A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio

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Topics of the presentation

- An insurance portfolio consists of various contracts; it is heterogeneous.
- In the first example it is shown which effects the heterogeneity has got for the different policies: One contract can benefit from another contract.
- We want to give an opportunity to analyse and measure the interaction of different contracts.
- Definition of a collective bonus/ collective malus to identify contracts, which subsidize in expectation (ex ante) or have subsidized (ex post) other contracts
- Relationship between a balanced balance sheet and the concept of collective bonus
- Usage of the theoretical results to show how much one contract benefit from another contract, for example if they share the default risk of the policy sponsor

Motivation

– Insurance portfolios are heterogeneous, i.e. consist of many different contracts
  - In a typical German life insurance portfolio annual return guarantees range from 0% to 4%.
  - Re-designed variations of participating products only provide some guaranteed interest rate at contract's maturity.

– Interdependence between different insurance contracts, because of
  - joint management, for example sharing the same bonus reserve
  - same credit risk of the insurance sponsor
  - investment of the premiums in the same reference portfolio

- We provide a framework to analyse such interdependence.
- We give an opportunity to check, if a contract is actually calculated fair in a heterogeneous insurance portfolio.

Agenda

Agenda

Motivating example

- Focus on the financial part of the products
- Interdependence or profit sharing due to biometric or expense results are neglected.
- Insurer’s portfolio consists of two single premium insurance contracts (A and B) with a duration of T=10 years.
- Both contracts are concluded at time $t_0 = 0$.
- The single premiums $P_0^A = 1$ and $P_0^B = 1$ are invested in the same reference portfolio *F*.
- Policyholder account at maturity of contract A (**guaranteed maturity benefit**):

$$L_{10}^A = P_0^A \cdot \max\left(e^{g_A \cdot 10}; \prod_{i=1}^{10} \max\left(1; 90\% \cdot \frac{F_i}{F_{i-1}}\right)\right)$$

- Policyholder account at maturity of contract B (**year-by-year guarantee**):

$$L_{10}^B = P_0^B \cdot \prod_{i=1}^{10} \max\left(e^{g_B}; 90\% \cdot \frac{F_i}{F_{i-1}}\right)$$

Motivating example

- At time T=10
  - $L_{10}^{A|B}$ is paid out to the policyholder.
  - the shareholder gets the difference between the value of the reference portfolio and the policyholder accounts:

$$E_{10} := F_{10} - L_{10}^{A} - L_{10}^{B}$$

- The financial market is arbitrage free and complete.
- Value of an insurance contract (with risk-free asset $(B_t)_{t\geq 0}, \mathrm{B}_0 = 1; \mathrm{Q}$: equivalent pricing measure):

$$V_0^{A|B} = E_Q\left[L_{10}^{A|B} \cdot B_{10}^{-1}\right]$$

- We call a contract “fair” if:

$$V_0^{A|B} = L_0^{A|B} \left(= P_0^{A|B} = 1\right)$$

- Black-Scholes model ($\mathrm{r} = 4\%$, $\sigma = 16\%$) for the reference portfolio F:
  - Individually fair guarantee of contract A is $g_A = 2.88\%$.
  - Individually fair guarantee of contract B is $g_B = 0.81\%$.

Motivating example

– The insurance portfolio should be priced fair as a total, i.e.

$$E_Q\left[\left(L_{10}^A + L_{10}^B\right) \cdot B_{10}^{-1}\right] = P_0^A + P_0^B$$

– To demonstrate the interaction of the contracts, the guaranteed rate of the first contract g_A is changed which lead to situation where contract A is not fair any more, e.g.

$$E_Q[L_{10}^A \cdot B_{10}^{-1}] > P_0^A.$$

➔ The policyholder of contract A in expectation receives a benefit higher than the single premium paid.

➔ Because of the assumption that the whole insurance portfolio should be priced fair, the policyholder of contract B has to pay a price for this.

The policyholder of contract A gets a **collective bonus** and the policyholder of contract B a **collective malus**.

Agenda

The concept of the collective bonus

Model Setup

– Simplified market consistent balance sheet of the insurance company:

ASSETS	LIABILITIES
market value of the assets A_t	**present value of future profits** $PVFP_t$
	best estimate of liabilities BE_t

– In a first step, we do not require that the total value of assets and liabilities equal each other, i.e. assets and liabilities are defined independent of each other.

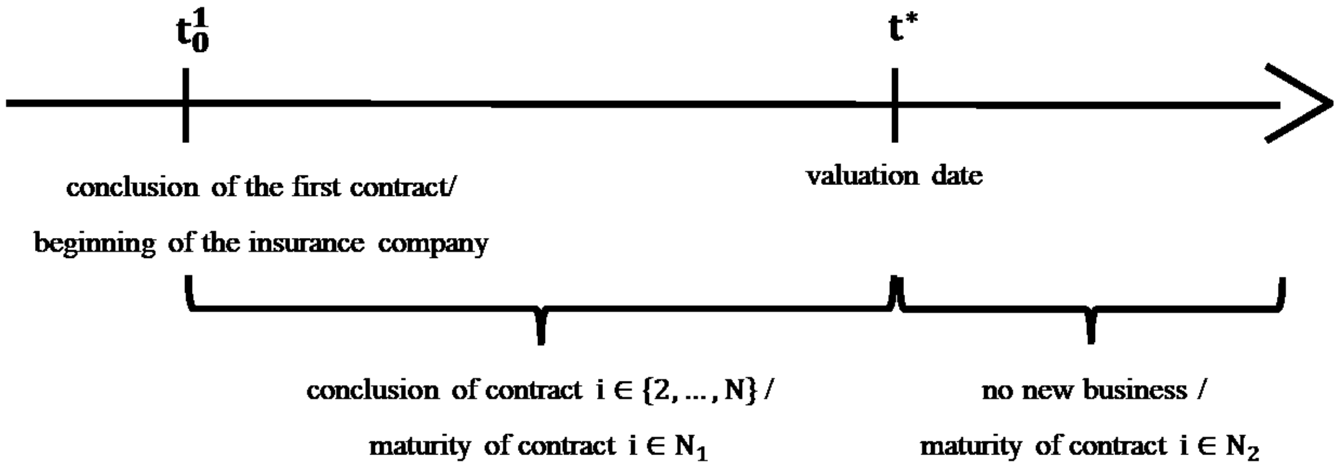
– The values are calculated on the probability space (Ω,F,Q) where (Ω,F) is a measurable space and Q the equivalent pricing measure in this space.

The concept of the collective bonus

Model Setup

Best Estimate of Liabilities

- The insurance company is considered at time t^*.
- Insurer's portfolio consists of N potentially different contracts.
- Any new business after time t^* is not considered here.
- The contract $i \in \{1, \dots, N\}$ is concluded at time $t_0^i \in \mathbb{N}$ and has a term of $T^i \in \mathbb{N}$ years.
- At time t^* …
  - …the contracts $i \in N_1, N_1 \subseteq \{1, \dots, N\}$ already matured.
  - …the insurance portfolio consists of the contracts $i \in N_2, N_2 := \{1, \dots, N\} \backslash N_1$.



The concept of the collective bonus

Model Setup

Best Estimate of Liabilities

- The term of the contracts is discretized in full years.
- Annual premium payments, beginning of the year: A premium of contract $i \in \{1, \dots, N\}$ at time t is denoted by $P_t^i > 0$.
- Charges, surrender, solvency capital or mortality are neglected.
- Policyholder account L_t^i at time $t = t_0^i$ is equal to the first premium payment.
- Each year, the policyholder account is changed by the function f and the new premium.

$$L_t^i = \begin{cases} \sum_{l=t_0^i}^{t} P_l^i \cdot \prod_{j=l}^{t-1} \left(1 + f_{j+1}^i\right), \mathrm{t} \in \left\{t_0^i, \dots, t_0^i + T^i - 1\right\} \\ \sum_{l=t_0^i}^{t_0^i + T^i - 1} P_l^i \cdot \prod_{j=l}^{t_0^i + T^i - 1} \left(1 + f_{j+1}^i\right), \qquad \mathrm{t} = t_0^i + T^i \\ 0, \qquad t < t_0^i \text{ and } t > t_0^i + T^i \end{cases}$$

- The annual credit f would typically not coincide with the insurance company's market return.
- At time $t_0^i + T^i$, the maturity benefit $L_{t_0^i + T^i}^i$ is paid out to the policyholder.

The concept of the collective bonus

Model Setup

Best Estimate of Liabilities

- The financial market is arbitrage-free and complete.
- Using the equivalent martingale measure Q and the risk-free asset $(B_t)_{t\geq t^*}$ with $B_{t^*}=1,$ the best estimate of liabilities at time t^* is defined as follows:

$$BE_{t^*} = \sum_{i\in N_2} E_Q\left[B^{-1}_{t_0^i+T^i}\cdot L^i_{t_0^i+T^i} - \sum_{j=t^*+1}^{t_0^i+T^i-1} B_j^{-1}\cdot P_j^i\right]$$

Present Value of Future Profits

- The cash flows of the shareholder in the period $(j-1,j)$ are denoted by X_j.
- Consistent with the valuation of liabilities, cash flows to or from the shareholder are directly before the considered time t^*.

$$PVFP_{t^*} = E_Q\left[\sum_{j=t^*+1}^{T} B_j^{-1}\cdot X_j\right]$$

- T is the expiration of the last contract of the insurance company.

The concept of the collective bonus

Model Setup

Market Value of Assets

- The premiums of all contracts are invested in the same reference portfolio $(F_t)_{t \geq t_0^1}$.
- We do not specify the assets in the portfolio, but we assume that the net market return of the reference portfolio in any year t is denoted by $\frac{F_t}{F_{t-1}}$.
- At time $t = t_0^1$ the first premiums $P_{t_0^1}$ are paid; no initial payment by the shareholder:

$$A_{t_0^1} = P_{t_0^1}$$

- Market value of assets at time t^*:

$$A_{t^*} = \left(A_{t^*-1} - \sum_{i \in N_1} L^i_{t_0^i + T^i} \cdot 1_{\{t^*-1 = t_0^i + T^i\}} \right) \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*}$$

- The indicator function $1_{\{.\}}$ is one if and only if contract i ends at time $t^* - 1$.
- The payment to the policyholder is directly after the considered time t^*.

The concept of the collective bonus

Model Setup

Simplified market consistent balance sheet of the insurance company based on the model:

ASSETS	LIABILITIES
market value of the assets $A_{t^*} = \left(A_{t^*-1} - \sum_{i\in N_1} L^i_{t^i_0+T^i} \cdot 1_{\{t^*-1=t^i_0+T^i\}}\right)\frac{F_{t^*}}{F_{t^*-1}} + \sum_{i\in N_2} P^i_{t^*} - X_{t^*}$	**present value of future profits** $PVFP_{t^*} = E_Q\left[\sum_{j=t^*+1}^{T} B_j^{-1} \cdot X_j\right]$
	best estimate of liabilities $BE_{t^*} = \sum_{i\in N_2} E_Q\left[B^{-1}_{t^i_0+T^i} \cdot L^i_{t^i_0+T^i} - \sum_{j=t^*+1}^{t^i_0+T^i-1} B_j^{-1} \cdot P^i_j\right]$

The total value of assets not necessarily equals the total value of liabilities.

Agenda

The concept of the collective bonus

Definition of the collective bonus and relation to the balance sheet

- The main idea of the collective bonus is to quantify, whether an insurance contract **has earned** (ex post) or **in expectation will earn** (ex ante) more than an investment in the reference portfolio.
- The collective bonus of contract $i \in \{1, \dots, N\}$ at time t^* consists of two parts:
  - **Ex post collective bonus**:

$$CB_{t^*}^i(\text{ex post}) = L_{t^*}^i - \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}}$$

  - **Ex ante collective bonus**:

$$CB_{t^*}^i(\text{ex ante}) = E_Q\left[\sum_{j=t^*}^{t_0^i+T^i-1} B_{j+1}^{-1} \cdot L_j^i \cdot \left(1 + f_{j+1}^i - \frac{F_{j+1}}{F_j}\right)\right]$$

- If $t^* > t_0^i + T^i$ the collective bonus is equal to the ex post collective bonus.

The concept of the collective bonus

Definition of the collective bonus and relation to the balance sheet

– Collective bonus of the shareholder:

- **Ex ante**: present value of future profits represents all future gains or losses of the shareholder

$$CB_{t^*}^{sh}(\text{ex ante}) = PVFP_{t^*}$$

- **Ex post**: all previous cash flows $X_j,\ j \leq t^*$, are paid out; there is no equity account at time t^*

$$CB_{t^*}^{sh}(\text{ex post}) = \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}$$

– Relation of the concept of the collective bonus to the balance sheet items $A_t, PVFP_t$ and BE_t focussing on the identity

$$A_{t^*} = PVFP_{t^*} + BE_{t^*}$$

Proposition: Assume the model setup. At time t^*: The market value of assets is equal to the sum of present value of future profits and best estimate of liabilities if and only if the sum of the collective boni is zero, i.e.

$$A_{t^*} = BE_{t^*} + PVFP_{t^*} \Leftrightarrow CB_{t^*} + CB_{t^*}^{sh} = 0.$$

Agenda

Applications

Motivating example

- Contract A (guaranteed maturity benefit):

$$L_{10}^{A} = \max\left(e^{g_A \cdot 10}; \prod_{i=1}^{10} \max\left(1; 90\% \cdot \frac{F_i}{F_{i-1}}\right)\right)$$

- Contract B (year-by-year guarantee):

$$L_{10}^{B} = \prod_{i=1}^{10} \max\left(e^{g_B}; 90\% \cdot \frac{F_i}{F_{i-1}}\right)$$

- The contracts share the same reference portfolio F.
- Black Scholes model for the development of assets with a risk free rate of interest $r = 4\%$ and a volatility of the risky asset of $\sigma = 16\%$
- If the contracts are priced fairly on a collective basis, $E_Q\left[\left(L_T^A + L_T^B\right) \cdot D_T\right] = P_0^A + P_0^B$, fair parameter combinations of both contracts depend on each other.
- We change the guarantee g_A of contract A:

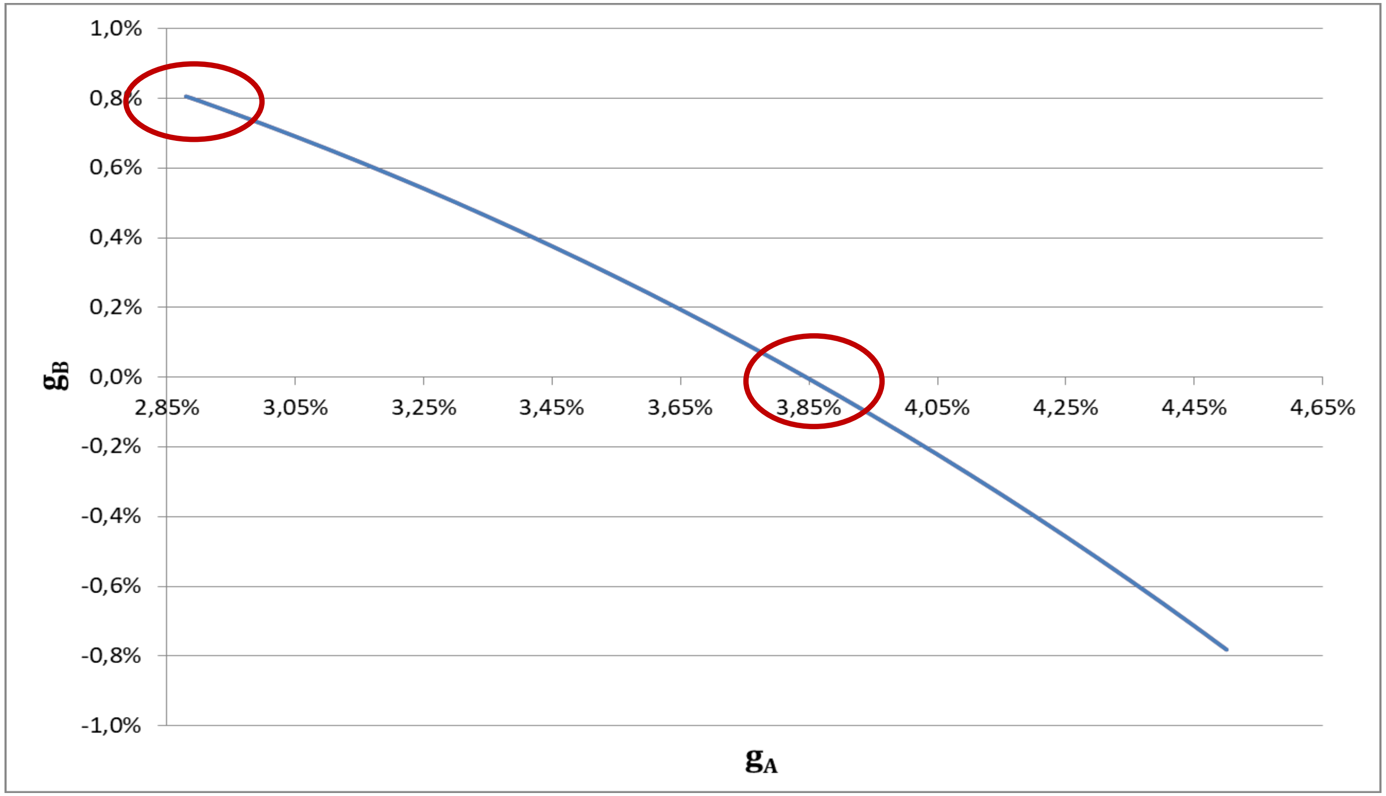
➔ This must be compensated by a change of the guarantee of contract B.

Applications

Motivating example

Pairs of guarantees (g_A, g_B), so that the insurance portfolio is priced fair, i.e.

$$E_Q\left[\left(L_T^A + L_T^B\right) \cdot D_T\right] = P_0^A + P_0^B.$$



Applications

Motivating example

– Calculation of the collective bonus of contract A and B at contract's inception, i.e. $t^* = 0$, depending on the guarantee g_A, resp. g_B.

g_A	**2.88%**	**3.00%**	**3.20%**	**3.40%**	**3.60%**	**3.80%**	**4.00%**	**4.20%**	**4.40%**
CB_0^A	0.00%	0.54%	1.52%	2.59%	3.75%	4.99%	6.32%	7.73%	9.23%
g_B	**0.81%**	**0.73%**	**0.58%**	**0.42%**	**0.24%**	**0.05%**	**-0.17%**	**-0.40%**	**-0.65%**
CB_0^B	0.00%	-0.54%	-1.52%	-2.59%	-3.75%	-4.99%	-6.32%	-7.73%	-9.23%

– The collective boni is given as the percentage of the single premium of each contract.

The collective bonus of contract A and the collective bonus of contract B offset each other.

Agenda

Applications

CB of an insurance contract added to existing an insurance portfolio

- Hieber et al (2016): *Fair valuation of cliquet-style return guarantees in a heterogeneous life insurance portfolio*
  - **Policyholder account**:
    - Single premium; conclusion at t_0^i; duration T^i
    - Annually guaranteed minimum return g^i

$$L_{t+1}^i = L_t^i \cdot \max\left(\exp\left(g^i\right), \left(\frac{F_{t+1}}{F_t}\right)^{\alpha^i(t+1)}\right);\ L_{t_0^i}^i = P_{t_0^i}^i$$

  - **Terminal bonus**:

$$TB_{t_0^i+T^i} := \beta^i \cdot \max\left(P_{t_0^i}^i \cdot \frac{F_{t_0^i+T^i}}{F_{t_0^i}} - L_{t_0^i+T^i}^i; 0\right)$$

  - **Time of default**:

$$\tau := \inf\{t \in \{1, \dots, T | A_t < \sum_{i=1}^{N} L_t^i\}$$

  - **In case of default**:

$$\frac{L_\tau^i}{L_\tau} \cdot A_\tau$$

Applications

CB of an insurance contract added to existing an insurance portfolio

– Life insurance portfolio at time $t^* = 0 =$ 01/01/2015:

i	$e^{g^i} - 1$	start (t_0^i)	maturity $(t_0^i + T^i)$	L_0^i	β^i	α^i	$P_{t_0^i}^i$
1	1.75 %	01/01/2014	31/12/2034	8000	0.30	0.90	7681
2	2.25 %	01/01/2010	31/12/2029	17000	0.20	0.90	13969
3	2.75 %	01/01/2005	31/12/2024	13000	0.20	0.90	8773
4	3.25 %	01/01/2001	31/12/2023	14000	0.20	0.90	7871
5	4.00 %	01/01/1995	31/12/2023	20000	0.10	0.90	7512
6	3.50 %	01/01/1990	31/12/2022	18000	0.10	0.90	4987

– At time t^*:

- Liabilities: 90,000
- Equity: 18,000
- Market value of the assets: 108,000

– Reference portfolio F: Black Scholes model $(r = 3\%$ and $\sigma = 20\%)$

- Share λ of the risk-free asset; $(1 - \lambda)$ of the risky asset
- $\lambda = 90\%$

Applications

CB of an insurance contract added to existing an insurance portfolio

– Ex ante collective bonus:

$$CB_0^i(ex\ ante) := E_Q\left[\sum_{j=0}^{t_0^i+T^i-1} \mathrm{B}_{\mathrm{j+1}}^{-1} \cdot L_j^i \cdot \left(1 + f_{j+1}^i - \frac{F_{j+1}}{F_j}\right)\right]$$

$CB_0^1(\mathbf{ex\ ante})$	$CB_0^2(\mathbf{ex\ ante})$	$CB_0^3(\mathbf{ex\ ante})$	$CB_0^4(\mathbf{ex\ ante})$	$CB_0^5(\mathbf{ex\ ante})$	$CB_0^6(\mathbf{ex\ ante})$
90.07	564.30	597.69	944.58	2273.77	1393.67

– Every contract has an ex ante collective bonus. The shareholder has to compensate this and has an ex ante collective malus: $PVFP_0 = -5{,}864$

In expectation the shareholder will subsidize the policyholders.

Applications

CB of an insurance contract added to existing an insurance portfolio

– New individually fair priced contracts are added to the existing insurance portfolio:
  - Contract A: Time to maturity $T^* = 20$, end date $31/12/2034$; $(\alpha^*, \beta^*, P_0^*, g^*) = (0.80, 0.20, 8000, 2.37\%)$.
  - Contract B: Time to maturity $T^* = 20$, end date $31/12/2034$; $(\alpha^*, \beta^*, P_0^*, g^*) = (0.90, 0.20, 8000, 1.55\%)$.

i	A	1	2	3	4	5	6	$PVFP_0$
CB_0^i (ex ante)	435.48 (435.48)	89.43 (-0.64)	562.41 (-1.89)	595.15 (-2.54)	941.34 (-3.24)	2268.86 (-4.91)	1388.87 (-4.80)	-6281.54 (-417.46)

The existing insurance portfolio and especially the shareholder subsidizes the new contract A.

Applications

CB of an insurance contract added to existing an insurance portfolio

– New individually fair priced contracts are added to the existing insurance portfolio:
  - Contract A: Time to maturity $T^* = 20$, end date $31/12/2034$; $(\alpha^*, \beta^*, P_0^*, g^*) = (0.80, 0.20, 8000, 2.37\%)$.
  - Contract B: Time to maturity $T^* = 20$, end date $31/12/2034$; $(\alpha^*, \beta^*, P_0^*, g^*) = (0.90, 0.20, 8000, 1.55\%)$.

i	B	1	2	3	4	5	6	$PVFP_0$
CB_0^i (ex ante)	-0.10 (-0.10)	90.82 (0.75)	565.78 (1.48)	598.17 (0.48)	945.06 (0.48)	2274.50 (0.73)	1393.98 (0.31)	-5868.21 (-4.13)

- If contract B is added the ex ante collective boni of the contracts of the existing portfolio are almost the same.
- Contract B seems to be 'fairer' for the collective and the shareholder than A.

Agenda

Conclusion

- An insurance portfolio consists of various contracts; it is heterogeneous.
- In the first example it is shown which effects the heterogeneity has got for the different policies: One contract can benefit from another contract.
- We want to give an opportunity to analyse and measure the interaction of different contracts.
- Definition of a collective bonus/ collective malus to identify contracts, which subsidize in expectation (ex ante) or have subsidized (ex post) other contracts
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- Usage of the theoretical results to show how much one contract benefit from another contract, for example if they share the default risk of the policy sponsor

Thank you for your attention!

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