The Impact of Mortality Shocks on Modeling and Insurance Valuation as Exemplified by COVID-19

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The trend of improving longevity is erratically interrupted by mortality shocks.

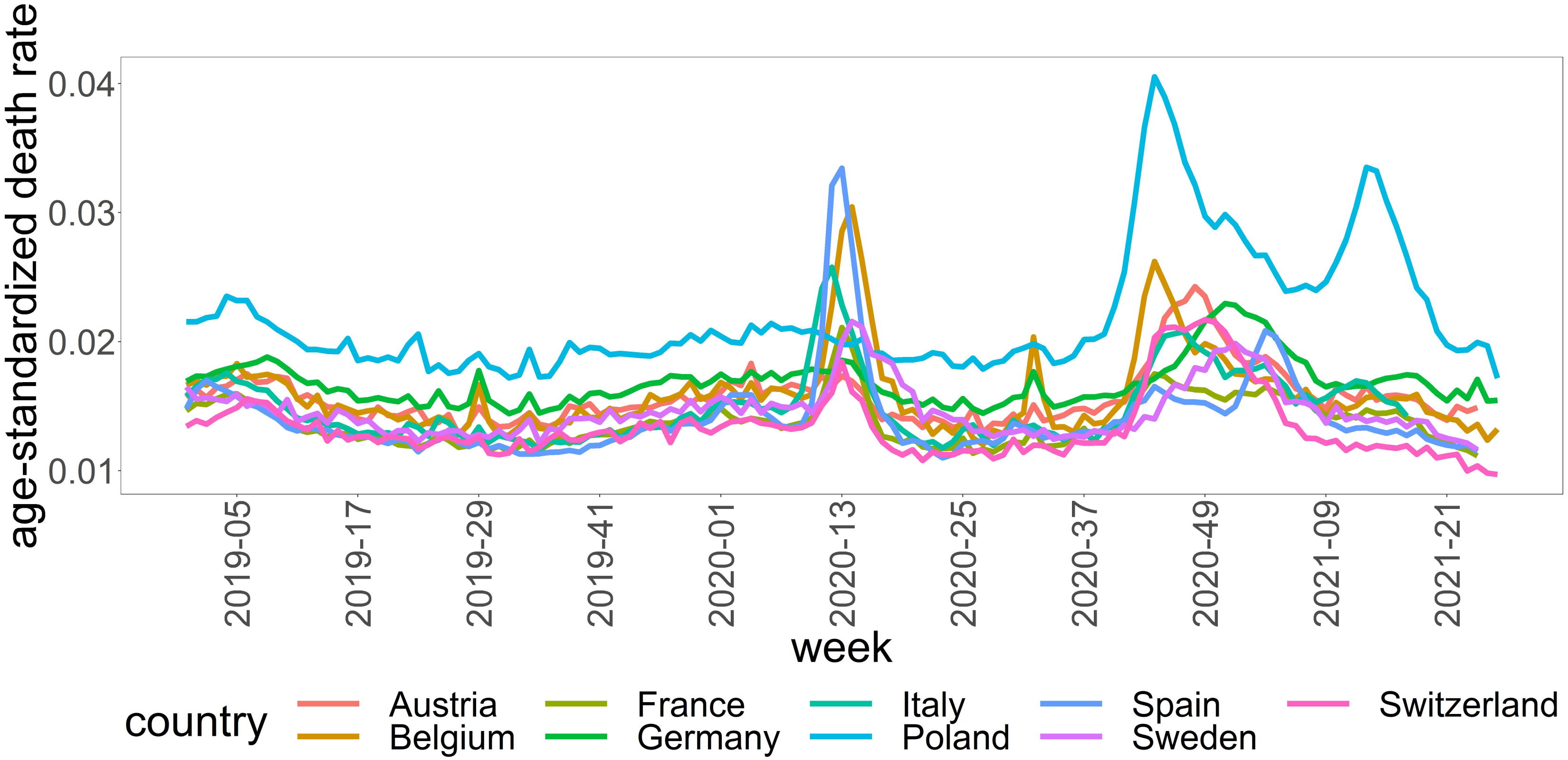

Figure: Weekly, country-specific age-standardized death rates in 2019, 2020 and early 2021.

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The possibility of shocks is highly relevant for mortality modeling.

How much has COVID-19 influenced mortality in 2020?

- significant excess mortality on a weekly and also yearly scale,
- in terms of yearly mortality improvements, 2020 is among the worst 10 years for all countries in our data set.

How much impact does this have on the Lee-Carter model?

- numerical example: decrease in annuity values of up to 9%, increase in life insurance values of up to 29%,
- uncertainty related to forecasts strongly increases when taking 2020 into account.

How can extreme mortality events be handled?

- outlier adjustment,
- deviating from the normal distribution assumption for period effect increments.

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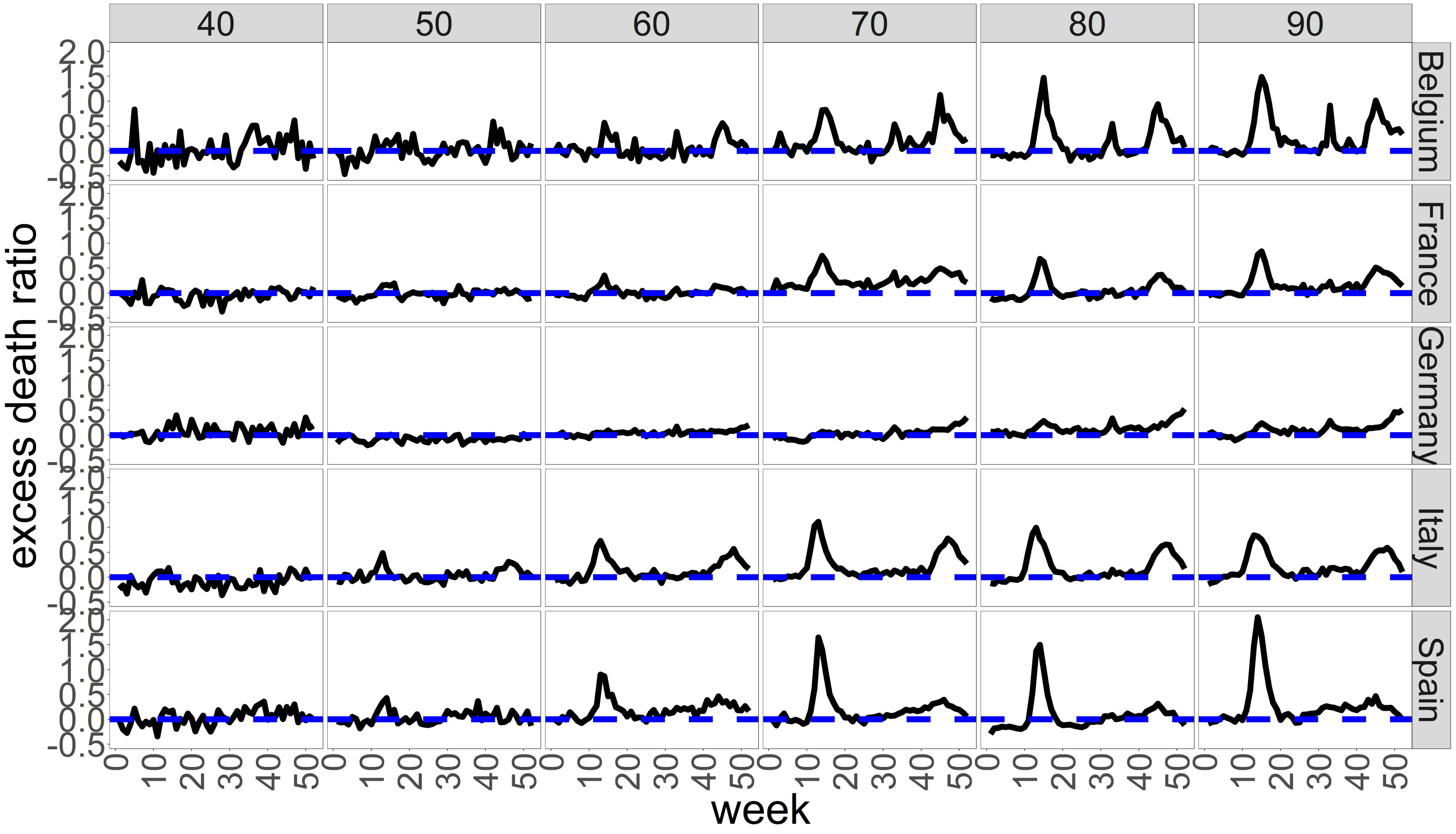
We consider data of 9 European countries to prove this empirically.

- data sources: Human Mortality Database [2021, HMD] for yearly death rates $m^i_{x,t} = \frac{\#(\text{Deaths})}{\text{Exposure}}$, Short Term Mortality Fluctuations [2021, STMF] for weekly death counts $D^i_{x,t,w}$ (age x, year t, population i, week w),
- weekly STMF death counts are aggregated to obtain yearly data if these are not available from the HMD
- age groups $x = [35, 39], [40, 44], \ldots, [85, 89], 90+$.

Table: Considered countries and available years.

Country	Available years
Austria	1947–2020
Belgium	1900–2020
France	1900–2020
Germany	1956–2020
Italy	1900–2020
Poland	1958–2020
Spain	1908–2020
Sweden	1900–2020
Switzerland	1900–2020

Excess mortality (at higher ages) is clearly visible on a weekly scale...



$$p^i_{x,2020,w} := \frac{D^i_{x,2020,w} - \bar{D}^i_{x,2016:2019,w}}{\bar{D}^i_{x,2016:2019,w}}$$

Figure: Weekly, country-specific excess death ratios $p^i_{x,2020,w}$ for different ages. Values above the zero line (blue, dashed) indicate excess mortality.

… and also on a yearly scale.

- Polish males and Spanish females have "lost" around 12 years of mortality development in 2020,
- German females have only "lost" around 5 years,
- improvement rates from 2019 to 2020 are among the worst 10 observed for almost every population in our data set.

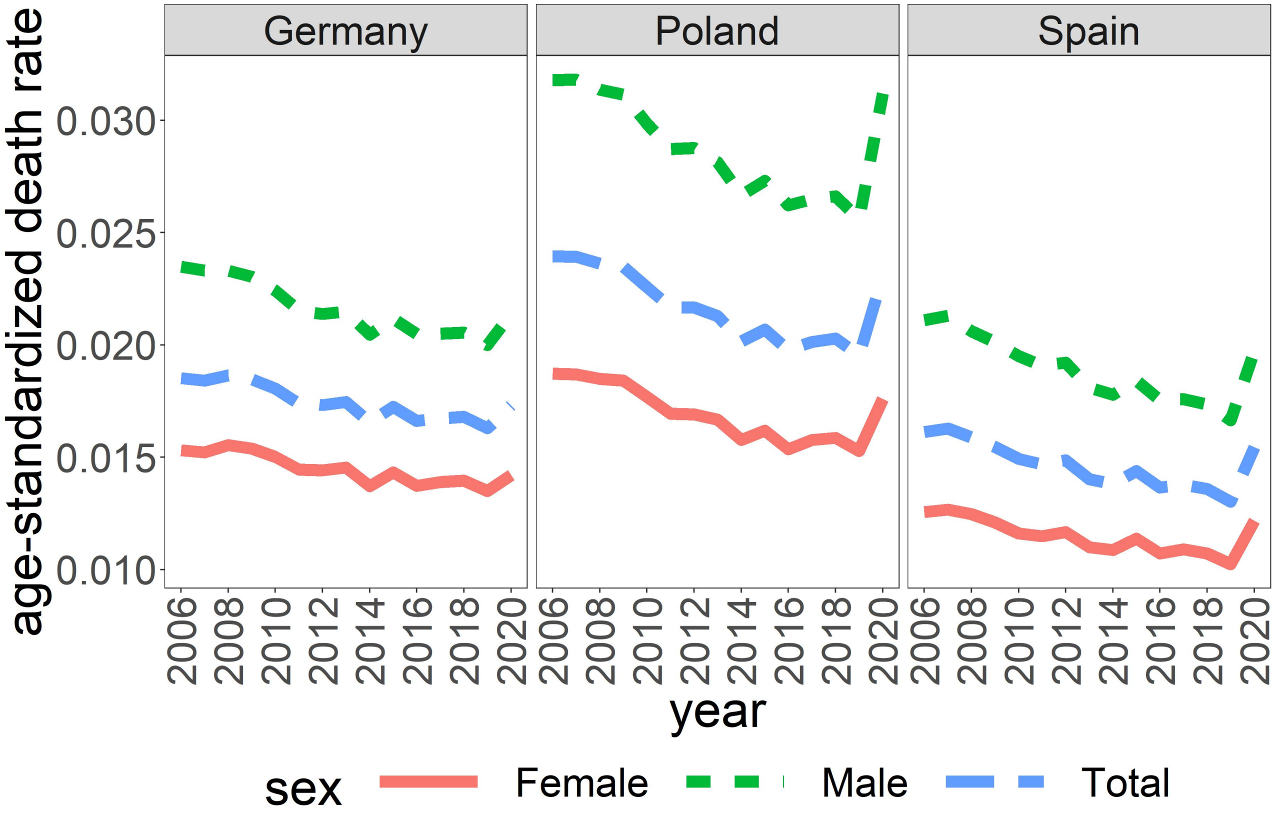


Figure: Yearly German, Polish and Spanish age-standardized death rates between 2006 and 2020 for females (red, solid), males (green, dash) and the total population (blue, long dash).

We analyze the impact of COVID-19 mortality on the Lee-Carter (LC) model...

Lee and Carter [1992]

$$\log m^i_{x,t} = \alpha^i_x + \beta^i_x \kappa^i_t + \varepsilon^i_{x,t}$$

with

- basic age structure of mortality α^i_x,
- period effects κ^i_t,
- age effects β^i_x,
- error terms $\varepsilon^i_{x,t}$.

… by comparing LC models calibrated on different scenarios (data sets).

1. Calibrate 2 LC models for evaluating the influence of a shock in the forecast jump-off year:
   - 1991 to 2020 (real data)
   - 1991 to 2019 (real data) $\cup$ LC-2019 estimate for 2020

2. Calibrate 2 LC models for evaluating the influence of a shock before the forecast jump-off year:
   - 1992 to 2020 (real data) $\cup$ LC-2019 estimate for 2021
   - 1992 to 2019 (real data) $\cup$ LC-2019 estimates for 2020 and 2021

A shock in the jump-off year leads to a change in point and interval forecasts ...

- $\hat{\kappa}_{2020}$ jumps upwards due to the mortality shock,
- using 2020 as the forecast jump-off year, this leads to a change in period effect drift and, thus, point forecasts,
- changes of up to 9% in annuity values and up to 29% in life insurance values,
- 95% prediction interval width increases as well (more than doubles in some cases).

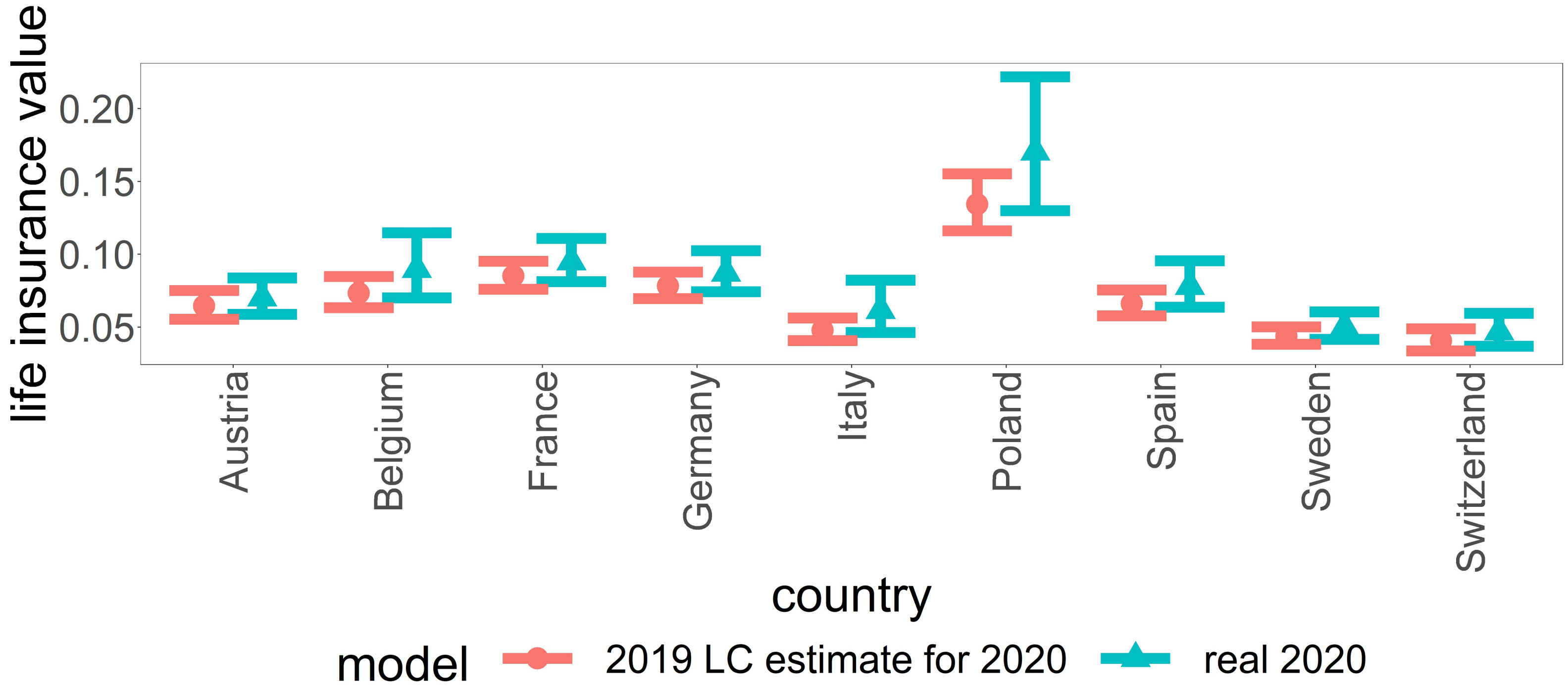


Figure: Country-specific 30-year life insurance values for 35-year old males (based on point and interval death rate forecasts), comparing an LC model trained on real data up to 2020 (blue triangle) and an LC model trained on real data up to 2019 and 2020 best estimates (red circle). Discount factor $v = \frac{1}{1.005}$.

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… while a shock before the jump-off year still leads to a change in interval forecasts.

- we make the (unrealistic) assumption that mortality reverts to "normal" levels in 2021,
- the 2020 shock has very little influence on point forecasts if 2021 is used as the jump-off year,
- but interval forecasts still widen substantially, by a factor of up to 2.58 for the annuity and up to 2.61 for the term assurance.

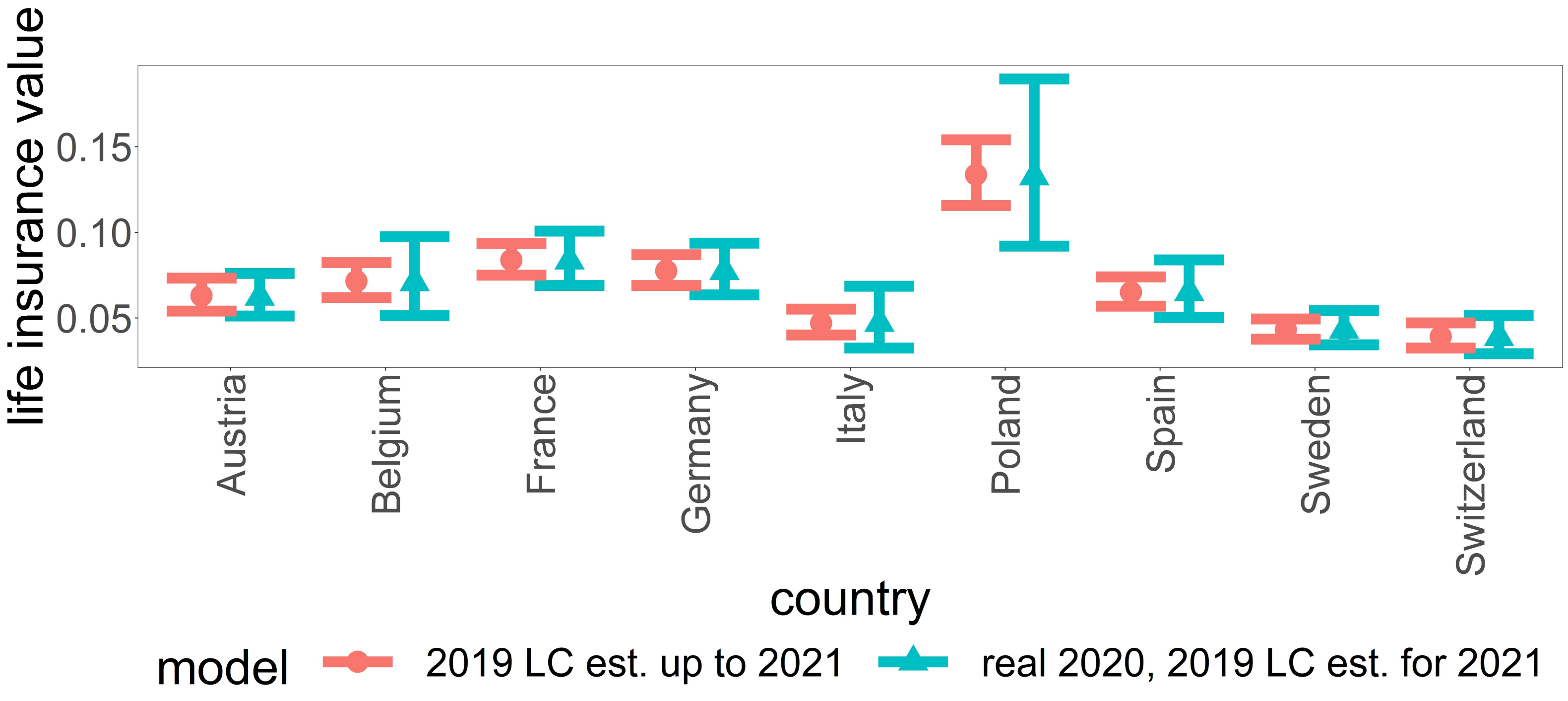

Figure: Country-specific 30-year life insurance values for 35-year old males (based on point and interval death rate forecasts), comparing an LC model trained on real data up to 2020 and 2021 best estimates (blue triangle) and an LC model trained on real data up to 2019 and 2020/2021 best estimates (red circle). Discount factor $v = \frac{1}{1.005}$.

There are several ways to treat mortality shocks in the modeling process.

- random walk with drift (or more general ARIMA model), i.e., normal distribution assumption for the total yearly log-mortality improvements (period effect increments) $\hat{\kappa}_{t+1} - \hat{\kappa}_t$,
- remove 2020 death counts from the data / replace by best estimate / remove COVID-19 deaths,
- outlier analysis and adjustment [Li and Chan, 2005, 2007],
- deviate from the normal distribution assumption: lognormal distribution, mixture distribution, jump process, regime switching.

We consider a mixture distribution based on the peaks-over-threshold method.

- Pickands-Balkema-de Haan: Under certain conditions $X|X > u$, the excess value of a random variable X over a threshold u, converges in distribution to a generalized Pareto distribution (GPD) as $u \to \infty$,
- model $\hat{\kappa}_{t+1} - \hat{\kappa}_t$ with normal distribution below u and GPD above u [Chen and Cummins, 2010],
- choose u from a set of candidate values (high quantiles) by profile likelihood maximization,
- model calibration by penalized maximum likelihood estimation.

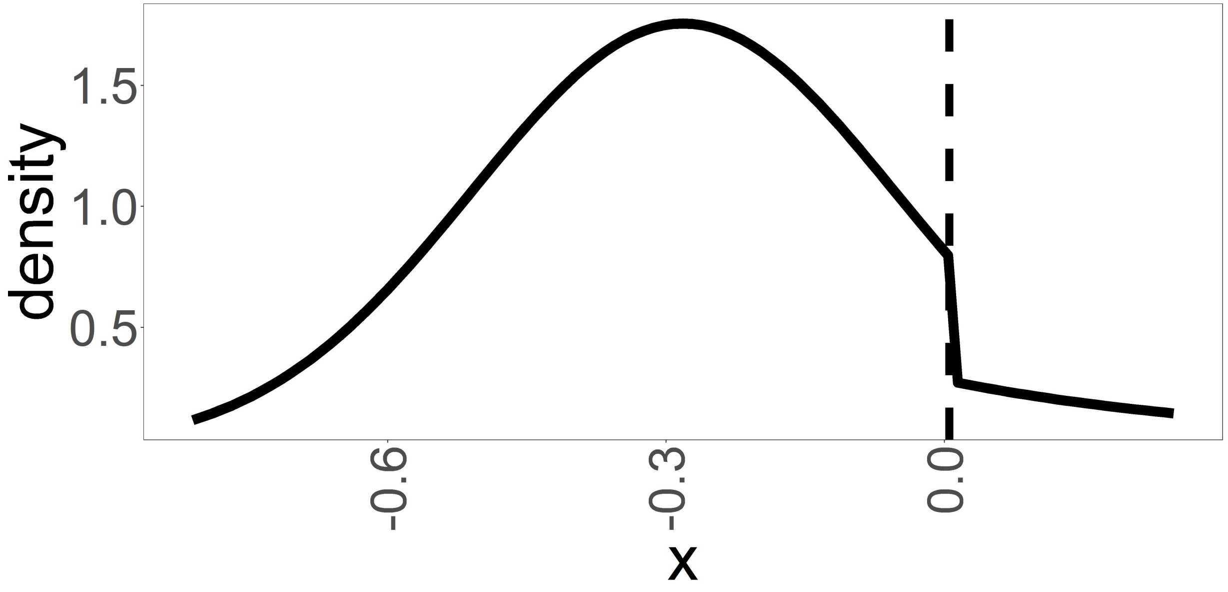

Figure: Density of a mixture model for Switzerland (1900–2020), $u = 0.005$, $\mu = -0.282$, $\sigma = 0.227$, $\xi = 0.094$, $\theta = 0.395$.

A jump model is a further alternative.

Chen and Cox [2009]

$$\hat{\kappa}_{t+1} - \hat{\kappa}_t = d + e_{t+1} + N_{t+1} Y_{t+1} - N_t Y_t$$

with $d \in \mathbb{R}$, $e_t \sim \mathcal{N}\left(0, \sigma^2\right)$, $Y_t \sim \mathcal{N}\left(m, s^2\right)$ and $N_t \sim B(1, p)$.

- transitory jumps are modeled but longer shock phases are possible if $N_t = N_{t+1} = 1$,
- model calibration by conditional maximum likelihood estimation,
- other distributions for the jump severity Y_t are possible, for example Pareto (upward jumps)/Beta (downward jumps) [Deng et al., 2012], which is inspired by the Kou and Wang [2004] option pricing model.

Regime switching allows for transitions between a "normal" and a "shock" state.

Milidonis et al. [2011]

$$\hat{\kappa}_{t+1} - \hat{\kappa}_t \sim \begin{cases} \mathcal{N}\left(d_1, (\sigma_1)^2\right) \text{ if } \rho_t = 1 \\ \mathcal{N}\left(d_2, (\sigma_2)^2\right) \text{ if } \rho_t = 2 \end{cases},$$

with binary Markov chain ρ_t.

- model calibration via maximum likelihood estimation,
- initialization such that state 2 is the shock state, i.e., we expect $d_2 > d_1$ and/or $\sigma_2 > \sigma_1$.

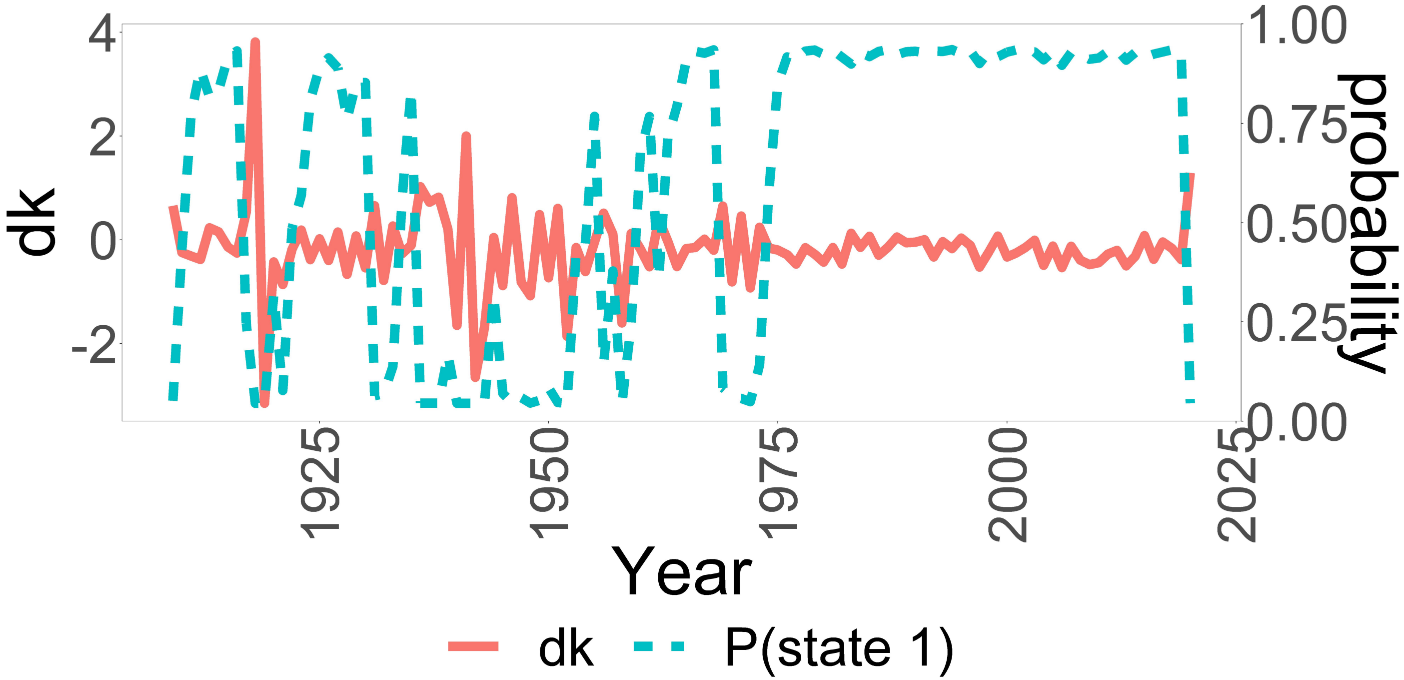

Figure: Regime switching model probability $\mathbb{P}(\rho_t = 1)$ for Spain, 1908–2020.

Calibrating the non-shock parameters on a shorter time period improves forecasts.

- originally, all model parameters calibrated on the same data set, e.g., years 1900-2020,
- but: LC assumes β_x, d, σ constant over the whole calibration period,
- idea: calibrate only "shock parameters" on a long time period and remaining LC parameters on a shorter time period,
- this approach clearly improves point forecasts in a backtest.

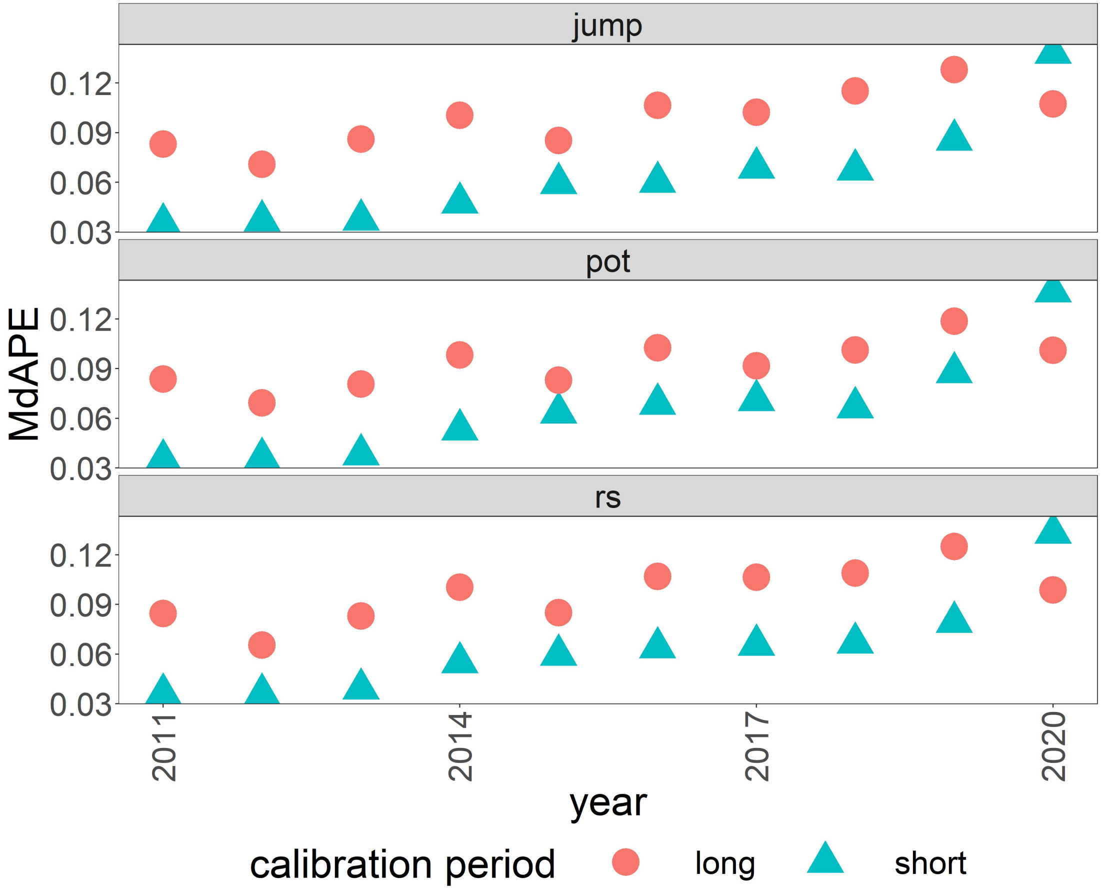

Figure: Relative errors in point forecasts, comparing calibration on 1981–2010 (blue triangles) or all available data (red circles).

The mixture distribution approach yields the best goodness of fit among "realistic" models.

- goodness of fit measured by

$$\mathsf{BIC} := -2\,\mathsf{L}_{\max} + \log\left(n_{\mathrm{obs}}\right) \cdot \mathsf{n}_{\mathrm{par}},$$

  striking a balance between high likelihood and parsimony,

- intervention model has low BIC but ignores possibility of events similar to COVID-19 in the future,

- regime switching and jump model are strongly penalized for their higher number of parameters,

- mixture distribution approach ("pot") works very well in most countries.

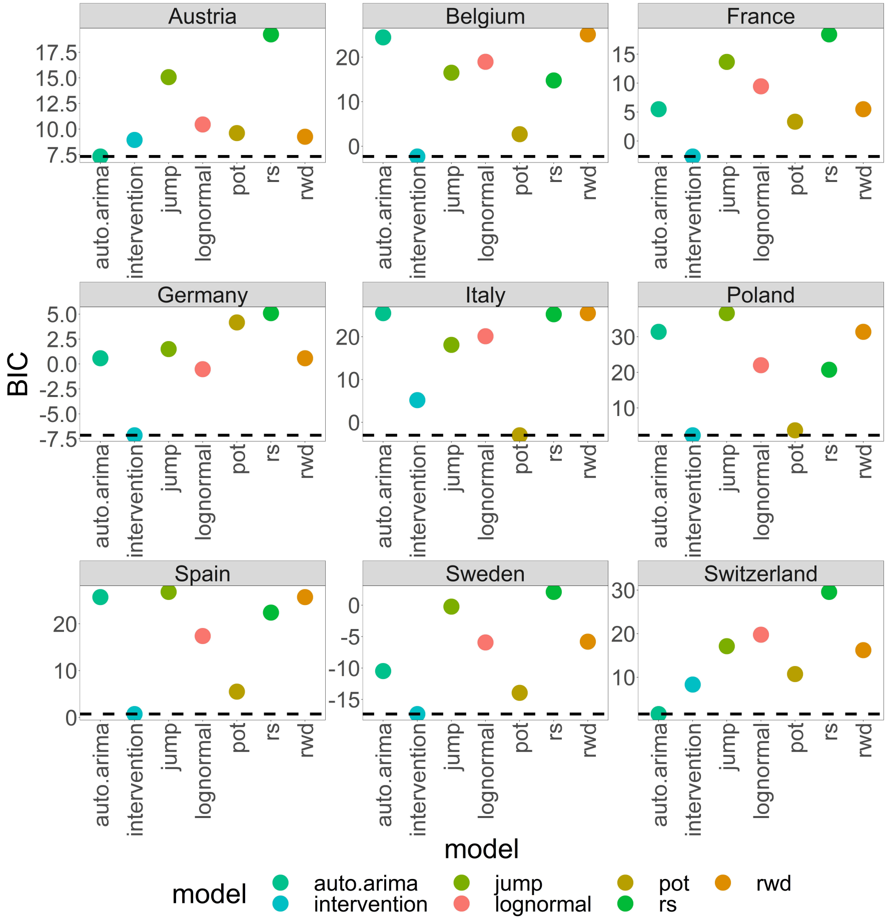

Figure: BIC for models calibrated on data from 1991 to 2020. The black dashed line is at the level of the best-fitting model.

A backtest shows that the normal distribution underestimates prediction uncertainty.

- calibrate different models on yearly data from 1981 to 2010,
- perform out-of-sample evaluation on data from 2011 to 2020,
- point forecast errors measured by

$$\mathrm{MdAPE}(t) := \mathrm{median}_{x,i}\left\{\frac{\left|\hat{m}^i_{x,t} - m^i_{x,t}\right|}{m^i_{x,t}}\right\} \cdot 100\%$$

  are similar and (plausibly) increasing over time,
- interval forecast errors measured by

$$\mathrm{PICP}(t) := \frac{1}{N}\sum_{x,i} \mathbb{1}_{\left\{m^i_{x,t} \in \left[\hat{m}^{i,\,\mathrm{lower}}_{x,t}, \hat{m}^{i,\,\mathrm{upper}}_{x,t}\right]\right\}}$$

  strongly depend on the model, with approaches based on a normal distribution assumption heavily underestimating prediction uncertainty.

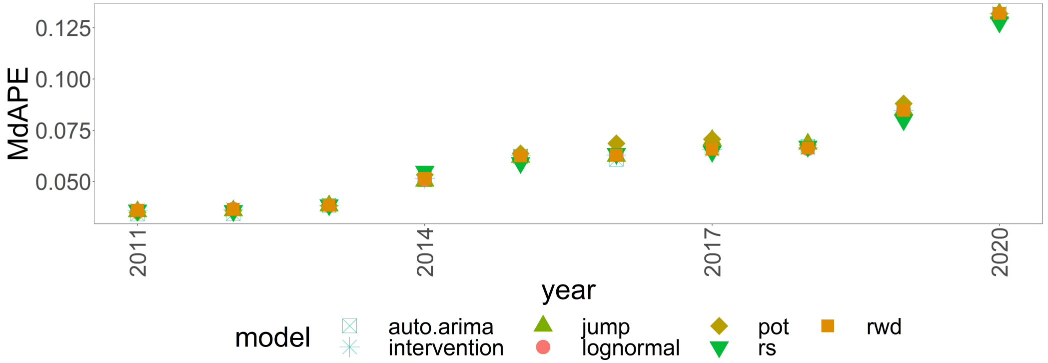

Figure: Relative errors in point forecasts.

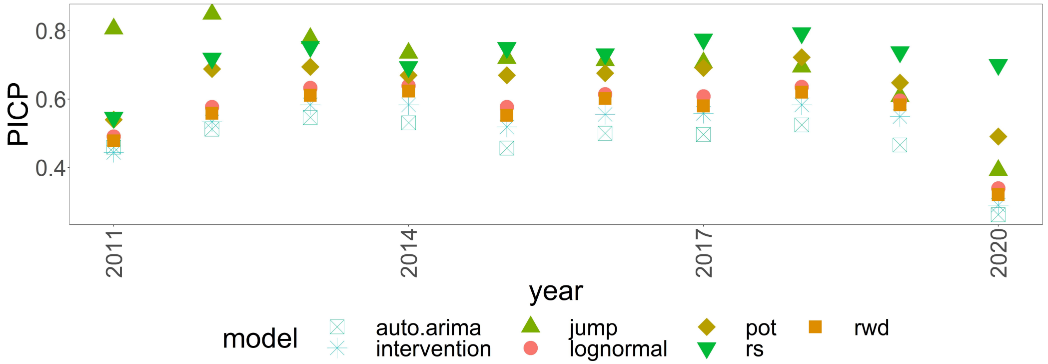

Figure: Prediction interval coverage probabilities.

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We have seen that mortality shocks have significant influence on the LC model and sketched some ways how to deal with this.

Further research (or patience) is needed regarding the questions

- how many COVID-19 deaths will be observed in the future and whether COVID-19 will cause new cohort effects due to selection [Cairns et al., 2020],
- how 2020, and possibly also 2021, mortality data should be treated in other mortality models,
- how age-specific impacts of mortality shocks could be modeled as well.

For more details, see our preprint at `ssrn.com/abstract=3835907`.

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