Demand for retirement products: An analysis of individual welfare

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Motivation I

Aging society:

complex
challenging
societally important

- One possible way to deal with longevity risk: Innovative retirement products such as Group self-annuitization, pooled annuity funds and tontines ([Piggott et al., 2005], [Valdez et al., 2006], [Stamos, 2008], [Sabin, 2010], [Donnelly et al., 2013, Donnelly et al., 2014], and [Milevsky and Salisbury, 2015]).

Motivation II: an observation

- Actuarial literature on retirement products:
  - Many innovative retirement products whose benefits depend on the mortality realizations have been discussed in the last several years (e.g. [Milevsky and Salisbury, 2015] and [Chen et al., 2019])
- (Public) economic literature on retirement products
  - annuity puzzle: mainly regular annuities are discussed (e.g. [Yaari, 1965], [Davidoff et al., 2005], [Hu and Scott, 2007], [Bommier et al., 2011])
- This paper:
  - We aim to combine these two streams of the literature
  - We focus on the changes in an agent's and social planners' choices for retirement products (if mortality-linked products are additionally provided), and wealth transfers among agents.

What do we specifically analyze in this paper?

- We extend [Bommier et al., 2011]
  - [Bommier et al., 2011] study the demand for regular annuities under temporal risk aversion: Individuals might be risk-averse with respect to their future lifetime.
- by allowing access to an innovative plan depending on the realized survival probabilities, like tontines (in addition to traditional annuities)
- and (in social planner's problem considering two groups of individuals), allowing for differential mortality, wealth and safety loadings
- we study the demand for retirement products both from individual and social planner's viewpoint

Main questions answered in the paper

- Under temporal risk neutrality and actuarially fair pricing, it is optimal to invest all wealth in annuities ([Yaari, 1965]).

  How is the optimal demand for the retirement product under temporal risk aversion?
- Will the introduction of tontines crowd out the existence of annuities?
  - Do tontines gain or lose attractiveness under temporal risk aversion and can they be a beneficial supplement to annuities for agents exhibiting temporal risk aversion?
- In the social planner's problem, how does the wealth transfer occur? What are crucial deciding factors?

Selected results

- Temporal risk aversion can **increase the demand** for tontines even in an actuarially fair pricing framework.
- It is almost always optimal for individuals to invest in both tontines and annuities $\rightarrow$ no crowding-out effect
- Mostly, the living-longer-type benefits from the living-shorter-type ([Weil and Fisher, 1974], [Brown, 2002] and [Bommier et al., 2011] for annuities)
  - Note: it holds if the wealth level of the living-longer-type does not exceed that of the the living-shorter-type.
  - However: If more wealth is assigned to the living-longer type, the wealth transfer can be reversed.

What comes as next?

- Main assumptions
- Individual retirement problems
  - a market exclusively with tontines
  - a market exclusively with annuities
  - a market with annuities and tontines
- Government's problem
  - a market exclusively with tontines
  - a market exclusively with annuities
  - a market with annuities and tontines

Assumption about remaining lifetime

- Let T_x be the random remaining lifetime of an agent aged x years and $\{\mu_{x+t}\}_{t\geq 0}$ be the (possibly stochastic) force of mortality process of the agent.
- Further, we define the conditional survival probabilities $S(t) := e^{-\int_0^t \mu_{x+s}\mathrm{d}s}$.
- We assume that

$$S'(t) = \frac{\mathrm{d}S(t)}{\mathrm{d}t} < 0$$

  and that $S(\infty) = 0$.
- We define the survival curve: $s(t) = \mathbb{E}\left[S(t)\right]$

Products under consideration: Annuity and Tontine

- Annuity: Deterministic payoff $c(t)$ until death.
- Tontine with fully diversified unsystematic mortality risk: Payoff $d(t)/S(t)$ until death, where $d(t)$ is deterministic.
  - This is the tontine design introduced by [Milevsky and Salisbury, 2015] with an infinitely large pool size.
- An agent who purchases both products receives the payoff

$$\mathcal{C}(t) = c(t) + \frac{d(t)}{S(t)}.$$

Expected value principle used for premium calculation

- Assume that $r(t)$ is a deterministic interest rate curve.
- Premium of the tontine:

$$P_0^d = (1+\delta_d)\int_0^\infty e^{-\int_0^t r(s)\mathrm{d}s}\cdot d(t)\mathrm{d}t.$$

- Premium of the annuity:

$$P_0^a = (1+\delta_a)\int_0^\infty e^{-\int_0^t r(s)\mathrm{d}s}\cdot s(t)\cdot c(t)\mathrm{d}t.$$

- An agent purchasing both pays $P_0^a + P_0^d$.

Individual preferences

- Following [Bommier et al., 2011], the expected lifetime felicity is then given by

$$EU(\boldsymbol{C}) = \mathbb{E}\left[\Phi\left(\int_0^{T_x} \alpha(t)u(C(t))\mathrm{d}t\right)\right].$$

  - where $\alpha(t)$ is a subjective discount factor, $u(\cdot)$ a strictly increasing and concave utility function, and $\boldsymbol{C} = \{C(t)\}_{t\geq 0}$ a (possibly stochastic) payoff of a retirement product.
  - Temporal risk aversion characterized by Φ: Φ is a twice differentiable function satisfying $\Phi'' \leq 0$ and $u(\cdot)$ a CRRA utility function.

Individual problem with tontines

- A policyholder who purchases a tontine with an initial wealth W_0 faces the following optimization problem:

$$\max_{d(t)} \mathbb{E}\left[\Phi\left(\int_0^{T_x} \alpha(t) u\left(\frac{d(t)}{S(t)}\right) \mathrm{d}t\right)\right]$$

$$\text{subject to } (1+\delta_d)\int_0^{\infty} e^{-\int_0^t r(s)\mathrm{d}s} \cdot d(t)\mathrm{d}t \le W_0.$$

Approximation method

- Following [Bommier, 2006] (applied for annuity), we obtain the following additive approximation for the objective function in tontine case:

$$
\begin{aligned}
EU(\boldsymbol{C}) &\approx \mathbb{E}\left[\int_0^\infty S(t)^\gamma \beta(t)\alpha(t)u\left(d(t)\right)\mathrm{d}t,\right] \\
&= \int_0^\infty \kappa_\gamma(t)\alpha(t)u\left(d(t)\right)\mathrm{d}t,
\end{aligned}
$$

where $\kappa_\gamma(t) := \mathbb{E}\left[S(t)^\gamma \beta(t)\right]$, and

$$
\beta(t) := -\frac{1}{S(t)}\int_t^\infty S'(s)\cdot\Phi'\left(\int_0^s \alpha(\tau)d\tau\right)ds.
$$

Theorem: optimal tontine payoffs

The optimal tontine payoff is given by

$$d^*(t) = \left(\frac{(1+\delta_d)\,\lambda_d e^{-\int_0^t r(s)ds}}{\kappa_\gamma(t)\alpha(t)} \right)^{-\frac{1}{\gamma}},$$

where the Lagrangian multiplier λ_d is, for all $\gamma > 0$, given by

$$\lambda_d = \left(\frac{1+\delta_d}{W_0} \int_0^\infty e^{-\int_0^t r(s)ds} \cdot \left(\frac{(1+\delta_d)\, e^{-\int_0^t r(s)ds}}{\kappa_\gamma(t)\alpha(t)} \right)^{-\frac{1}{\gamma}} \mathrm{d}t \right)^{\gamma}.$$

For $\gamma \neq 1$, the expected discounted lifetime felicity is then given by

$$U_d = \frac{\lambda_d}{1-\gamma} \cdot \frac{W_0}{1+\delta_d}.$$

Numerical illustration: Stochastic mortality law

- Following e.g. [Lin and Cox, 2005], we apply a random shock ϵ taking values in $(-\infty, 1)$ to the Gompertz mortality rates:

$$\mu_{x+t} = (1-\epsilon)\frac{1}{b}e^{\frac{x+t-m}{b}},$$

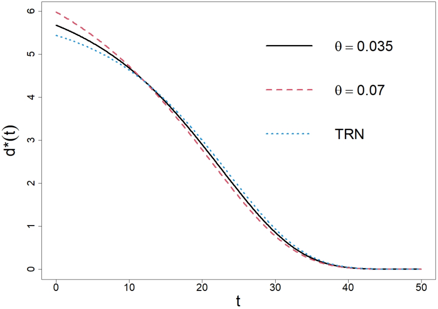
  where m is the modal age at death and b is the dispersion coefficient.

Numerical example

Initial wealth $W_0 = 100$	Utility function 1 $\Phi(y) = \frac{1}{\theta} - \frac{1}{\theta} e^{-\theta y}$, $\theta = 0.035$	Utility function 2 $u(y) = \frac{y^{1-\gamma}}{1-\gamma}$, $\gamma = 3$
Risk-free rate $r(t) = r = 0.01$	Subjective discount rate $\alpha(t) = e^{-rt}$	Safety loading $\delta_d = \delta_a = 0$
Initial age $x = 65$	Longevity shock $\epsilon \sim \mathcal{N}_{(-\infty,1)}\left(-0.0035, 0.0814^2\right)$	Gompertz law $m = 88.721$, $b = 10$

Table: Base case parameters. θ is chosen similarly to [Bommier et al., 2011]. We assume a constant risk-free interest rate close to zero (cf. [Statista, 2019]). $\delta_a = \delta_d = 0$ means that we are in an actuarially fair pricing framework. Concerning the longevity shock ϵ, we follow [Chen et al., 2019]. The Gompertz parameters are taken from [Milevsky and Salisbury, 2015].

Optimal tontine payoff



Optimal payout function $d^*(t)$ over time under temporal risk aversion and temporal risk neutrality (TRN).

Individual problem with annuities

- A policyholder who purchases an annuity with an initial wealth W_0 faces the following optimization problem:

$$\max_{c(t)} \mathbb{E}\left[\Phi\left(\int_0^{T_x} \alpha(t)u\left(c(t)\right)\mathrm{d}t\right)\right]$$

$$\text{subject to } (1+\delta_a)\int_0^{\infty} e^{-\int_0^t r(s)\mathrm{d}s}\cdot s(t)\cdot c(t)\mathrm{d}t \leq W_0.$$

Theorem: optimal annuity payoffs

$$c^*(t) = \left((1+\delta_a) \frac{\lambda_a e^{-\int_0^t r(s)ds}}{\overline{\beta}(t)\alpha(t)} \right)^{-\frac{1}{\gamma}},$$

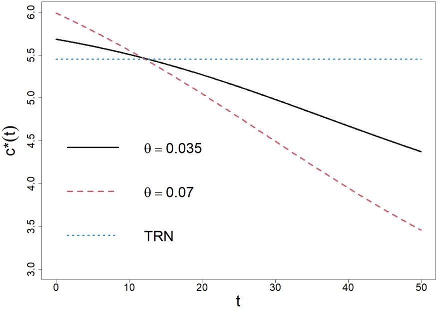
where the Lagrangian multiplier λ_a is, for all $\gamma > 0$, given by

$$\lambda_a = \left(\frac{1+\delta_a}{W_0} \int_0^\infty e^{-\int_0^t r(s)ds} \cdot s(t) \cdot \left(\frac{(1+\delta_a)\, e^{-\int_0^t r(s)ds}}{\overline{\beta}(t)\alpha(t)} \right)^{-\frac{1}{\gamma}} \mathrm{d}t \right)^{\gamma}.$$

with $\overline{\beta}(t) = -\frac{1}{s(t)} \int_t^\infty s'(s) \cdot \Phi' \left(\int_0^s \alpha(\tau) d\tau \right) ds$.
For $\gamma \neq 1$, the expected discounted lifetime felicity is then given by

$$U_a = \frac{\lambda_a}{1-\gamma} \cdot \frac{W_0}{1+\delta_a}.$$

Optimal annuity payoff



Optimal payout function $c^*(t)$ over time under temporal risk aversion and temporal risk neutrality (TRN)

Combined investment

- A policyholder who purchases an annuity along with a tontine with an initial wealth W_0 faces the following optimization problem:

$$\max_{c_{ad}(t),d_{ad}(t)} \mathbb{E}\left[\int_0^\infty S(t)\beta(t)\alpha(t)u\left(c_{ad}(t)+\frac{d_{ad}(t)}{S(t)}\right)\mathrm{d}t\right]$$

$$\text{subject to } (1+\delta_a)\int_0^\infty e^{-\int_0^t r(s)\mathrm{d}s}s(t)c_{ad}(t)\mathrm{d}t$$

$$+(1+\delta_d)\int_0^\infty e^{-\int_0^t r(s)\mathrm{d}s}\cdot d_{ad}(t)\mathrm{d}t \le W_0.$$

- Can be solved by numerical procedures.

Theorem

Let $\kappa_\gamma(t) := \mathbb{E}\left[S(t)^\gamma \beta(t)\right]$ *and*

$$\overline{\beta}(t) := -\frac{1}{s(t)} \int_t^\infty s'(s) \cdot \Phi' \left(\int_0^s \alpha(\tau) \mathrm{d}\tau \right) \mathrm{d}s.$$

(i) *If and only if*

$$\delta_a \leq \underline{\delta}_a := \inf_{t \geq 0} \frac{\overline{\beta}(t)}{\mathbb{E}\left[\beta(t)\right]}(1+\delta_d) - 1, \tag{1}$$

the optimal solution is given by $c_{ad}(t) = c^*(t)$, $d_{ad}(t) = 0$, *i.e. it is optimal to invest all the initial wealth in the optimal annuity.*

Theorem

(ii) *If and only if*

$$\delta_a \geq \overline{\delta}_a := \sup_{t \geq 0} \frac{\kappa_{\gamma+1}(t)}{\kappa_\gamma(t)s(t)}(1+\delta_d) - 1, \tag{2}$$

the optimal solution is given by $d_{ad}(t) = d^*(t)$, $c_{ad}(t) = 0$, *i.e. it is optimal to invest all the initial wealth in the optimal tontine.*

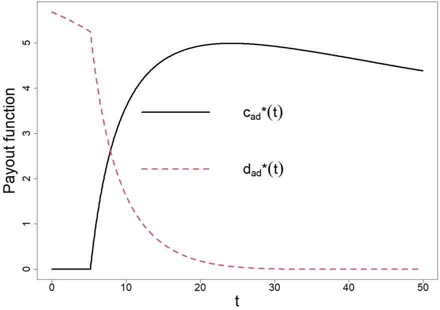
(iii) *If both condition* (1) *and* (2) *are not fulfilled, it is optimal for the agent to invest positive fractions of wealth in both the annuity and the tontine.*

Some numbers: critical bounds of the annuity loading

θ	TRN	0.035	0.07	0.105	0.14
$\underline{\delta}_a$	0	-0.0014	-0.0027	-0.0039	-0.0051
$\overline{\delta}_a$	∞	∞	∞	∞	∞

Analysis of the bounds (1) and (2) depending on the temporal risk aversion θ.

Optimal annuity and tontine payoff



Optimal payout functions in the portfolio for an agent exhibiting temporal risk aversion.

Government's problem: assumption

- We consider two groups of agents $i = H, L$, where we assume

$$\mu_{x+t}^{H}(\omega) < \mu_{x+t}^{L}(\omega)$$

for all $\omega \in \Omega$, i.e. agents of type H have (on average) a longer remaining lifetime than type-L-agents.

- We introduce the survival probabilities for a member of the groups as $s_i(t) = \mathbb{E}\left[S_i(t)\right] = \mathbb{E}\left[e^{-\int_0^t \mu_{x+t}^{(i)}}\right]$, $i = H, L$.
- n_H and n_L are fractions of agents in the two groups.

Social planner's optimization problem with tontines

The problem of the social planner is then given by

$$
\begin{aligned}
&\max_{d_L(t), d_H(t)} n_L \int_0^\infty \kappa_\gamma^L(t)\alpha(t)u\left(d_L(t)\right)\mathrm{d}t \\
&+ n_H \int_0^\infty \kappa_\gamma^H(t)\alpha(t)u\left(d_H(t)\right)\mathrm{d}t \\
\text{subject to } & n_L(1+\delta_d^L)\int_0^\infty e^{-\int_0^t r(s)ds}\cdot d_L(t)\mathrm{d}t \\
&+ n_H(1+\delta_d^H)\int_0^\infty e^{-\int_0^t r(s)ds}\cdot d_H(t)\mathrm{d}t \\
&\leq n_L W_0^L + n_H W_0^H.
\end{aligned}
$$

Theorem

The optimal tontine payoffs are given by

$$d_i^*(t) = \left(\frac{\left(1+\delta_d^{(i)}\right) \lambda_d^G e^{-\int_0^t r(s)ds}}{\kappa_\gamma^{(i)}(t)\alpha(t)} \right)^{-\frac{1}{\gamma}},$$

where the Lagrangian multiplier λ_d^G is given by

$$\lambda_d^G = \left(\left(n_L W_0^L + n_H W_0^H\right)^{-1} \left(n_L \left(1+\delta_d^H\right) \int_0^\infty e^{-\int_0^t r(s)ds} \cdot \left(\frac{\left(1+\delta_d^L\right) e^{-\int_0^t r(s)ds}}{\kappa_\gamma^L(t)\alpha(t)} \right)^{-\frac{1}{\gamma}} \mathrm{d}t \right.\right.$$
$$\left.\left. + n_H \left(1+\delta_d^H\right) \int_0^\infty e^{-\int_0^t r(s)ds} \cdot \left(\frac{\left(1+\delta_d^H\right) e^{-\int_0^t r(s)ds}}{\kappa_\gamma^H(t)\alpha(t)} \right)^{-\frac{1}{\gamma}} \mathrm{d}t \right) \right)^{\gamma}.$$

The collective expected discounted lifetime felicity is then given by

$$U_d^G = \frac{\lambda_d^G}{1-\gamma} \left(n_H W_0^H + n_L W_0^L \right).$$

A numerical illustration

We rely on the base case parameters summarized in Table 1 and consider the additional parameters summarized in Table 2.

Pool size	Safety loadings	Modal ages
$n_H = n_L = 0.5$	$\delta_d^{(i)} = \delta_a^{(i)} = 0$	$m_H = 88.721,\ m_L = 84$

Table: Base case parameter setup.

Numerical illustration for wealth transfer I

	H	L
$W_0^H = W_0^L = 100$, $n_H = n_L = 0.5$		
$\theta = 0.035$	53.60	46.40
TRN	54.41	45.59
$W_0^H = 200$, $W_0^L = 100$, $n_H = n_L = 0.5$		
$\theta = 0.035$	80.40	69.60
TRN	81.61	68.39

Present values of consumption for agents of the two groups.

Conclusion

- We study the optimal demand for retirement products in the presence of temporal risk aversion and markets offering annuities and mortality-linked products like tontines.
  - for a single agent
  - for a social planner
- Risk aversion with respect to the time of death can **increase the demand** for tontines even in an actuarially fair pricing framework.
  - Agents subject to temporal risk aversion prefer to invest positive fractions of wealth in both annuities and tontines to full annuitization even in perfect annuity markets.
- The wealth ratio of the two groups turns out to be the main driving factor for the direction of the wealth transfers.

Many thanks for your attention!

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