Modeling the Reserving Cycle Using Fourier Methods

IAA Colloquia 2021

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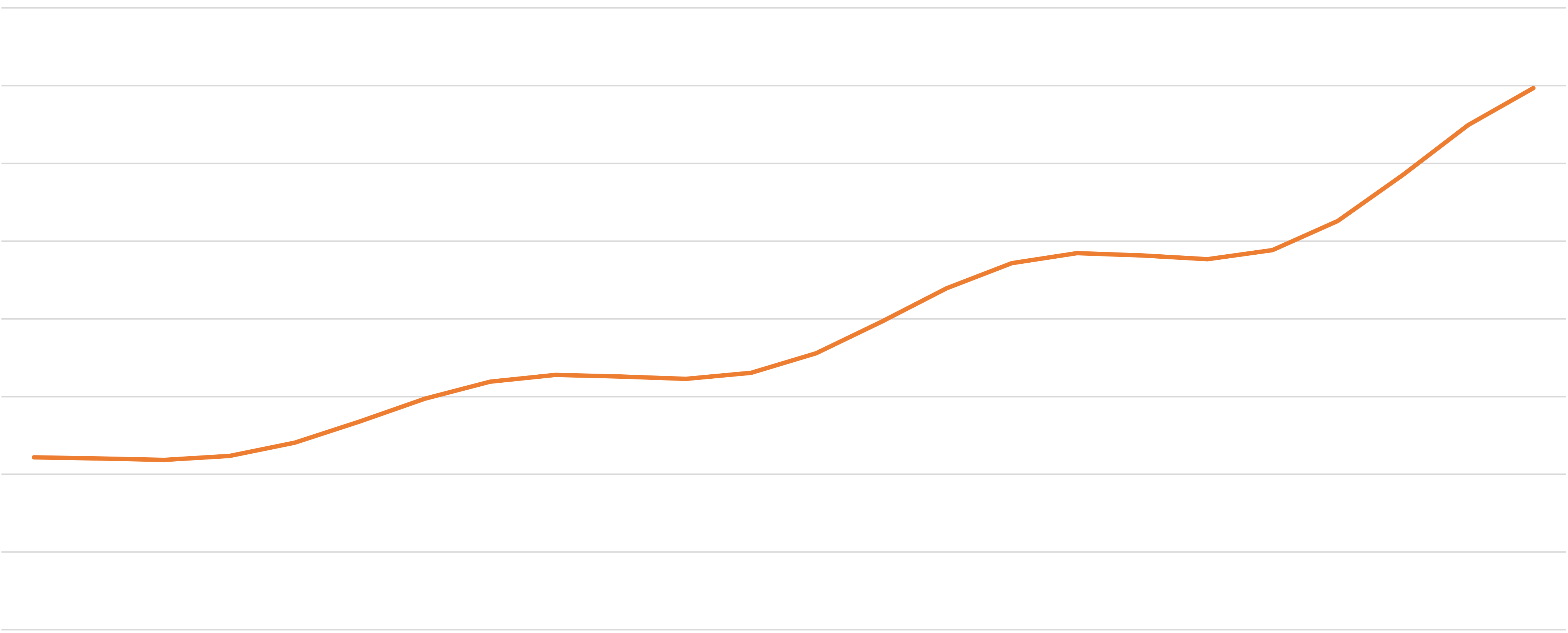
Constant Inflation

Variable Inflation



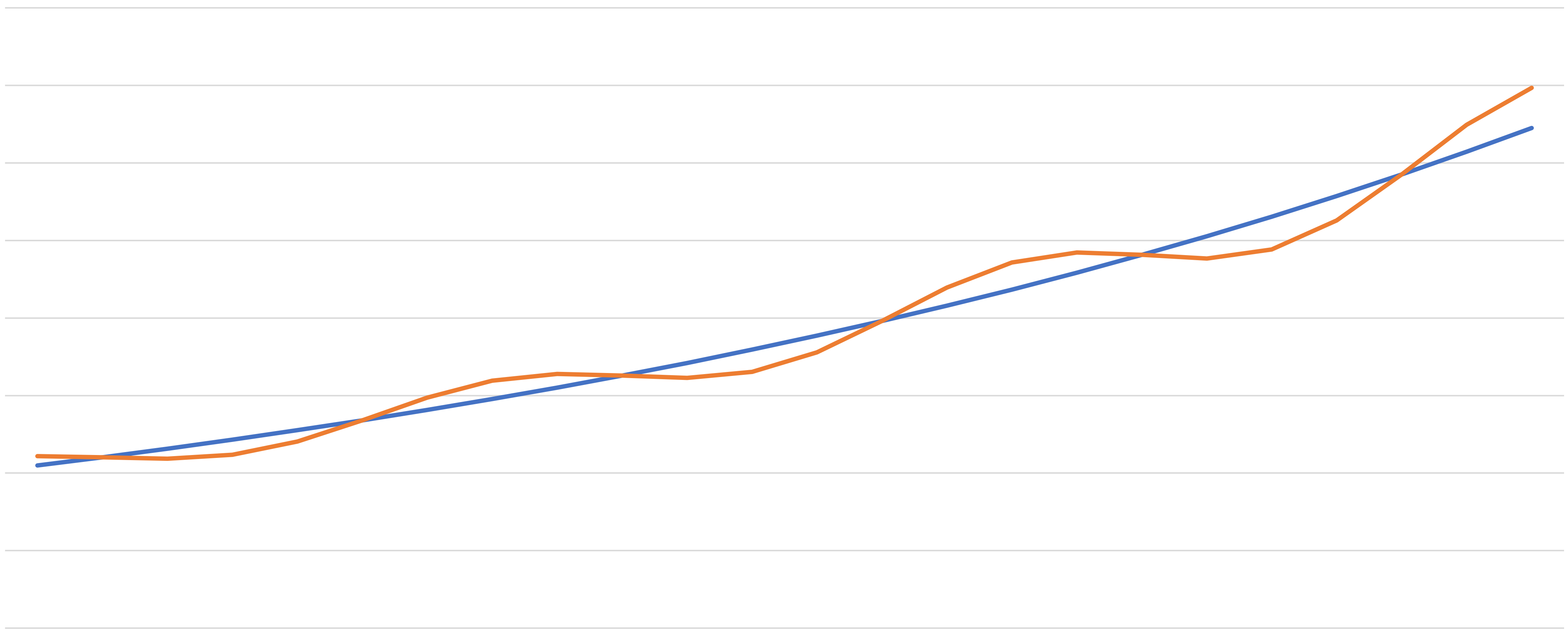
Inflation Risk in Reserving

- Variations in inflation cause diagonal effects in our triangles

- These variations in inflation and the associated diagonal effects constitute systemic risk in loss reserving. They are not diversifiable as they are often caused by external events.

- Can we model and quantify this systemic risk?

Trend/Cycle



Modeling the Trend/Cycle

- In most actuarial analyses we focus on modeling the exponential trend
- In this case our focus is on modeling the cycle
- I have chosen to use a Fourier Series because it is an appropriate method for modeling cyclical processes

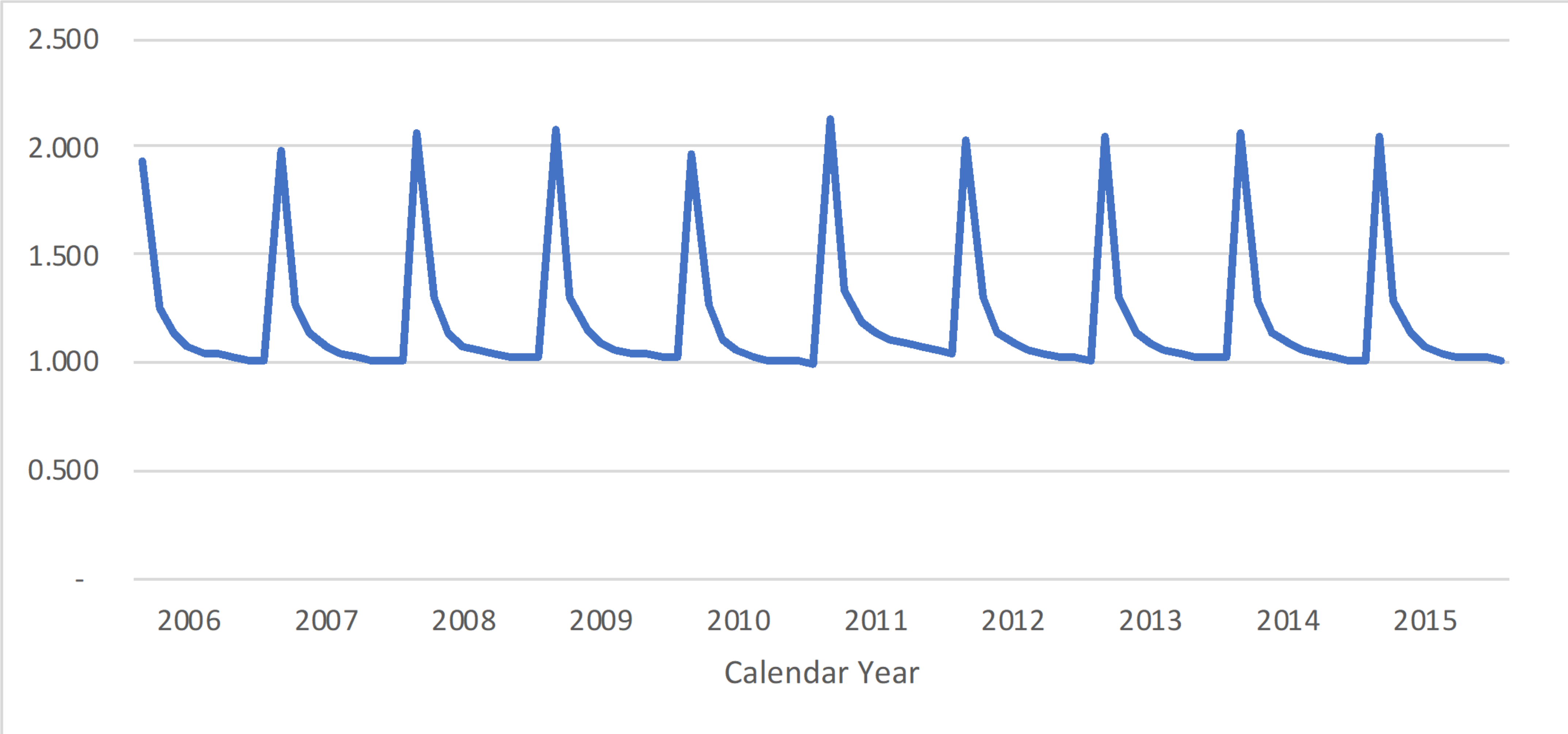
Data Organization

- We want to model the diagonal effects on a two-dimensional data array by AY and age
- The data must be reorganized as a periodic sequence so that we can compute the coefficients of a Fourier Series
- Start with a parallelogram of data and consider each CY diagonal as an individual cycle, i.e. sort it by CY and age

US Workers Compensation
Paid Link Ratios

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
AY									
1997									1.011
1998								1.018	1.010
1999							1.026	1.013	1.022
2000						1.037	1.019	1.021	1.036
2001					1.048	1.025	1.033	1.034	1.002
2002				1.082	1.044	1.043	1.042	1.005	1.052
2003			1.141	1.084	1.058	1.043	1.012	1.061	1.019
2004		1.261	1.138	1.084	1.061	1.014	1.075	1.026	1.022
2005	1.931	1.274	1.136	1.092	1.026	1.087	1.029	1.027	1.016
2006	1.981	1.297	1.161	1.053	1.101	1.038	1.036	1.020	1.017
2007	2.056	1.307	1.113	1.135	1.058	1.044	1.029	1.022	
2008	2.072	1.267	1.198	1.088	1.058	1.040	1.026		
2009	1.971	1.344	1.143	1.089	1.058	1.033			
2010	2.122	1.299	1.142	1.089	1.046				
2011	2.030	1.297	1.148	1.078					
2012	2.049	1.289	1.134						
2013	2.068	1.284							
2014	2.041								

US Workers Compensation
Paid Link Ratios as a Periodic Sequence



A Note on Data Type

In the previous slides I chose to use AY link-ratios as the input data. Link ratios mod-out exposure changes and the average trend.

Partial severities and partial pure premiums will also work, because they also mod-out exposure changes. But an additional step is required to find the least-squares trend.

Steps in Fourier Analysis

1. Calculate Fourier Coefficients a_n and b_n

2. Separate Signal from Noise

3. Calculate Variance Using Parseval's Equation

Separating Signal from Noise
(by Analogy to Color Vision)

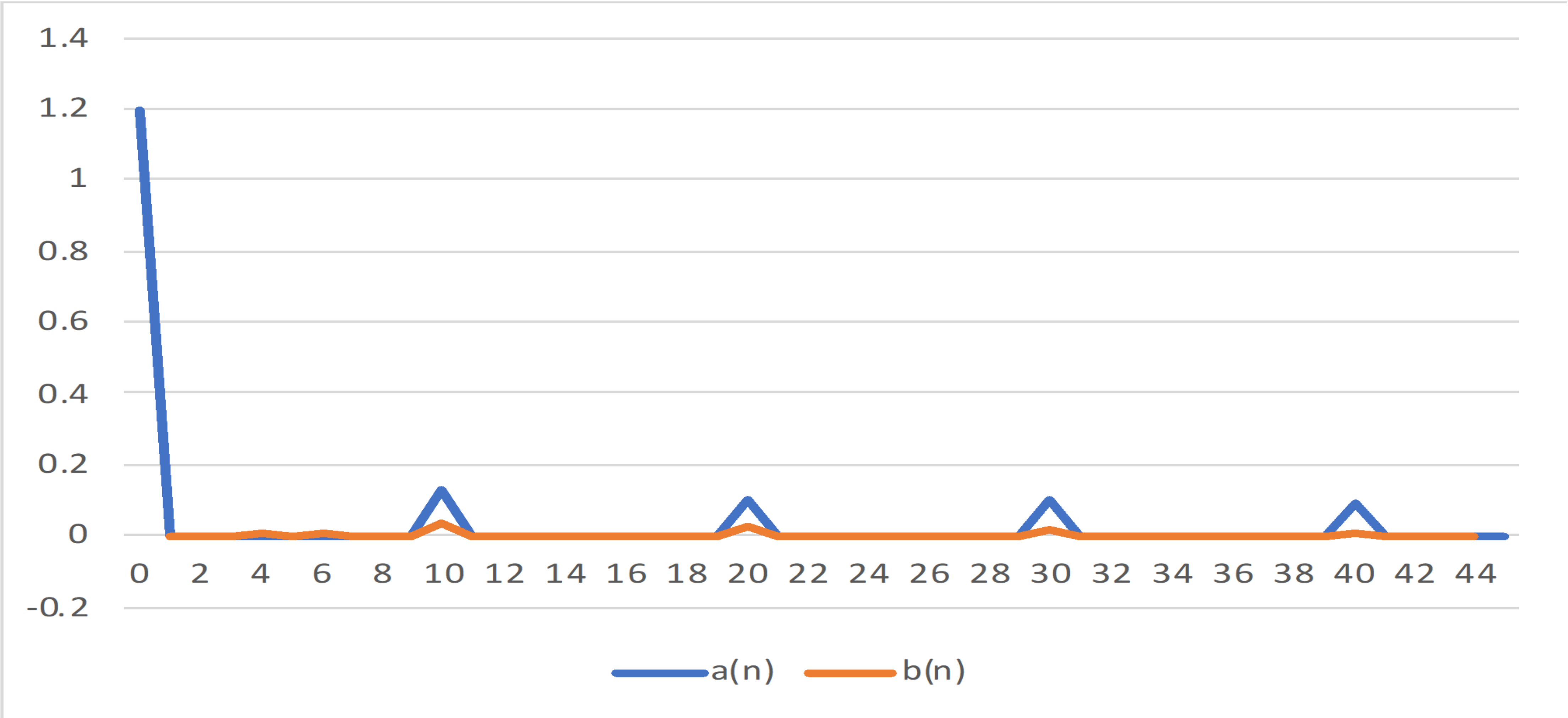
- Find something green

  Look for a particular frequency or a narrow band of frequencies

- What color is it?

  Identify the frequencies that are the most significant

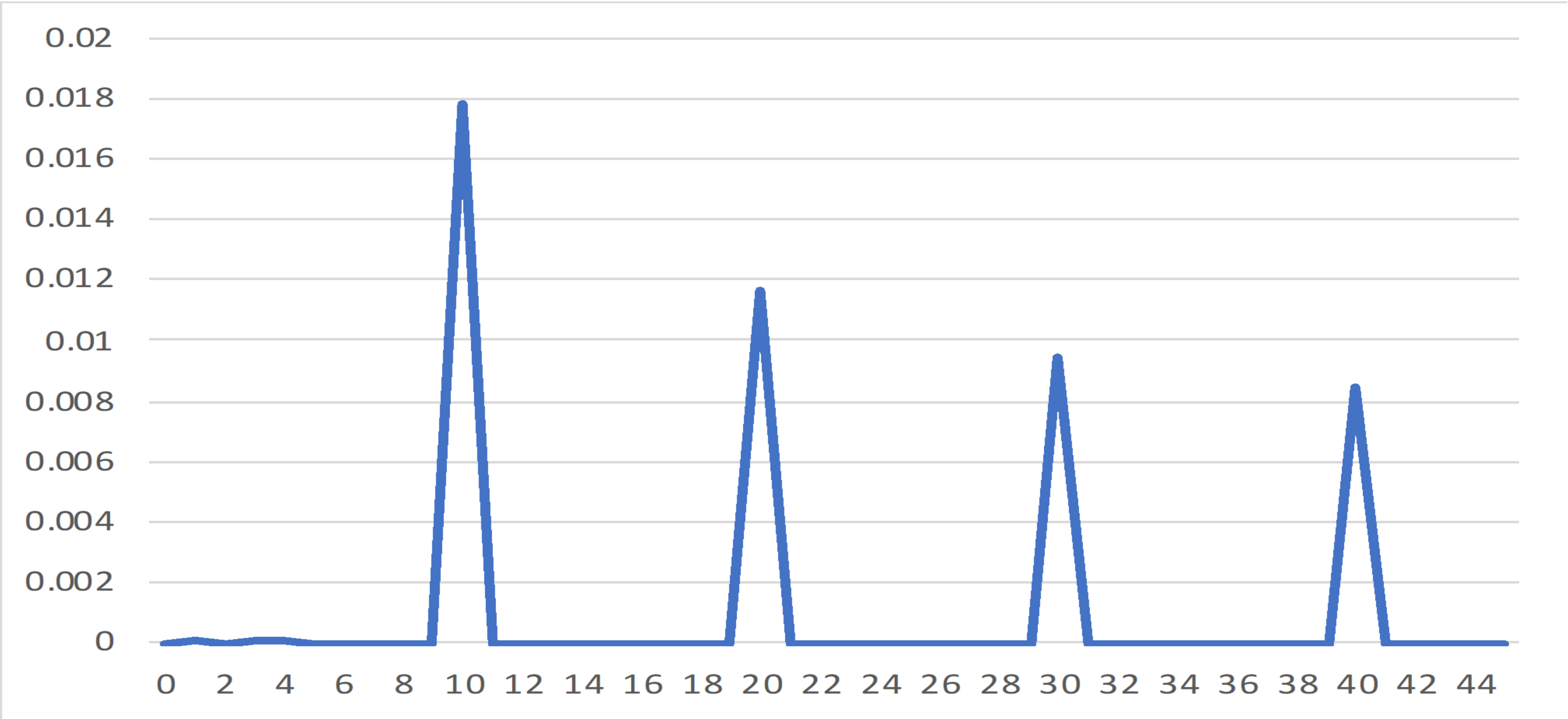
Fourier Coefficients for WC Link-Ratios



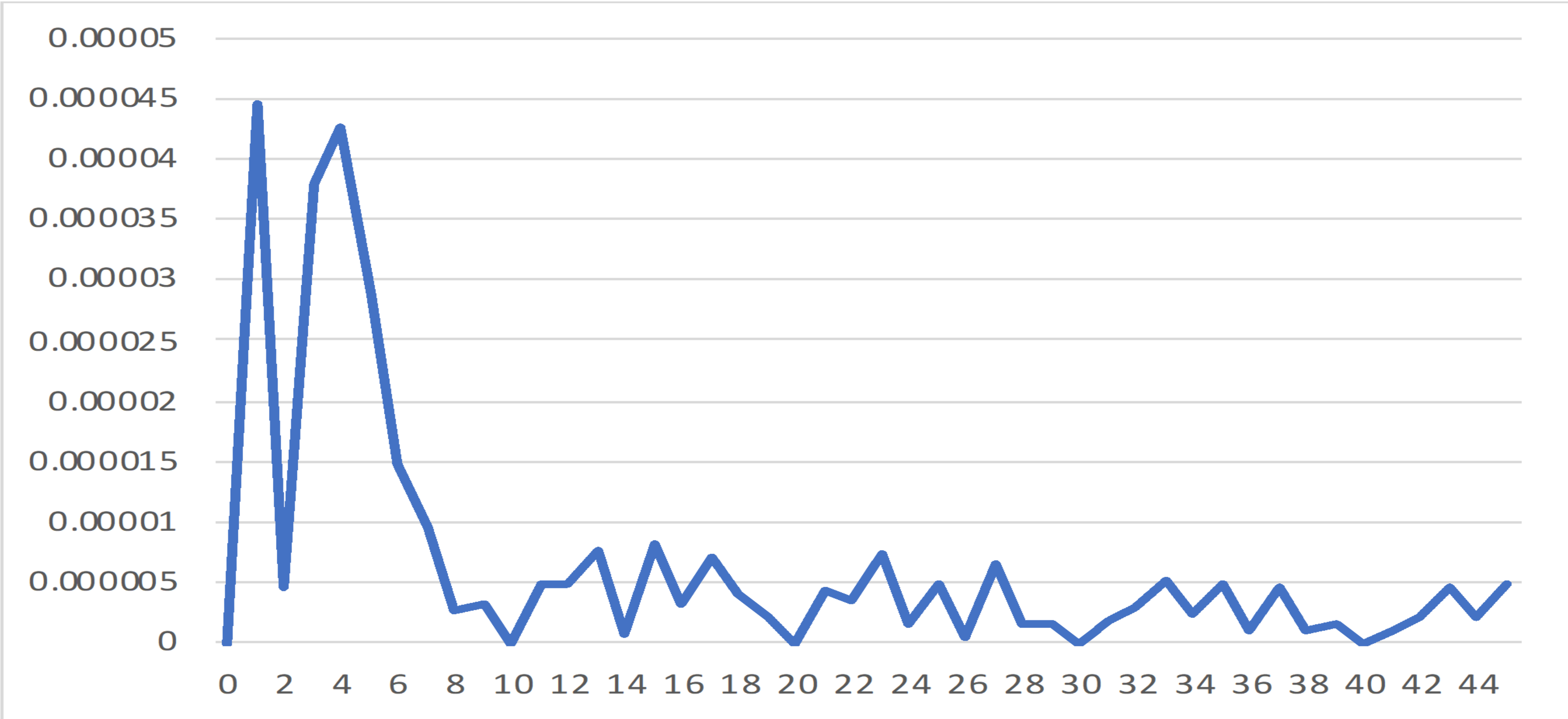
Modifications to Aid Interpretation

- Omit the constant term a_0
- Square the coefficients
- Add $a_n^2 + b_n^2$ term-by-term

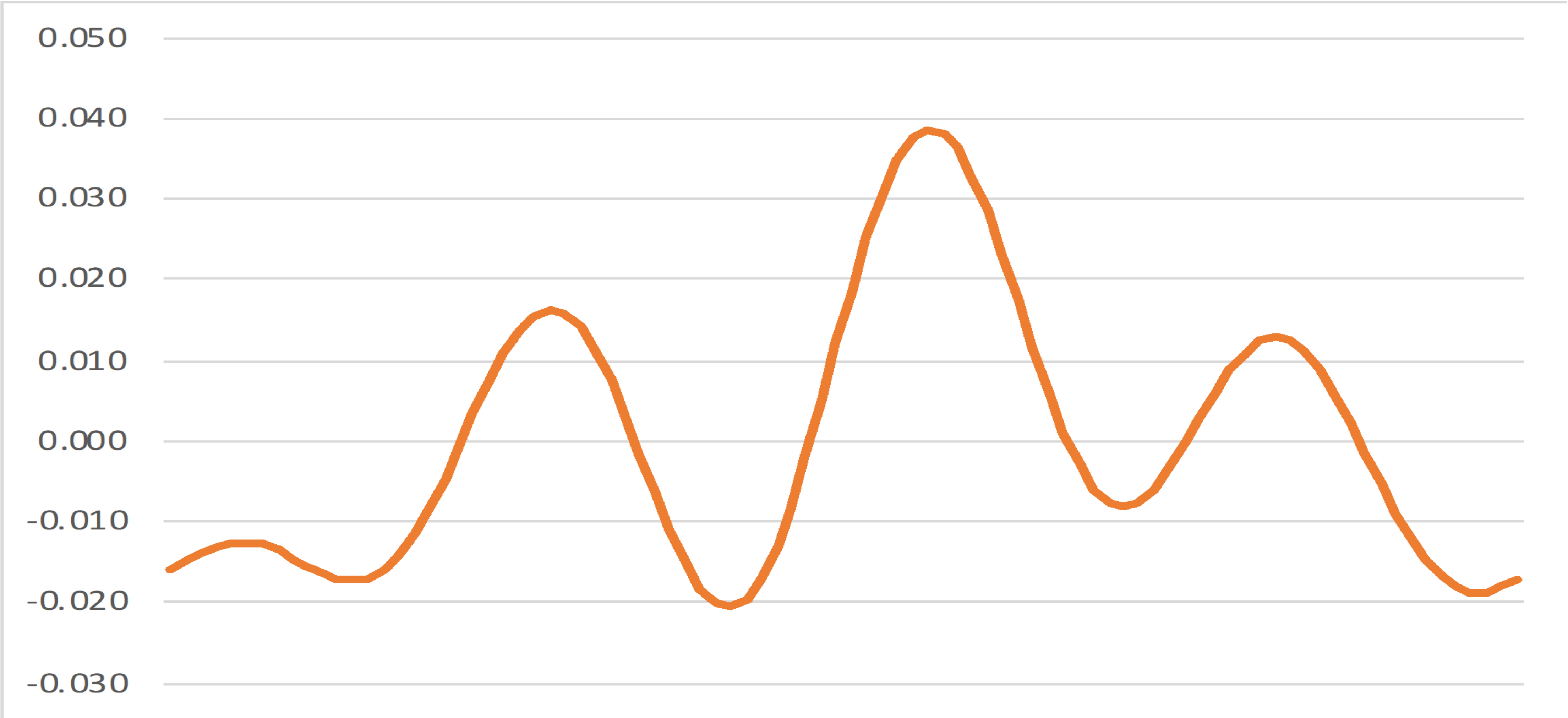
$(a_n^2 + b_n^2)$ for WC Link Ratios



$(a_n^2 + b_n^2)$ for Noise Frequencies



Modeled Reserving Cycle – 4 Terms



Parseval's Equation

$$Total\ Sum\ of\ Squares = \sum_{n>0} ({a_n}^2 + {b_n}^2)$$

Systemic Reserve Risk

$$\frac{SS\ Cycle}{SS\ Noise} = .94$$

For Nationwide WC, the cyclical component is the dominant component of noise, as we should expect. For individual companies, the significance of the cyclical component depends on the size of the company, line of business, retained limits, and the time period chosen.

I use the Fourier Series as a Visualization of the Fourier Transform

The Discrete Fourier Transform (DFT) interpolates data with a complex wave function. But working in the complex plane is difficult to visualize.

A Fourier Series interpolates data with a real wave function.

- From a Fourier Series to DFT is a variation of Euler’s formula:

$$e^{ix} = \cos x + i \sin x$$

- From the DFT to Fourier Series is a variation on the identities

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2}$$

Why Should We Learn to Use Transforms in Actuarial Science?

Because we already think this way sometimes:

Return Time = Wavelength