How do tax certainty measures affect optimal investment under cash flow and tax uncertainty?

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Agenda

Taxation for multi-national companies

- Multi-national companies can shift profits worldwide and benefit from favorable taxation and/or differences in taxation across countries.
- Not only the tax rates differ between countries but also the way taxable income is defined.
- However, these taxation rules also include uncertainty, unclear definitions, legal risks for the multi-national companies.

Taxation for multi-national companies

- Cash-flow risk and tax uncertainty are not only a matter of tax avoidance. Many tax issues are exposed to severe tax uncertainty:
  - New business models (e.g. R&D, digital business) lead to unclear tax issues (not yet in the tax lax, intangible valuation).
  - New business models are also innovative, leading to high cash-flow risks.
- There are examples regarding also tax avoidance (see next slide) - these are not our focus but illustrate the issues that may arise.

"The US Tax Court will decide whether Facebook owes more than \$9 billion to the Internal Revenue Service (IRS) in a landmark case."

(International Tax Review 2020, Josh White)

Problem: In 2010, Facebook valued its intangible assets to \$6.5 billion; tax authorities claim its value to be \$21 billion. This difference would lead to \$9 billion additional tax.

The problem was raised years later in 2018.

Taxation for multi-national companies

- Court rulings and policy changes may lead to huge payments later on (after a tax audit...) – also for small/medium-sized firms that are not "tax-aggressive".
- Many countries offer so-called **Advance Tax Rulings (ATRs)**.

Advance Tax Rulings (ATRs)

Advance tax rulings are agreements to promote clarity and consistency regarding the application of the tax law for both taxpayers and the tax authority.
Companies usually pay a fee to receive clarity on tax issues.

What do we model?

- We consider R&D investments as illustrative example:
  - High cash-flow risk.
  - Considerable tax uncertainty.
- We explicitly model uncertainty in **both** net cash-flows and tax rate.
- We consider loss offset restrictions.

Loss offset restrictions

Countries provide rules how gains can be offset by prior losses to reduce taxation. Restrictions do not allow to offset certain losses or allow it with a time delay only (i.e. depreciation rules).

Research questions

We provide a theoretical model to analyze research questions related to taxation, ATRstax uncertainty and optimal investment amount:

- Do ATRs encourage risky investment? What is the role of loss offset restrictions and ATR fees?
- How should tax authorities price ATR agreements (i.e. set ATR fees)? How does this relate to the tax policy (i.e. tax rate, loss offset restrictions)?
- What is the willingness to pay for ATRs for investors? What is the role of ATR fee and risk aversion?

We aim to reveal relationships between ATRs, tax uncertainty, loss offset restrictions and optimal investment amount.

Related literature

We build on two streams of literature: (1) **taxations policy, ATRs, Advanced Pricing Agreements (APR)** and (2) **optimal investment**.

Some (out of many ...) results in (1):

- There is empirical evidence that tax uncertainty attenuates investment (e.g. Jacob et al. (2021)) and risk taking (e.g. Dharmapala & Hines (2009), Osswald & Sureth-Sloane (2020)).
- For investors, ATRs have also several disadvantages, e.g. increased inspection, detection and expertise of tax examiners (e.g., Givati (2009)).
- Closest to our study is Diller et al. (2018), looking at the effect of ATRs on optimal investment in a model with deterministic cash-flows, no loss-offset restrictions and no risk aversion.

Related literature

In the **optimal asset allocation literature** (2), results are mainly based on cash-flow uncertainty:

- Started with the seminal works Merton (1969,1971).
- Summarized, e.g., in the book Korn (1997).

Literature on (after-tax) utility maximiztion problems is scarce:

- Seifried (2010) optimizes after-tax cash-flows for financial investment.
- Chen, Hieber & Nguyen (2019) optimize after-tax payoffs from equity-linked life insurance.

Both work on a simple, deterministic proportional tax rate.

Agenda

Framework: Investment decision

Investment horizon $[0, T]$, $T < \infty$, fixed filtered probability space $(\Omega, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbb{P})$.
Investor with initial wealth X_0 invests in...:

(1) ... a risk-free government bond $B_t := e^{rt}$.

(2) ... a risky project (for example, R&D) with dynamics

$$\mathrm{d}S_t = S_t\left(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\right), \; S_0 > 0 \text{ is given}.$$

μ, the rate of return on the risky investment, and σ, the volatility with $\mu, \sigma > 0$, are constants and W is a $\mathbb{P}$- Brownian motion.

Framework: Investment decision

The amount invested in the risky (R&D) project is $\theta \in [0, x_0]$. Firm dynamics are:

$$\begin{aligned} \mathrm{d}X_t^\theta &= \theta \frac{\mathrm{d}S_t}{S_t} + (X_t^\theta - \theta)\, r\, \mathrm{d}t \\ &= \big(rX_t^\theta + \big(\mu - r\big)\theta\big)\mathrm{d}t + \sigma\, \theta \mathrm{d}W_t, \quad X_0 \text{ is given.} \end{aligned} \tag{1}$$

or, solving this:

$$X_T^\theta = e^{rT} X_0 + \theta\big(\mu - r\big) \int_0^T e^{r(T-s)} \mathrm{d}s + \sigma\theta \int_0^T e^{r(T-s)} \mathrm{d}W_s\,. \tag{2}$$

Framework: Investment decision

This implies that X_T^θ is normally distributed with mean and variance

$$\mathbb{E}[X_T^\theta] = e^{rT}X_0 + \theta(\mu - r)\int_0^T e^{r(T-s)}\mathrm{d}s\,, \tag{3}$$

$$\mathrm{Var}[X_T^\theta] = \mathrm{Var}\left[\sigma\theta\int_0^T e^{r(T-s)}\mathrm{d}W_s\right] = \sigma^2\theta^2\int_0^T e^{2r(T-s)}\mathrm{d}s\,. \tag{4}$$

(In theory, values X_T^θ turn negative; but the probability is for reasonable parameter choices small. Our results also work for negative X_T^θ.)

Proportional tax: Profits

The amount $X_T^\theta - X_0 \geq 0$ is subject to a (stochastic!) proportional tax rate $\tilde{\tau}_p$, after-tax the net-payoff reduces to

$$(1 - \tilde{\tau}_p)(X_T^\theta - X_0)\,.$$

We describe $\tilde{\tau}_p$ as a binary random variable:

$$\tilde{\tau}_p = \begin{cases} \delta_p \cdot \tau & \text{with probability } d \\ \tau & \text{with probability } 1 - d \end{cases} \tag{5}$$

where $d \in (0, 1)$ is the probability of a tax audit increasing the tax rate by $\delta_p \geq 1$.

Proportional tax: Losses

The amount $X_T^\theta - X_0 < 0$ is subject to a (stochastic!) proportional tax rate $\tilde{\tau}_l$, after-tax the net-payoff reduces to

$$(1 - \tilde{\tau}_p)(X_T^\theta - X_0)\,.$$

We describe $\tilde{\tau}_l$ as a binary random variable:

$$\tilde{\tau}_l = \begin{cases} \delta_l \cdot \lambda \cdot \tau & \text{with probability } d \\ \lambda \cdot \tau & \text{with probability } 1 - d. \end{cases} \tag{6}$$

where $\lambda \in [0, 1]$ is the tax loss offset parameter, $d \in (0, 1)$ is the probability of a tax audit decreasing tax offset by $\delta_l \in (0, 1)$.

Requesting an ATR?

- **Procedure 1 (no ATR)**: The after-tax net payoff is simply:

$$\tilde{X}_T^{(1)} = \begin{cases} (1 - \tilde{\tau}_p)(X_T^\theta - X_0), & \text{if } X_T^\theta \geq X_0\,, \\ (1 - \tilde{\tau}_l)(X_T^\theta - X_0), & \text{if } X_T^\theta < X_0\,. \end{cases} \tag{7}$$

- **Procedure 2 (ATR)**: Initiating an ATR, the investor pays the fixed cost F_0 upfront. This replaces the random tax rates $(\tilde{\tau}_p, \tilde{\tau}_l)$ by fixed tax rates $(\eta_p \cdot \tau, \eta_l \cdot \tau)$. After-tax net payoff:

$$\tilde{X}_T^{(2)} = \begin{cases} (1 - \eta_p \cdot \tau)(X_T^\theta - X_0) - F, & \text{if } X_T^\theta \geq X_0\,, \\ (1 - \eta_l \cdot \tau)(X_T^\theta - X_0) - F, & \text{if } X_T^\theta < X_0\,, \end{cases} \tag{8}$$

where we accrue the ATR fee F_0 to obtain $F := F_0 e^{rT}$.

Optimal investment decision

Let us consider a risk-averse investor whose preferences can be described by exponential utility. The corresponding optimal investment problem of the firm under **Procedure 1** and **2** is then given by:

$$\max_{\theta \in [0, X_0]} EU^{(i)}(\theta) := \max_{\theta \in [0, X_0]} \mathbb{E}\left[-\frac{1}{\gamma} \exp\left\{ -\gamma \tilde{X}_T^{(i)} \right\} \right], \; i = 1, 2, \qquad (9)$$

s.t. $X^{\theta} = (X_t^{\theta})_{t \in [0,T]}$ follows (1).

We compare the optimal utility from both procedures in terms of **certainty equivalents**.

Technical lemmas, notation

For $a > 0$, $b \in$, the functions $g_1(a,b)$ and $g_2(a,b)$ are defined as:

$$
\begin{aligned}
g_1(a,b,\theta) &:= \mathbb{E}\Big[\exp\Big\{-\gamma(1-b)X_T^\theta\Big\}\cdot \mathbb{1}_{\{X_T^\theta \geq a\}}\Big] \\
&= \exp\left\{-\gamma(1-b)\mathbb{E}[X_T^\theta] + \frac{1}{2}\gamma^2(1-b)^2\,\mathrm{Var}[X_T^\theta]\right\} \\
&\quad \cdot \Phi\left(-\frac{a-\mathbb{E}[X_T^\theta]}{\sqrt{\mathrm{Var}[X_T^\theta]}} - \gamma(1-b)\sqrt{\mathrm{Var}[X_T^\theta]}\right), \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
g_2(a,b,\theta) &:= \mathbb{E}\Big[\exp\Big\{-\gamma(1-b)X_T^\theta\Big\}\cdot \mathbb{1}_{\{X_T^\theta < a\}}\Big] \\
&= \exp\left\{-\gamma(1-b)\mathbb{E}[X_T^\theta] + \frac{1}{2}\gamma^2(1-b)^2\,\mathrm{Var}[X_T^\theta]\right\} \\
&\quad \cdot \Phi\left(\frac{a-\mathbb{E}[X_T^\theta]}{\sqrt{\mathrm{Var}[X_T^\theta]}} + \gamma(1-b)\sqrt{\mathrm{Var}[X_T^\theta]}\right). \qquad (11)
\end{aligned}
$$

Theorem (Optimal investment amount Procedure 1)

Under Procedure 1, the expected utility as a function of the investment amount $\theta \in [0, X_0]$ *is given by*

$$
\begin{aligned}
EU^{(1)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\left\{-\gamma \tilde{X}_T^{(1)}\right\}\right] \\
&= -\frac{d}{\gamma}\Big(e^{\gamma(1-\delta_p\tau)X_0} g_1(X_0, \tau\delta_p, \theta) + e^{\gamma(1-\lambda\tau\delta_l)X_0} g_2(X_0, \lambda\tau\delta_l, \theta)\Big) \\
&\quad - \frac{1-d}{\gamma}\left(e^{\gamma(1-\tau)X_0} g_1(X_0, \tau, \theta) + e^{\gamma(1-\lambda\tau)X_0} g_2(X_0, \lambda\tau, \theta)\right).
\end{aligned}
$$

where The optimal amount $\theta^* \in [0, X_0]$ *solving* (9) *for Procedure 1 is either* 0, X_0, *or determined implicitly and uniquely by solving*

$$\frac{\partial EU^{(1)}(\theta)}{\partial \theta} = 0\,. \tag{12}$$

Theorem (Optimal investment amount Procedure 2)

Under Procedure 2, the expected utility as a function of the investment amount $\theta \in [0, X_0]$ *is given by*

$$
\begin{aligned}
EU^{(2)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\left\{-\gamma \tilde{X}_T^{(2)}\right\}\right] \\
&= \frac{-e^{\gamma F}}{\gamma}\left(e^{\gamma(1-\tau\eta_p)X_0} g_1(X_0, \tau\eta_p, \theta) + e^{\gamma(1-\lambda\tau\eta_l)X_0} g_2(X_0, \lambda\tau\eta_l, \theta)\right)
\end{aligned}
$$

where $F := F_0 e^{rT}$. *The optimal amount* $\theta^{**} \in [0, X_0]$ *solving* (9) *for Procedure 2 is either* 0, X_0 *or determined implicitly and uniquely solving*

$$
\frac{\partial EU^{(2)}(\theta)}{\partial \theta} = 0\,. \tag{13}
$$

Agenda

Investor perspective

We aim to determine the **critical ATR fee** F_0^* where the investor is indifferent between Procedure 1 (no ATR) and Procedure 2 (ATR).

- In Procedure 1, the investor obtains optimally $EU^{(1)}(\theta^*)$.
- In Procedure 2, the investor obtains optimally $EU^{(2)}(\theta^{**})$.

The critical fee level F_0^* is determined such that

$$EU^{(1)}(\theta^*) = EU^{(2)}(\theta^{**}) .$$

Given previous results, this is explicitly given by

$$F_0^* = \frac{e^{-rT}}{\gamma} \ln \frac{EU^{(1)}(\theta^*)}{h(\tau, \eta_p, \eta_l, \theta^{**})} , \tag{14}$$

with $h(\tau, \eta_p, \eta_l, \theta^{**}) := \frac{-1}{\gamma}\left[e^{\gamma(1-\tau\eta_p)X_0} g_1(X_0, \tau\eta_p, \theta^{**}) - e^{\gamma(1-\lambda\tau\eta_l)X_0} g_2(X_0, \lambda\tau\eta_l, \theta^{**})\right]$.

Tax authority perspective

Tax authorities have an expected tax income in Procedure 1:

$$\begin{aligned} ER^{(1)}(\theta) = \tau \cdot (d \cdot \delta_p + (1-d)) \cdot \mathbb{E}\big[(X_T^{\theta} - X_0)\mathbb{1}_{\{X_T^{\theta} \geq X_0\}}\big] \\ - \lambda\tau \cdot (d \cdot \delta_l + (1-d)) \cdot \mathbb{E}\big[(X_0 - X_T^{\theta})\mathbb{1}_{\{X_T < X_0\}}\big], \end{aligned} \quad (15)$$

and in Procedure 2:

$$\begin{aligned} ER^{(2)}(\theta) = \eta_p\tau \cdot \mathbb{E}\big[(X_T^{\theta} - X_0)\mathbb{1}_{\{X_T^{\theta} \geq X_0\}}\big] \\ - \lambda\eta_l\tau \cdot \mathbb{E}\big[(X_0 - X_T^{\theta})\mathbb{1}_{\{X_T^{\theta} < X_0\}}\big] + F_0 e^{rT}. \end{aligned} \quad (16)$$

Again, we determine the **critical ATR fee F_0^{**}** where the tax authority is indifferent between the two procedures:

$$ER^{(1)}(\theta^*) = ER^{(2)}(\theta^{**}).$$

When should an ATR be implemented?

. . . and what is the ATR fee the tax authority may want to choose?

- The investor optimally chooses in ATR if the fee is $\leq F_0^*$.
- The tax authority may want to only offer ATRs such that the fee is $\geq F_0^{**}$.

$\Longrightarrow$ Any fee in $[F_0^{**}, F_0^*]$ leads to a "good" ATR deal.
$\Longrightarrow$ There may be situations where no such fee exists.

We identify settings where negative ATR fee make sense.

Agenda

Parameter choice

In our baseline scenario, we assume

$$\mu = 0.06, \quad r = 0.02, \quad \sigma = 0.15, \quad \gamma = 0.02 \text{ (risk aversion)}, \tag{17}$$

$$T = 5, \quad X_0 = 100, \quad \tau = 15\% \text{ (base tax rate)}, \quad F_0 = 0.00\,. \tag{18}$$

The rate of return μ on risky investment and the volatility σ are chosen to achieve a reasonable Sharpe ratio, i.e. a Sharpe ratio of $26.67\% \approx (\mu - r)/\sigma$.

We define the certainty equivalent:

$$CE^{(1)}(\theta^*) = -\frac{1}{\gamma} \ln \left(-\gamma \cdot EU^{(1)}(\theta^*) \right) + X_0\,,$$

$$CE^{(2)}(\theta^{**}) = -\frac{1}{\gamma} \ln \left(-\gamma \cdot EU^{(2)}(\theta^{**}) \right) + X_0\,.$$

Optimal investment amount: Comparison

#	$(\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l)$	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
		$\lambda = 0.9,\ \gamma = 0.02$			
(1)	$(4.00, 1.00, 0.40, 2.20, 1.00)$	91.80	113.40	95.34	113.93
(2)	$(4.00, 1.00, 0.50, 2.50, 1.00)$	90.11	112.39	94.08	112.94
(3)	$(4.00, 1.00, 0.60, 2.80, 1.00)$	88.31	111.41	92.41	111.92
(4)	$(3.00, 1.00, 0.40, 1.80, 1.00)$	95.26	115.00	96.52	115.22
(5)	$(3.00, 1.00, 0.50, 2.00, 1.00)$	94.62	114.35	96.00	114.58
(6)	$(3.00, 1.00, 0.60, 2.20, 1.00)$	93.96	113.71	95.34	113.93

- With Procedure 2 (ATR), the optimal risk amount is higher.
- With Procedure 2 (ATR), the certainty equivalent is higher.

Optimal investment amount: Comparison

#	$(\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l)$	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
		$\lambda = 0,\ \gamma = 0.02$			
(1)	$(4.00, 1.00, 0.40, 2.20, 1.00)$	77.83	112.56	80.85	113.01
(2)	$(4.00, 1.00, 0.50, 2.50, 1.00)$	76.07	111.61	79.42	112.07
(3)	$(4.00, 1.00, 0.60, 2.80, 1.00)$	74.18	110.68	77.62	111.11
(4)	$(3.00, 1.00, 0.40, 1.80, 1.00)$	81.18	114.05	82.28	114.24
(5)	$(3.00, 1.00, 0.50, 2.00, 1.00)$	80.43	113.43	81.63	113.63
(6)	$(3.00, 1.00, 0.60, 2.20, 1.00)$	79.65	112.82	80.85	113.01

- If losses cannot be offset ($\lambda = 0$), the optimal investment amount is much lower (compare to previous slide).
- ATRs in combination with a high loss offset can strongly encourage investment. ATR fees can turn negative und specific conditions.

Critical ATR fee level and risk aversion

risk aversion	F_0^*
$\gamma = 0.02$	0.13
$\gamma = 0.03$	0.11
$\gamma = 0.04$	0.10
$\gamma = 0.05$	0.10
$\gamma = 0.06$	0.10
$\gamma = 0.07$	0.11
$\gamma = 0.08$	0.11
$\gamma = 0.09$	0.11
$\gamma = 0.10$	0.12

The effect of risk aversion on the investor's critical fee level F_0^*.

We observe that this relation is non-linear. Investors with a very high and very low risk aversion accept the highest fee levels.

Critical ATR fee level and tax level

tax rate multiplier	F_0^{**}
$\eta_p = 1.30$, $\eta_l = 1.00$	0.27
$\eta_p = 1.35$, $\eta_l = 0.95$	0.12
$\eta_p = 1.40$, $\eta_l = 0.90$	-0.02
$\eta_p = 1.45$, $\eta_l = 0.85$	-0.16
$\eta_p = 1.50$, $\eta_l = 0.80$	-0.30
$\eta_p = 1.55$, $\eta_l = 0.75$	-0.44
$\eta_p = 1.60$, $\eta_l = 0.70$	-0.58
$\eta_p = 1.65$, $\eta_l = 0.65$	-0.72
$\eta_p = 1.70$, $\eta_l = 0.60$	-0.85

Figure: Critical fee level F_0^{**} for different tax rate multipliers

Notes: We use the set of parameters of the baseline scenario, i.e., $\mu = 0.06$, $r = 0.02$, $\sigma = 0.15$, $T = 5$, $X_0 = 100$, $\tau = 15\%$ and $(\delta_p,\ \delta_l,\ d) = (3.0, 0.5, 0.2)$ and a loss offset parameter $\lambda = 0.5$. We assume a risk aversion level of $\gamma = 0.04$.

Comparison to constant proportional tax

In the case of an ATR with symmetric taxation ($\eta := \eta_p = \eta_l = 1$, $\lambda = 1$), the optimal investment decision under exponential utility is well-studied. The optimal investment amount θ^{**} is a function of the adjusted Sharpe ratio (ASR):

$$\text{ASR} := \frac{\mu - r}{\sigma^2} \,,$$

A constant tax rate τ leads to the optimal investment amount

$$\theta^* = \theta^{**} = \frac{\mu - r}{\gamma\sigma^2(1-\tau)} \frac{\int_0^T e^{r(T-s)}\,\mathrm{d}s}{\int_0^T e^{2r(T-s)}\,\mathrm{d}s} = \frac{\text{ASR}}{\gamma(1-\tau)} \frac{\int_0^T e^{r(T-s)}\,\mathrm{d}s}{\int_0^T e^{2r(T-s)}\,\mathrm{d}s} \,.$$

Comparison to constant proportional tax

To analyze the difference between constant and asymmetric taxation (loss-offset), we choose investment pairs $(\mu, \sigma^2) = \left(\text{ASR} \cdot \sigma^2 + r, \sigma^2\right)$ and fix the adjusted Sharpe ratio ASR ≈ 1.777778.

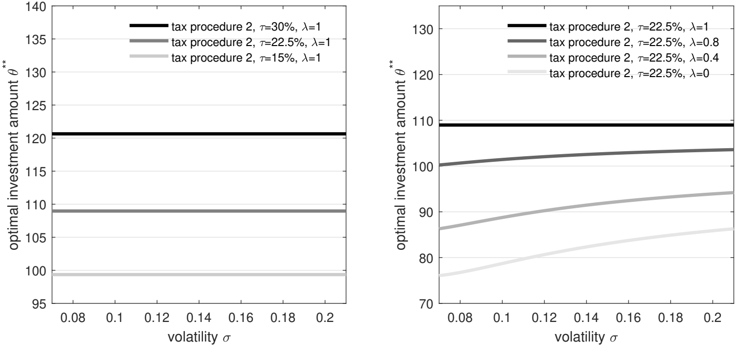

Figure: Optimal investment amount θ^{**} in Procedure 2 (with ATR).

Summary/conclusion

- We provide a theoretical model that models both cash-flow and tax uncertainty, combined with loss-offset restrictions.
- We give guidance for the choice of ATR fees, taking both the view of a risk-averse investor and tax authorities.
- We find that ATRs in combination with small loss offset restrictions encourage investment.

Summary/conclusion

- Surprisingly, there is a non-monotone relation between critical fee levels and investor's risk aversion.
- Optimal investment amounts under loss offset restrictions ("asymmetric taxation") may be very different from the case of a simple proportional tax ("symmetric taxation").
- Negative ATR fees can be optimal.

Thank you!

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