Life reinsurance under perfect and asymmetric information

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Motivation

- Default risk of life insurers, e.g., due to various forms of guaranteed payment to policyholders in life insurance contracts
- Various ways to hedge the default risk, e.g.:
  - Investment strategies (CPPI or OBPI)
  - **Reinsurance** to cover (partially) the default risk
- Possible advantages of reinsurance
  - Reinsurance companies have extensive expertise in navigating financial markets
  - Other risks arising from life insurance contract transferred to reinsurer (e.g. longevity risk)

Motivation

- Non-life reinsurance is widely used in practice and analyzed in academia
  - 2022: 51% of the gross premium income of Munich Re corresponds to non-life reinsurance (cf. Munich Re (2022), p.60)
  - Schmidli (2006), Shiu (2011), Asimit *et al.* (2015), Horáková *et al.* (2021), Zanotto & Clemente (2022), etc.
- Life reinsurance is becoming more emerging, but it is still in developmental stage
  - 2022: 20% of the gross premium income of Munich Re corresponds to life and health reinsurance (cf. Munich Re (2022), p.60)
  - Escobar-Anel *et al.* (2022): Reinsurance-investment problem under value-at-risk and no-short-selling constraint
  - Havrylenko *et al.* (2022): Reinsurance-investment problem modeled as a Stackelberg game
  - Chen *et al.* (2023): Information asymmetry in longevity risk transfer
- We focus on the impact of the insurer's portfolio risk on the reinsurance contract

Research questions and main findings

- Research questions
  - (a) What is the optimal reinsurance contract under full information?
  - (b) How does the insurer's investment strategy influences the optimal reinsurance contract?
  - (c) How does asymmetric information between the reinsurer and the insurer influence the optimal reinsurance contract?

- Main findings
  - (a) Optimal reinsurance is no, partial or full reinsurance depending on the default risk
  - → If the default risk is large enough, the insurer purchases reinsurance
  - (b) A higher investment strategy results in a higher reinsurance cover
  - (c) Additional information costs due to asymmetric information (higher reinsurance premium and lower reinsurance share)

Overview

Model: Payment structure

Life insurance contract between insurer and policyholder

- Finite time horizon $[0, T]$, $T < \infty$
- Initial asset value of the insurer: $A_0 = L_0 + E_0$
  - Initial contribution of policyholder $L_0 > 0$
  - Initial contribution of equity holder $E_0 > 0$
  - Initial premium share of policyholder $\alpha \in (0, 1)$, i.e., $L_0 = \alpha A_0$
- Asset value of insurer with initial asset value A_0 at time t: $A^{A_0}(t)$
- Benefits to policyholder are paid at maturity T by insurer
  - Insurer aims to offer a guarantee to the policyholder
  - Guaranteed interest rate $g \in (0, r]$ with r is the risk-free rate

  $\rightarrow$ Guaranteed payoff: $G_T = \alpha A_0 e^{gT}$

Model: Payment structure

Two possible cases:

(1) $A^{A_0}(T) \geq G_T$

- Insurer performs well
- Policyholder receives at least guaranteed payment G_T
- Surplus participation: $\delta(\alpha A^{A_0}(T) - G_T)^+$ with $\delta \in [0, 1]$

(2) $A^{A_0}(T) < G_T$

- Insurer does not perform well
- Without external guarantor, the policyholder receives less than the guaranteed payment G_T

$\rightarrow$ Policyholder receives $A^{A_0}(T)$ and insurer has nothing left

Terminal payout to PH

$$\Psi(A^{A_0}(T)) = G_T + \delta(\alpha A^{A_0}(T) - G_T)^+ - \underbrace{(G_T - A^{A_0}(T))^+}_{\text{Default option}}$$

Model: Payment structure

Reinsurance contract between the insurer and the reinsurer

- Reinsurance premium $p \geq 0$ paid at time 0
- Default option $(G_T - A^{A_0}(T))^+$ paid at maturity T
- Reinsurance share $q \in [0, 1]$: reinsurer covers $100 \cdot q\%$ of policyholder's default
- Insurer's initial asset value: $a_0(p, q) := A_0 - pq$
- Insurer's terminal asset value

$$\tilde{A}^{a_0(p,q)}(T) := A^{a_0(p,q)}(T) - G_T - \delta C(\alpha A^{a_0(p,q)}(T)) + P(A^{a_0(p,q)}(T))$$

  with $C(S(T))$ and $P(S(T))$ is call and put option with underlying S and strike price G_T
- Policyholder's terminal payout

$$\Psi(A^{a_0(p,q)}(T)) = G_T + \delta C(\alpha A^{a_0(p,q)}(T)) - (1 - q)P(A^{a_0(p,q)}(T))$$

Model: Asset dynamics

- Financial market

$$dS_0(t) = rS_0(t)dt$$
$$dS_1(t) = S_1(t)(\mu dt + \sigma dW(t))$$

with W Brownian motion on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$, $\mu > r$ expected return of risky asset and $\sigma > 0$ volatility of risky asset

- Investment strategy of insurer given by constant $\pi \in (0,1]$
- Insurer's asset value with initial value $a_0(p,q)$ and investment strategy π at time t

$$dA^{a_0(p,q),\pi}(t) = A^{a_0(p,q),\pi}(t)((r + \pi(\mu - r))dt + \pi\sigma dW(t))$$
$$A^{a_0(p,q),\pi}(0) = a_0(p,q)$$

Model: Preference measure

- Aim of insurer: maximize its mean-variance preference of its terminal asset value $\tilde{A}^{a_0(p,q),\pi}(T)$ with respect to the reinsurance share

- Objective

$$J(\tilde{A}^{a_0(p,q),\pi}(T)) := \mathbb{E}[\tilde{A}^{a_0(p,q),\pi}(T)] - \frac{\gamma}{2} Var(\tilde{A}^{a_0(p,q),\pi}(T))$$

with $\gamma > 0$ describing the insurer's risk aversion.

Model: Parameter selection

Parameter	Notation	Value
Interest rate	r	0.9%
Drift coefficient for S_1	μ	6.6%
Diffusion coefficient for S_1	σ	21.6%
Initial asset value of insurer	A_0	100
Proportion of initial contributions of PHs	α	93%
Guaranteed interest rate	g	0.9%
Risk aversion parameter of insurer	γ	0.05
Profit-sharing parameter of insurer	δ	0.9
Time horizon	T	20

Table: Parameter setting for numerical analysis

Full information: Optimization problem

- Reinsurer has full information about insurer's portfolio risk
- Reinsurance premium: given through financial fair price of default option

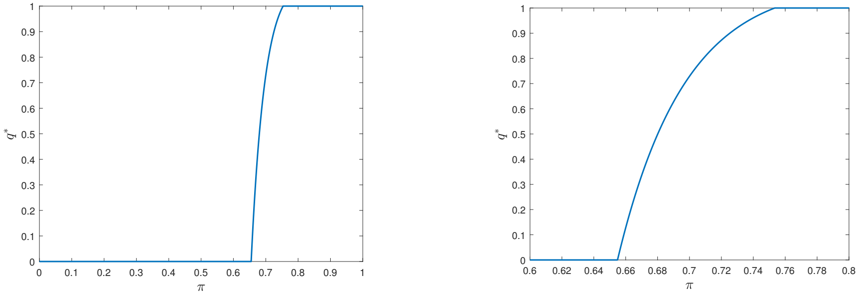
$$p = e^{-rT}\mathbb{E}_{\mathbb{Q}}[(G_T - A^{a_0(p,q),\pi}(T))^+] \tag{1}$$

  where $\mathbb{Q}$ is the risk-neutral measure
- Optimization problem

$$\sup_{q\in[0,1]} J(\tilde{A}^{a_0(p,q),\pi}(T)) \text{ s.t. (1) holds and } p \in [0, e^{-rT}G_T]$$

- Optimal reinsurance share
  - Reinsurance premium: there exist function $h : [0,1] \to [0, e^{-rT}G_T]$ s.t. $p = h(q)$ (Implicit Function Theorem)
  - No closed-form solution available
  - Existence of optimal reinsurance share
  - Sufficient conditions under which unique optimal reinsurance share exists
  - Conditions under which optimal reinsurance share is continuous and increasing regarding insurer's investment strategy

Full information: Numerical analysis



(a) q^* with respect to $\pi \in (0, 1]$.

(b) q^* with respect to $\pi \in [0.6, 0.8]$.

Figure: Optimal reinsurance share q^* with respect to investment strategy π.

Asymmetric information: Model

- Reinsurer has asymmetric information about insurer's portfolio risk
  - High-risk insurer (H) and low-risk insurer (L): $0 < \pi_L < \pi_H \leq 1$
  - Distribution of the types of insurers: $100 \cdot \varepsilon\%$ low-risk insurer with $\varepsilon \in (0, 1)$
  - Reinsurer does not know insurer's type, but the distribution of the types of insurers
- Reinsurer's expected profit resulting from reinsurance contract

$$\varepsilon EP_{R,L}(p, q_L) + (1 - \varepsilon) EP_{R,H}(p, q_H)$$

  - $EP_{R,i}(p, q_i) := \mathbb{E}[A_R^{q_i p, \pi_R}(T) - q_i(G_T - A_i^{a_0(p,q_i),\pi_i}(T))^+]$ is expected profit resulting from reinsurance contract with insurer of type $i \in \{L, H\}$
  - A_R^{qp,π_R} denotes reinsurer's asset process with initial value qp and constant investment strategy π_R, and
  - $A_i^{a_0(p,q_i),\pi_i}$ denotes asset process of insurer of type i with initial value $a_0(p, q)$ and investment strategy π_i

Asymmetric information: Optimization problem

- Reinsurer offers only one reinsurance premium to both types of insurers
  - Aim of reinsurer: maximize expected profit resulting from reinsurance contract
  - Upper limit: $\overline{p} := e^{-rT} G_T$
  - Lower limit: $\underline{p} := e^{-rT} \mathbb{E}_{\mathbb{Q}}[(G_T - A_L^{A_0,\pi_L}(T))^+]$

- Stackelberg game between the reinsurer and the insurer

$$\sup_{p \in [\underline{p},\overline{p}]} \varepsilon EP_{R,L}(p, q_L^*(p)) + (1-\varepsilon) EP_{R,H}(p, q_H^*(p))$$

$$\text{s.t. } q_i^*(p) = \underset{q_i \in [0,1]}{\arg\max}\, J(\tilde{A}_i^{a_0(p,q_i),\pi_i}(T)) \; \forall i \in \{L, H\}$$

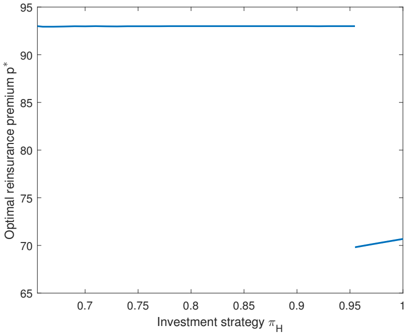
Asymmetric information: Optimization problem

- Procedure of solving Stackelberg game
  1. For every $p \in [\underline{p}, \overline{p}]$, insurer of type i selects optimal reinsurance share $q_i^*(p)$
  2. Given the optimal response $q_i^*(p)$ of both types of insurers, reinsurer selects optimal reinsurance premium p^*.
  3. Stackelberg equilibrium is given by $(p^*, q_L^*(p^*), q_H^*(p^*))$

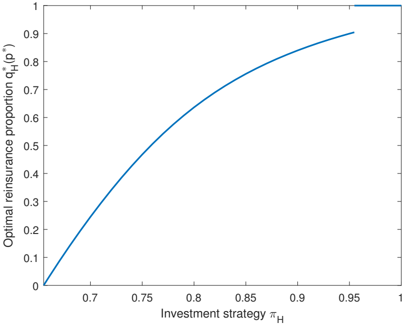
- Stackelberg equilibrium
  - No closed-form solution available
  - Conditions under which a Stackelberg equilibrium exists

- Numerical analysis: Three different cases
  1. $\pi_L < \pi_H < 65.49\% \Rightarrow q_L^*(p^*) = q_H^*(p^*) = 0$
  2. $\pi_L < 65.49\% \leq \pi_H \Rightarrow q_L^*(p^*) = 0$ and $q_H^*(p^*) > 0$
  3. $65.49\% \leq \pi_L < \pi_H \Rightarrow q_L^*(p^*), q_H^*(p^*) > 0$

Asymmetric information: Numerical analysis



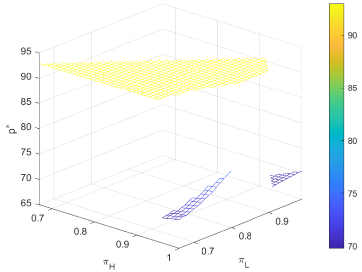
(a) Optimal reinsurance premium p^* with respect to π_H.



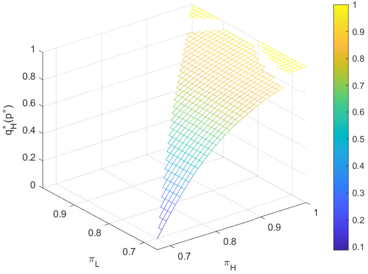
(b) Optimal reinsurance share $q_H^*(p^*)$ with respect to π_H.

Figure: Only high-risk insurer participates in reinsurance contract: $\pi_L < 65.49\%$ and $\pi_H \in [65.49\%, 100\%]$.

Asymmetric information: Numerical analysis



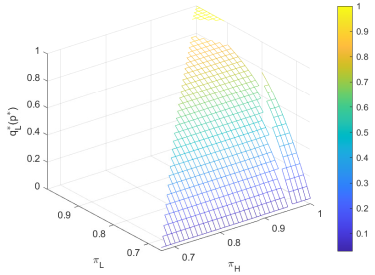
(a) Optimal reinsurance premium p^* with respect to π_L and π_H.



(b) Optimal reinsurance share $q_H^*(p^*)$ with respect to π_L and π_H.

Figure: Both types of insurers participate in reinsurance contract: $\pi_L \in [66\%, 99\%]$ and $\pi_H \in [67\%, 100\%]$ with $\pi_L < \pi_H$.

Asymmetric information: Numerical analysis



(c) Optimal reinsurance share $q_L^*(p^*)$ with respect to π_L and π_H.

Figure: Both types of insurers participate in reinsurance contract: $\pi_L \in [66\%, 99\%]$ and $\pi_H \in [67\%, 100\%]$ with $\pi_L < \pi_H$.

Conclusion

- Optimal life reinsurance under perfect and asymmetric information and impact of insurer's investment strategy on optimal reinsurance arrangement
  - Optimal reinsurance is (partial) reinsurance if investment strategy is large enough
  - Higher reinsurance cover if the portfolio risk increases
  - At least high-risk insurer prefers partial reinsurance $\Rightarrow$ maximum reinsurance premium
  - High-risk insurer full reinsurance $\Rightarrow$ lower reinsurance premium optimal
  - Asymmetric information leads to additional information costs (larger reinsurance premium and lower reinsurance share)

- Outlook
  - Reinsurer offers two different contracts to the two types of insurers (principal-agent model)

Thank you for your attention!

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