES_{\mathbb{Q}}} = (I(\lambda_{\epsilon_{\mathbb{Q}}}\xi_T) + D_T)\mathbb{1}_{\xi_T < \xi_{\epsilon_{\mathbb{Q}}}} \quad \text{if } \xi_{\epsilon_{\mathbb{Q}}} \leq \xi^{\widetilde{L}}_{\epsilon_{\mathbb{Q}}}, \tag{7}$$

$$X^{ES_{\mathbb{Q}}} = (I(\lambda_{\epsilon_{\mathbb{Q}}}\xi_T) + D_T)\mathbb{1}_{\xi_T < \xi^{\widetilde{L}}_{\epsilon_{\mathbb{Q}}}} + L\mathbb{1}_{\xi^{\widetilde{L}}_{\epsilon_{\mathbb{Q}}} \leq \xi_T < \xi_{\epsilon_{\mathbb{Q}}}} \quad \text{if } \xi_{\epsilon_{\mathbb{Q}}} > \xi^{\widetilde{L}}_{\epsilon_{\mathbb{Q}}}, \tag{8}$$

where $\xi^{\widetilde{L}}_{\epsilon_{\mathbb{Q}}} = U'(\widetilde{L} - (D_T - L))/\lambda_{\epsilon_{\mathbb{Q}}}$, $\widetilde{L}$ is the tangent point with respect to $D_T - L$, $\xi_{\epsilon_{\mathbb{Q}}}$ is defined through $\mathbb{E}[L\xi_T \mathbb{1}_{\xi_T \geq \xi_{\epsilon_{\mathbb{Q}}}}] = \epsilon_{\mathbb{Q}}$, $\xi^{\widehat{D}_T} = U'(\widehat{D}_T - D_T)/\lambda_{\epsilon_{\mathbb{Q}}}$ and $\lambda_{\epsilon_{\mathbb{Q}}}$ is obtained by solving $\mathbb{E}[\xi_T X_T^{ES_{\mathbb{Q}}}] = x_0$.

b) If $D_T < L \leq \widehat{D}_T$, the optimal solution to is:

$$X_T^{ES_{\mathbb{Q}}} = (I(\lambda_{\epsilon_{\mathbb{Q}}}\xi_T) + D_T)\mathbb{1}_{\xi_T < \xi_{\epsilon_{\mathbb{Q}}}} \quad \text{if } \xi_{\epsilon_{\mathbb{Q}}} < \underline{\xi}_{\epsilon_{\mathbb{Q}}}, \tag{9}$$

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where $\underline{\xi}_{\epsilon_{\mathbb{Q}}}$ is defined through $U'(L - D_T)/\lambda_{\epsilon_{\mathbb{Q}}}$.

$ES_{\mathbb{P}}$ constraint:

a) If $L \leq D_T$, the optimal solution to is:

$$X_T^{ES_{\mathbb{P}}} = (I(\lambda_{\epsilon_{\mathbb{P}}}\xi_T) + D_T)\mathbb{1}_{\xi_T < \xi_{\epsilon_{\mathbb{P}}}} \quad \text{if } \xi_{\epsilon_{\mathbb{P}}} \leq \xi_{\epsilon_{\mathbb{P}}}^{\widetilde{L}}, \tag{11}$$

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where $\xi_{\epsilon_{\mathbb{P}}}^{\widetilde{L}} = U^{'}(\widetilde{L} - (D_T - L))/\lambda_{\epsilon_{\mathbb{P}}}$, $\widetilde{L}$ is the tangent point with respect to $D_T - L$, $\xi_{\epsilon_{\mathbb{P}}}$ is defined through $\mathbb{E}[L\mathbb{1}_{\xi_T \geq \xi_{\epsilon_{\mathbb{P}}}}] = \epsilon_{\mathbb{P}}$, and $\lambda_{\epsilon_{\mathbb{P}}}$ is obtained by solving $\mathbb{E}[\xi_T X_T^{ES_{\mathbb{P}}}] = x_0$.

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where $\underline{\xi}_{\epsilon_{\mathbb{P}}}$ is defined by $U^{'}(L - D_T)/\lambda_{\epsilon_{\mathbb{P}}}$.

VaR constraint:

a) If $L \leq D_T$, the optimal solution is:

$$X_T^{VaR} = (I(\lambda_\alpha \xi_T) + D_T)\mathbb{1}_{\xi_T < \bar{\xi}_\alpha} \quad \text{if } \xi^{\widetilde{L}} \geq \bar{\xi}_\alpha, \tag{15}$$

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where $\xi^{\widetilde{L}} = U'(\widetilde{L} - (D_T - L))/\lambda_\alpha$, $\widetilde{L}$ is the tangent point with respect to $D_T - L$, $\xi^{\widehat{D}_T} = U'(\widehat{D}_T - D_T)/\lambda_\alpha$ and λ_α is defined via the budget constraint $\mathbb{E}[X_T^{VaR}\xi_T] = x_0$.

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where $\underline{\xi}_\alpha = U(L - D_T)/\lambda_\alpha$, $\xi^{\widehat{D}_T} = U'(\widehat{D}_T - D_T)/\lambda_\alpha$ and λ_α is obtained by solving $\mathbb{E}[X_T^{VaR}\xi_T] = x_0$.

AVaR constraint:

A useful lemma:

i)
$$\frac{1}{\alpha}\int_0^{\alpha} VaR_{X_T}(\beta)\mathrm{d}\beta \geq \epsilon_0 \iff \mathbb{E}[(L-X)^+] \leq \alpha(L-\epsilon_0), \quad L \equiv q_X(\alpha)$$

where $q_X(\alpha)$ denotes the $\alpha-$quantile of the terminal wealth, i.e., $q_X(\alpha) := \sup\{x|\mathbb{P}(X_T < x) < \alpha\}$.

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ii) The function $f : L \to E[(L-X)^+] - \alpha(L-\epsilon_0)$ attains its minimum at $L = q_X(\alpha)$.

iii)
$$\{X_T|\mathbb{E}[X_T|AVaR_{X_T}(\alpha)] \geq \epsilon_0\} = \bigcup_{L\geq\epsilon_0}\{X_T|E[(L-X_T)^+] \leq \alpha(L-\epsilon_0)\}.$$

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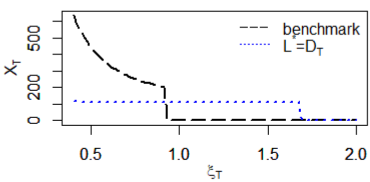
Theorem

Let $X_T^{ES,L}$ denote the optimal solution of the ES constrained problem

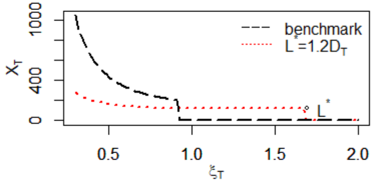
$$\left[\max_{X_T} \mathbb{E}[U(X_T - D_T)^+], \quad s.t. \quad \mathbb{E}[\xi_T X_T] \le x_0, \quad \mathbb{E}[(L - X_T)^+] \le \alpha(L - L^{'})\right].$$

For each given initial wealth x_0, there exists $L^* \ge L^{'}$ such that the optimal solution of the AVaR constrained problem X_T^{AVaR} coincides with the optimal solution of the ES-constrained problem, i.e.,

$$X_T^{AVaR} = X_T^{ES,L^*}.$$



(a) $L^* = D_T$.



(b) $L^* = 1.2D_T$.

Figure: **The optimal wealth under the risk constraint.**

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- The optimal wealth shows a two-region/three-region distribution depending on the parameters;
- The probability of default is effectively reduced by the regulation;
- All four risk constraints can lead to the same optimal wealth;

Example: A Black Scholes market

Assuming $\alpha = 1\%$ and $L^{'} = 0.9D_T$, the AVaR constraint is given by

$$\frac{1}{0.01}\int_0^{0.01} VaR_{X_T}(\beta)\mathrm{d}\beta \geq 0.9D_T.$$

The optimal solution under the AVaR constraint is the same as the optimal solution under the $ES_{\mathbb{P}}$ constraint:

$$\mathbb{E}[(L^* - X_T)^+] \leq (L^* - L^{'})\alpha.$$

The magnitude of L^* depends on the initial wealth. We consider two possible values to illustrate the equivalence among the risk constraints.

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Suppose $L^* = D_T$,

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- the equivalent $ES_{\mathbb{P}}$ constraint is

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- the equivalent $ES_{\mathbb{Q}}$ constraint is

$$\mathbb{E}[\xi_T(L^* - X_T)^+] \leq \epsilon_{\mathbb{Q}} = 0.00022\% D_T.$$

Example

	$\mathbb{P}(X_T \geq L^*) \geq \alpha^*$	$\epsilon_{\mathbb{P}}$	$\epsilon_{\mathbb{Q}}$
$L^* = D_T$	0.1%	$0.1\% D_T$	$0.00022\% D_T$
$L^* = 1.2 D_T$	0.25%	$0.3\% D_T$	$0.0015\% D_T$

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- the probability of default is effectively reduced by the regulation but the liability holders cannot be fully protected.

Thanks for your attention!

https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3920338

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