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Quantitative Assessment of Multi-Year Non-Life Insurance Risk

Dr. Lukas Hahn, SV SparkassenVersicherung Holding AG

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About me

- **Personal**
  - Born 1987 in Tönisvorst, Germany
- **Professional**
  - since 2019: data scientist at Big Data Lab of SV SparkassenVersicherung, Stuttgart
  - 2013-2019: actuarial consultant at Institute for Finance and Actuarial Sciences (ifa), Ulm
  - 2017-2019: lecturer of data analytics workshops at University of Ulm
- **Education**
  - 2019: Ph.D. in Insurance Mathematics (University of Ulm)
  - 2015: Master of Science in Mathematics and Management (University of Ulm)
  - 2014: Master of Mathematics in Statistics (University of Waterloo)
- **Involvement in the German Association of Actuaries (DAV)**
  - certified actuary
  - member of DAV working group Big Data in Life Insurance
  - lecturer in actuarial data science immersion and completion

Agenda

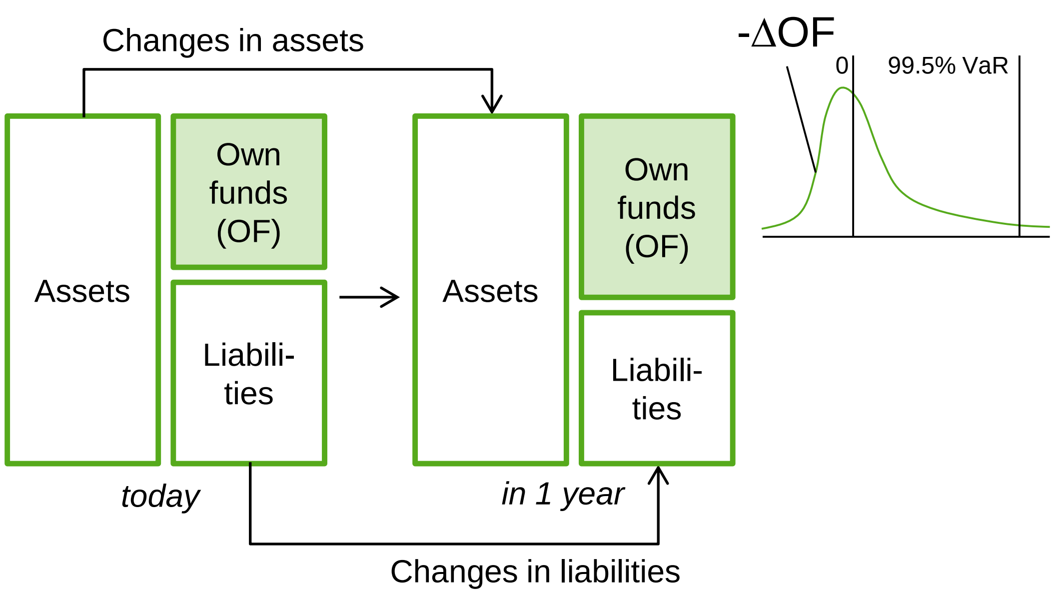
- Motivation
- Objectives and Research Questions
- Selected Research Results
- Summary

Motivation

- **Non-life insurance risk**
  - uncertainty in pricing and reserving
  - stochastic nature in claim occurrence, reporting, and settlement
  - focus on reserve and premium risk
- Sufficiency of claims reserves and premium provisions?
  - **Ultimo view** for safety loadings
    - Taylor and Ashe (1983); Verrall (1991); Mack (1993); Kaufmann et al. (2001); England and Verrall (2002, 2006)

Motivation

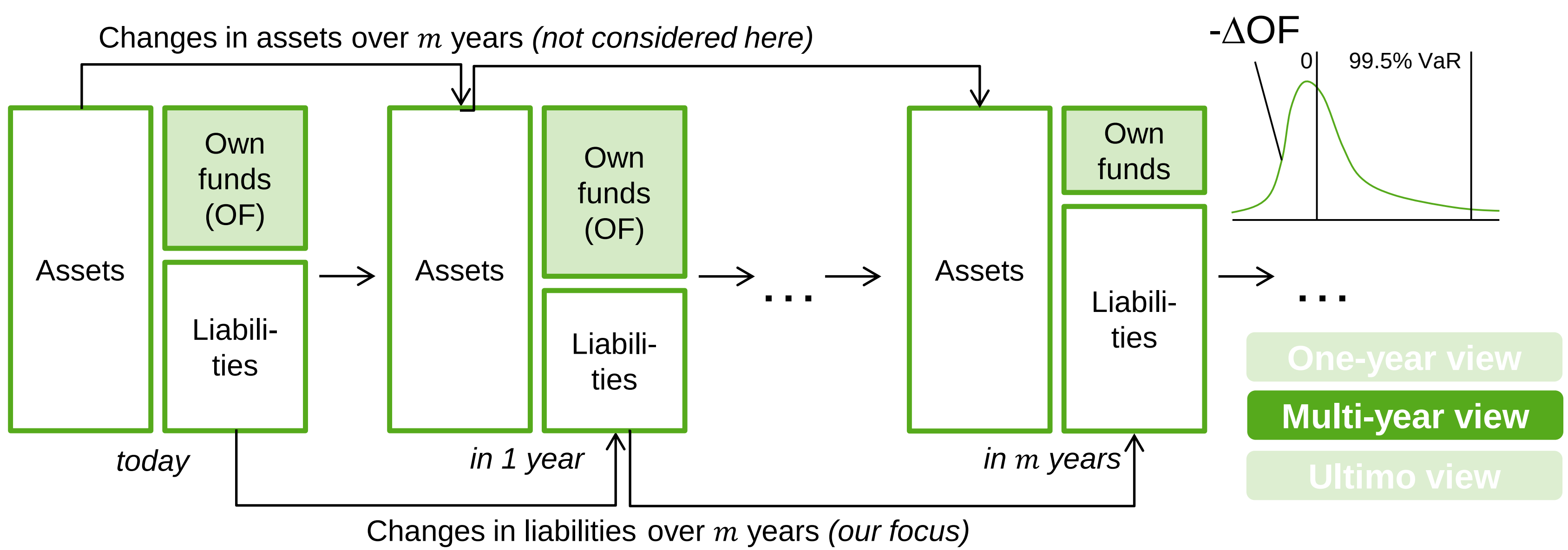
- **Non-life insurance risk**
  - uncertainty in pricing and reserving
  - stochastic nature in claim occurrence, reporting, and settlement
  - focus on reserve and premium risk
- Sufficiency of claims reserves and premium provisions?
  - **One-year view** for Solvency capital requirement (SCR) and consistent risk-based management
    - Merz and Wüthrich (2008); Ohlsson and Lauzeningks (2009)



One-year view
Multi-year view
Ultimo view

Motivation

- **Non-life insurance risk**
  - uncertainty in pricing and reserving
  - stochastic nature in claim occurrence, reporting, and settlement
  - focus on reserve and premium risk
- Sufficiency of claims reserves and premium provisions?
  - **Multi-year view** as a natural bridge between one-year and ultimo view for multi-year risk assessment
    - Diers (2009, 2011); England et al. (2019)



Motivation

- **Multi-year enterprise risk management:** determination and allocation of multi-year risk capital (Diers, 2009, 2011)
  - withstand financial losses over several years
  - while aiming for sustainable long-term profitability
- **Own Risk and Solvency Assessment (ORSA):** projection of Overall Solvency Needs (OSN)
  - medium or long-term perspective (3-5 years)
  - undertaking-specific risk profile, tolerance, and strategies
  - major effort for insurers applying Standard Formula (SF)

→ Demand for adequate, feasible multi-year risk projection models

Purposes
- core building block for multi-year internal models
- feasible stand-alone technique applicable in the ORSA
- future one-year solvency capital predictions for risk margins

Requirements
- beyond the standardized Solvency II framework
- consistent to company's best estimate reserving practices

Motivation

- ► **Quantification** of non-life insurance risk
  - (negative) multi-year claims development result (CDR)
  - difference in best estimate ultimates over time
    - Diers et al. (2013), generalizing Ohlsson and Lauzeningks (2009)
    - no discounting, inflation, stochasticity of future premiums/expenses, reinsurance, changes in risk margins, and catastrophic events
- ► Most common **reserving methods**
  - chain ladder method
    - closed-form second-order reserve risk estimator for overdispersed Poisson model (Matitschka, 2010) and bootstrap estimators for distribution-free chain ladder model (Diers et al., 2013)
  - incremental loss ratio method
    - closed-form second-order estimators for additive loss reserving model (Diers and Linde, 2013)

➔ Further tools for multi-year risk assessment required

Objectives and Research Questions

► **Estimators for most widely used reserving practice required**

- chain ladder method and customized versions
- embedded into (generalized) distribution-free model
  - Mack (1993, 1999)
- inherent claims dependence assumptions: analytical tractability?

Q1. How can a closed-form estimator be derived

- for the multi-year premium and reserve risk
- in terms of the prediction error in the multi-year CDR
- arising from a single homogeneous loss portfolio underlying Mack's (generalized) distribution-free chain ladder model
- taking into account the inherent dependence of consecutive cumulative claim payments?

→ *Research paper 1*

Objectives and Research Questions

► **Estimators for risk on company level required**

- existing techniques not applicable on aggregated portfolio
- bottom-up approach is also challenging
- multivariate reserving models implicitly account for correlations
- focus: multivariate chain ladder / additive loss reserving models
  - Braun (2004); Hess et al. (2006); Merz and Wüthrich (2009a)

Q2. **How can a corresponding multi-year premium and reserve risk estimator be derived**

- for the aggregated non-life insurance business
- in case of several dependent homogeneous loss portfolios
- all of which meet the assumptions of the additive loss reserving model?

→ *Research paper 2*

How can this estimator be generalized to the case in which each loss portfolio follows either the distribution-free additive and chain ladder reserving model?

→ *Research paper 3*

Objectives and Research Questions

► **Estimators based on full predictive distributions required**

- analytical approach: only standard deviations and approximations
- higher-order risk measures necessary for internal models and for ORSA (especially in case of deviations from SF assumptions)
- simulation of full predictive distribution: bootstrapping of claims triangles and stochastic re-reserving in integrated approach
    - England and Verrall (2006); Ohlsson and Lauzeningks (2009); Diers et al. (2013)

Q3. How can we simulate a full predictive distribution

- of the multi-year CDR on company level
- in a single integrated approach
- in case of several dependent homogeneous loss portfolios
- each following either the distribution-free additive and chain ladder reserving model?

→ *Research paper 3*

Objectives and Research Questions

Research Paper 1:

Q1. Closed-form estimator for Mack's chain ladder model

Addendum to 'The multi-year non-life insurance risk in the additive loss reserving model' [Insurance Math. Econom. 52(3) (2013) 590–598]: Quantification of multi-year non-life insurance risk in chain ladder reserving models

Joint work with D. Diers and M. Linde, Insurance: Mathematics and Economics (2016).

Research paper 2:

Q2. Closed-form estimator for multivariate additive loss model

Multi-year non-life insurance risk of dependent lines of business in the multivariate additive loss reserving model

Insurance: Mathematics and Economics (2017).

Research Paper 3:

Q2. Generalization for combined chain ladder / additive model

Q3. Simulation of full predictive distribution on company level

Quantification of multi-year non-life insurance risk: Analytical and simulation-based approaches in multivariate reserving models

Joint work with M. Linde, Working Paper.

Selected Research Results

Research Paper 1

Addendum to ‘The multi-year non-life insurance risk in the additive loss reserving model’ [Insurance Math. Econom. 52(3) (2013) 590–598]:
Quantification of multi-year non-life insurance risk in chain ladder reserving models

joint work with D. Diers and M. Linde
Insurance: Mathematics and Economics, 67(C): 187–199 (2016)

Model

Q1. Closed-form estimator for Mack's chain ladder model

Definition (Extended Mack chain ladder model)**.**

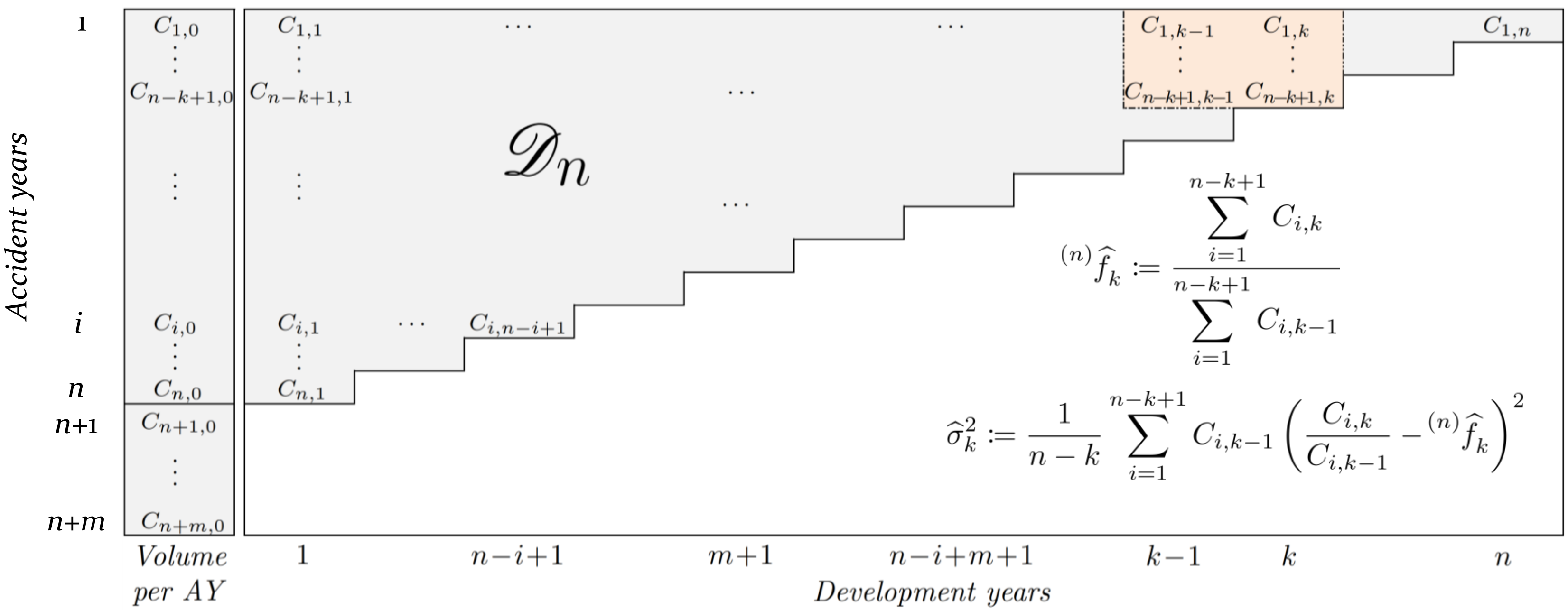
(CL1) $\{C_{i,1},\dots,C_{i,n}\}_{1\le i\le n+m}$ independent

(CL2) $\mathbb{E}\left[C_{i,k} \mid C_{i,0},\dots,C_{i,k-1}\right] = f_k C_{i,k-1}$ for all $i=1,\dots,n+m$

(CL3) $\mathbb{V}\left[C_{i,k} \mid C_{i,0},\dots,C_{i,k-1}\right] = \sigma_k^2 C_{i,k-1}$ for all $i=1,\dots,n+m$

Notation

$n\in\mathbb{N}$: number of historic accident years
$m\in\mathbb{N}_0$: number of future accident years
$T=n$: current point in time
$C_{i,k}>0$: cumulative payments
$C_{i,0}>0$: deterministic volume measures

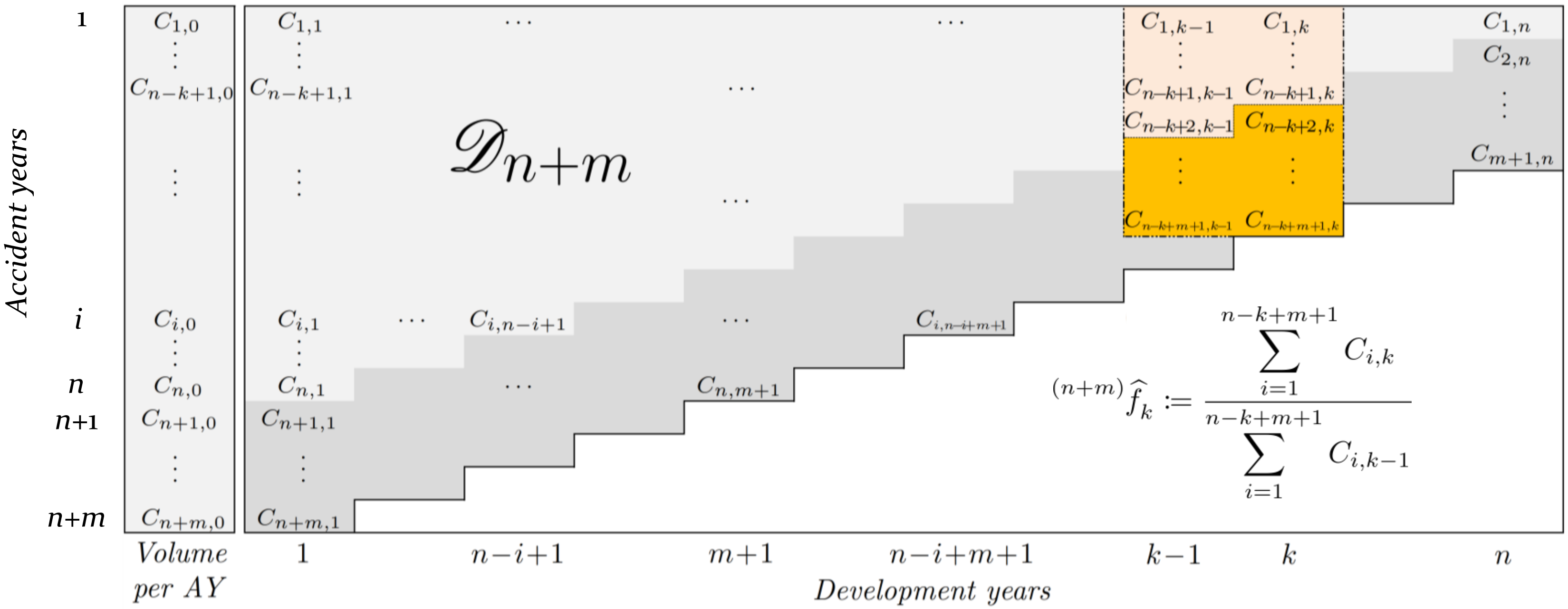


Ultimate prediction at $T=n$: ${}^{(n)}\widehat{C}_{i,k} := C_{i,\max\{n-i+1,0\}}\left(\prod_{l=\max\{n-i+2,1\}}^{k} {}^{(n)}\widehat{f}_l\right)$ ➔ ${}^{(n)}\widehat{U}_i := {}^{(n)}\widehat{C}_{i,n}$

Model

Q1. Closed-form estimator for Mack's chain ladder model

- Assumption: reserving method is re-applied after m years



Ultimate prediction at $T = n + m$: ${}^{(n+m)}\widehat{U}_i = C_{i,\min\{n-i+1+m,n\}} \left(\prod_{k=n-i+m+2}^{n} {}^{(n+m)}\widehat{f}_k\right)$

$$
\begin{aligned}
{}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i &:= {}^{(n)}\widehat{U}_i - {}^{(n+m)}\widehat{U}_i \\
&= C_{i,\max\{n-i+1,0\}} \left[\left(\prod_{k=\max\{n-i+2,1\}}^{n} {}^{(n)}\widehat{f}_k\right) - \left(\prod_{k=\max\{n-i+2,1\}}^{\min\{n-i+1+m,n\}} \boxed{F_{i,k}}\right)\left(\prod_{k=n-i+2+m}^{n} {}^{(n+m)}\widehat{f}_k\right)\right]
\end{aligned}
$$

$F_{i,k} := C_{i,k}/C_{i,k-1}$

Definition of risk

Q1. Closed-form estimator for Mack's chain ladder model

- ► Notion of risk: fluctuation of m-year CDR around predicted value 0

Definition (Conditional mean squared error of prediction)**.**

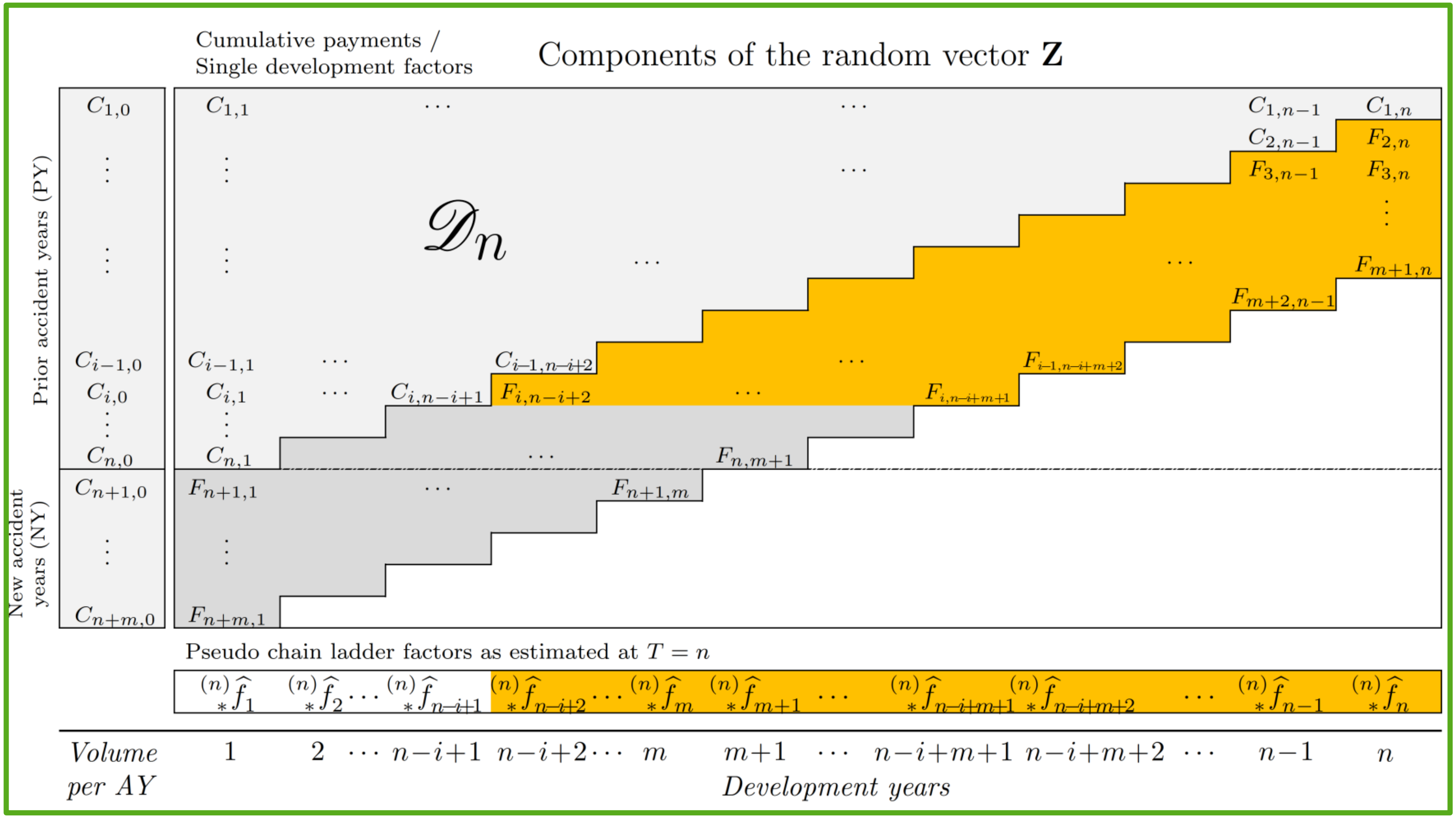
$$\mathrm{msep}_{{}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i \mid \mathscr{D}_n}(0) := \mathbb{E}\left[\left({}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i - 0\right)^2 \,\middle|\, \mathscr{D}_n\right] = \underbrace{\boxed{\mathbb{V}\left[{}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i \,\middle|\, \mathscr{D}_n\right]}}_{\text{process variance}} + \underbrace{\boxed{\mathbb{E}\left[{}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i \,\middle|\, \mathscr{D}_n\right]^2}}_{\text{estimation variance}}$$

  - ▪ Mack (1993), Merz and Wüthrich (2008)

- ► Derivation of estimator
  1. $\mathscr{D}_n$-measurability: conditional resampling (Buchwalder et al., 2006)
     - ▪ *pseudo* chain ladder factors ${}^{(n)}_{\;*}\widehat{f}_k$ with probability law consistent to (CL1–3) on $\mathscr{D}_n$
     - ▪ evaluate $\mathrm{msep}_{{}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i \mid \mathscr{D}_n}(0) = \mathbb{V}\left[{}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i \,\middle|\, \mathscr{D}_n\right]$
  2. Analytical tractability:
     - ▪ express ${}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i := {}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i(\mathbf{Z})$ via $\mathbf{Z} \subseteq \{{}^{(n)}_{\;*}\widehat{f}_k\}_{k=1,\ldots,n} \cup \{F_{i,k}\}_{n+1<i+k<n+m+1}$
     - ▪ use first-order Taylor expansion ${}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i(\mathbf{Z}) \approx {}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i(\boldsymbol{\mu}) + (\mathbf{Z}-\boldsymbol{\mu})^T\,\mathbf{D}(\boldsymbol{\mu})$ around $\boldsymbol{\mu} := \mathbb{E}[\mathbf{Z} \mid \mathscr{D}_n]$ with gradient vector D of ${}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i(\mathbf{Z})$
     - ▪ evaluate $\mathbb{V}\left[{}^{(n\to n+m)}_{\;\;*}\widehat{\mathrm{CDR}}_i(\mathbf{Z})\right] \approx \mathbf{D}(\boldsymbol{\mu}) \cdot \boldsymbol{\Sigma} \cdot \mathbf{D}(\boldsymbol{\mu})'$ where $\boldsymbol{\Sigma} := \mathbb{V}[\mathbf{Z} \mid \mathscr{D}_n]$
  3. Plug-in principle:
     - ▪ Replace unknown parameters with original estimates

Definition of risk

Q1. Closed-form estimator for Mack's chain ladder model



Analytical results

Q1. Closed-form estimator for Mack’s chain ladder model

- single accident year:

$$\widehat{\mathrm{msep}}_{{}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i|\mathscr{D}_n}(0) = \sum_{k=\max\{n-i+2,1\}}^{\min\{n-i+1+m,n\}} \widehat{C}_{i,k-1}\left(1+\frac{\widehat{C}_{i,k-1}}{\widehat{C}^{(n)}_{<,k-1}}\right)\widehat{\widehat{\sigma}}^2_k + \sum_{k=n-i+2+m}^{n} \frac{\widehat{C}^2_{i,k-1}}{\widehat{C}^{(n+m)}_{<,k-1}\,\widehat{C}^{(n)}_{<,k-1}}\left(\sum_{t=1}^{m} \widehat{C}_{n-k+1+t,k-1}\right)\widehat{\widehat{\sigma}}^2_k$$

- reserve risk: ${}^{(n\to n+m)}\widehat{\mathrm{CDR}}_{\mathrm{PY}} := \sum_{i=1}^{n} {}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i$

$$\widehat{\mathrm{msep}}_{{}^{(n\to n+m)}\widehat{\mathrm{CDR}}_{\mathrm{PY}}|\mathscr{D}_n}(0) = \sum_{k=2}^{m} \frac{\widehat{C}^{(n)}_{+,k-1}\,\widehat{C}^{(n)}_{\geq,k-1}}{\widehat{C}^{(n)}_{<,k-1}}\widehat{\widehat{\sigma}}^2_k + \sum_{k=m+1}^{n} \frac{\left(\widehat{C}^{(n)}_{+,k-1}\right)^2}{\widehat{C}^{(n+m)}_{<,k-1}\,\widehat{C}^{(n)}_{<,k-1}}\left(\sum_{t=1}^{m} \widehat{C}_{n-k+1+t,k-1}\right)\widehat{\widehat{\sigma}}^2_k$$

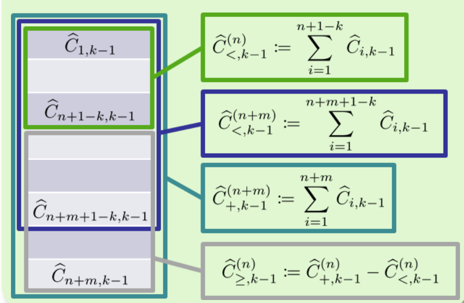
  - $m = 1$: Merz-Wüthrich formula
    - Merz and Wüthrich (2008)
  - $m = n - 1$: Mack formula
    - Mack (1993)

- premium risk: ${}^{(n\to n+m)}\widehat{\mathrm{CDR}}_{\mathrm{NY}} := \sum_{i=n+1}^{n+m} {}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i$
- non-life insurance risk: ${}^{(n\to n+m)}\widehat{\mathrm{CDR}} := \sum_{i=1}^{n+m} {}^{(n\to n+m)}\widehat{\mathrm{CDR}}_i$

$$\widehat{\mathrm{msep}}_{{}^{(n\to n+m)}\widehat{\mathrm{CDR}}|\mathscr{D}_n}(0) = \sum_{k=1}^{n} \frac{\left(\widehat{C}^{(n+m)}_{+,k-1}\right)^2}{\widehat{C}^{(n+m)}_{<,k-1}\,\widehat{C}^{(n)}_{<,k-1}}\left(\sum_{t=1}^{m} \widehat{C}_{n-k+1+t,k-1}\right)\widehat{\widehat{\sigma}}^2_k$$

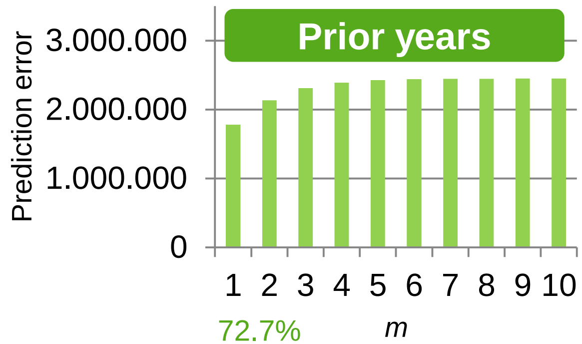
Notation

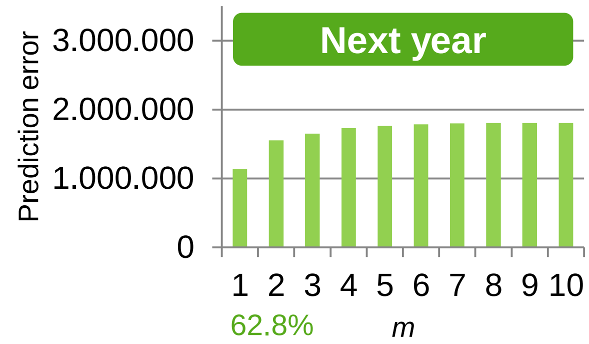
$$\widehat{\widehat{\sigma}}^2_k := \widehat{\sigma}^2_k\left(\prod_{l=k+1}^{n} {}^{(n)}\widehat{f}_l^{\,2}\right)$$

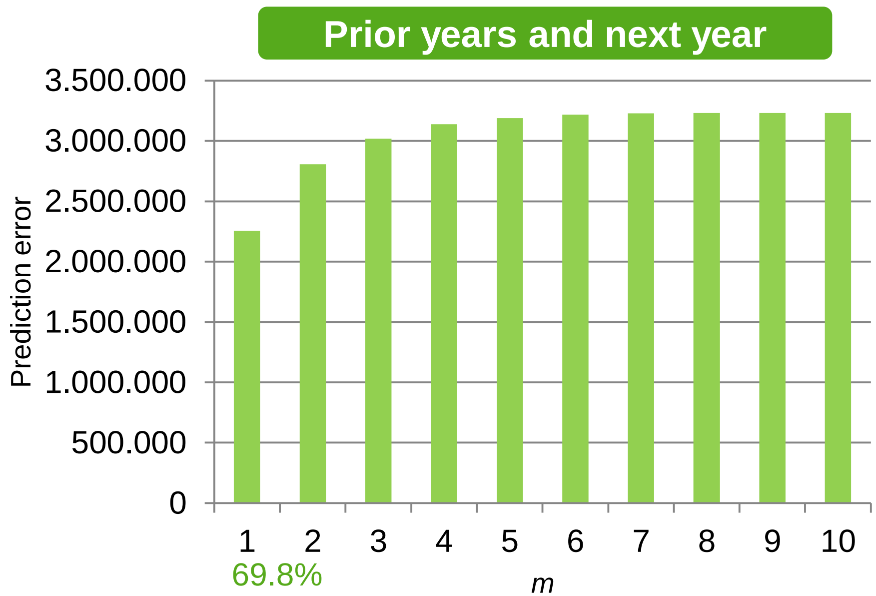


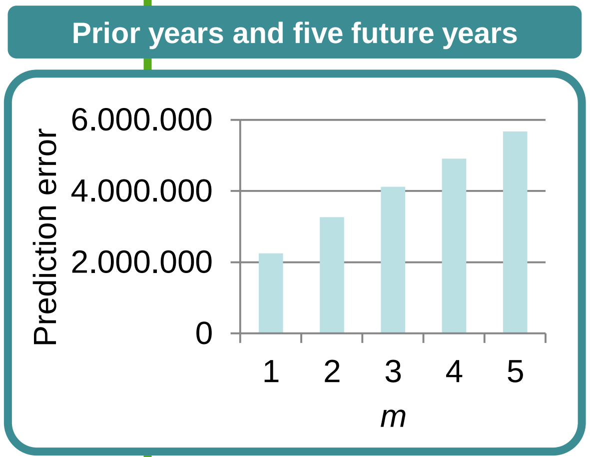
Case study

Q1. Closed-form estimator for Mack's chain ladder model









Selected Research Results

Research Paper 2

Multi-year non-life insurance risk of dependent lines of business in the multivariate additive loss reserving model

Insurance: Mathematics and Economics, 75: 71–81 (2017)

Summary of the Results of Research Paper 2

Q2. Closed-form estimator for multivariate additive loss model

Definition (Multivariate extended additive loss reserving model)**.**

(AL1) All incremental values $\mathbf{S}_{i,k}$ are pairwise uncorrelated.

(AL2) $\mathbb{E}(\mathbf{S}_{i,k}) = \mathrm{V}_i \mathbf{m}_k$ for all $k = 1, \ldots, n$.

(AL3) $\mathbb{V}(\mathbf{S}_{i,k}) = \mathrm{V}_i^{1/2} \Sigma_k \mathrm{V}_i^{1/2}$ for all $k = 1, \ldots, n$.

- ► Closed-form estimators for joint non-life insurance risk components

$$\mathbb{V}\left(\widehat{\mathrm{CDR}}_i^{(n \to n+m)}\right) = \mathrm{V}_i \left[\sum_{k=\max\{1,n-i+2\}}^{\min\{n+m-i+1,n\}} \left({}^{(n)}\mathrm{V}\Sigma_{\leq k}^{-1} + \mathrm{V}_i^{-1/2} \Sigma_k \mathrm{V}_i^{-1/2} \right) + \sum_{k=n+m-i+2}^{n} \left({}^{(n)}\mathrm{V}\Sigma_{\leq k}^{-1} - {}^{(n+m)}\mathrm{V}\Sigma_{\leq k}^{-1} \right) \right] \mathrm{V}_i.$$

  - analogy to univariate estimators
  - implicit linear correlations between portfolios
  - consistency to literature for special cases
    - Merz and Wüthrich (2009b)
  - identification of neglected sources of error in existing formulae
    - Ludwig and Schmidt (2010)
- ► Case study for motor and third-party liability business

Selected Research Results

Research Paper 3

Quantification of multi-year non-life insurance risk: Analytical and simulation-based approaches in multivariate reserving models

joint work with M. Linde
Working Paper

Summary of the Results of Research Paper 3

Q2. Generalization for combined chain ladder / additive model

- **Closed-form estimators for joint multi-year risk**

Q3. Simulation of full predictive distribution on company level

- Synchronous (non-)parametric multi-year “Mack bootstrap” algorithm
  - bootstrapping from several claims triangles
    - England and Verrall (1999, 2002, 2006); England (2002), Pinheiro et al. (2003); Taylor and McGuire (2007); Heberle et al. (2009, 2010)
  - stochastic re-reserving (“actuary-in-the-box”)
    - Ohlsson and Lauzeningks (2009); Björkwall et al. (2009)
  - multi-year view
    - Diers (2009); Diers et al. (2013)
  - both non-parametric and copula-based parametric versions
    - Taylor and McGuire (2007); Shi and Frees (2011); Côté et al. (2016)
- **Case study on two three porfolios within two lines of business**
  - risk evolution and SF assessment in light of ORSA process

Summary

Q1. Closed-form estimator for Mack's chain ladder model

- We have derived closed-form estimators for the conditional msep of the multi-year CDR using conditional resampling and Taylor expansions.
- The estimators build a natural bridge between well-known estimators for one-year and ultimo view.

Q2. Closed-form estimator for multivariate models

- We have generalized risk estimators to the joint company level in case of possibly dependent additive and chain ladder portfolios.
- The estimators allow to take into account and to express the implicit risk correlations among all loss portfolios.

Q3. Simulation of full predictive distribution on company level

- We have derived simulation algorithms using bootstrapping and stochastic re-reserving to obtain full predictive distributions of the joint multi-year CDR.
- Given realistic data among three portfolios, the algorithm is shown to be an adequate and feasible tool for risk assessment in the ORSA process.

Thank you for your attention.

Contact Details

Dr. Lukas J. Hahn
SV SparkassenVersicherung Holding AG
Löwentorstraße 65
70376 Stuttgart
Germany
+49 711 898-43765
lukas.hahn@sparkassenversicherung.de

Sparkassen Versicherung

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