european actuarial academy

The European knowledge centre for actuaries

ESG Rebase

Michael Leitschkis

EAA Week on actuview
February 2020 | online

Introduction (1/2)

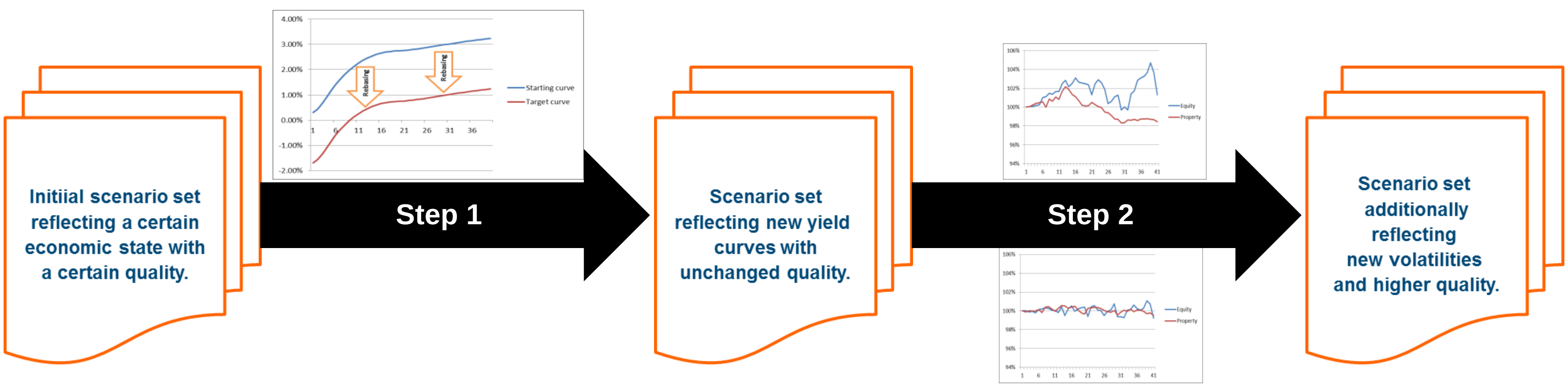
Some Use Cases for ESG Rebasing

- Produce ESG files for univariate and combined stresses
  - Input: Base ESG file (e.g. 1000 sims)
  - Input: Stress sizes
  - Output: Rebased ESG file (e.g. 1000 sims)

- Produce ESG files for proxy model calibrations
  - Input: Base ESG file (e.g. 1000 sims)
  - Input: Stress sizes (e.g. at 1-in-200 level)
  - Output: ESG file for proxy calibration

Introduction (2/2)

- Outline of ESG Rebasing
  - **Starting point:** Existing risk-neutral scenario set with x'000 paths
  - **Step 1:** Deterministic transformation of each of these paths to enforce new yield and spread curves keeping martingale tests and normal volatilities invariant.
  - **Step 2:** Perform ex-post change of measure via assigning weights to the individual paths to change the volatilities and improve martingale tests.



How to rebase nominal yields

Keep martingale test results invariant for all bonds

Approach: Rebase bond prices and discount factors as follows:

$$\overline{ZCB}_T(t) = ZCB_T(t) \times \frac{ZCB_t(0)}{\overline{ZCB}_t(0)} \times \frac{\overline{ZCB}_{T+t}(0)}{ZCB_{T+t}(0)}$$

$$\overline{Disc}_t = Disc_t \times \frac{ZCB_t(0)}{\overline{ZCB}_t(0)}$$

where

- $ZCB_T(t)$ is a price of a zero bond at time t which matures at time T
- the bar denotes “rebased” values
- $Disc_t$ denotes a stochastic discount factor for time t

How to rebase equity indices (1/2)

Preliminary remarks on equity returns

What does the return on some equity index I consist of?

- …of risk-free yield (rebased already)…
- *…and excess* return
  - in risk-neutral work, it has a *mean of zero*
  - say, standard deviation of excess return is σ
    - this determines implied volatility σ_I for options on index I

How to rebase equity indices (2/2)

Keep martingale test results invariant for equity indices

One can prove that the following approach does the trick:

$$\overline{Excret}_I(t) = Excret_I(t) \times \frac{\overline{\sigma_I}(t)}{\sigma_I(t)} \times \frac{ZCB_t(0)}{\overline{ZCB}_t(0)} \times \frac{\overline{ZCB}_{t-1}(0)}{ZCB_{t-1}(0)}$$

- The second multiplier captures the rebasing due to the changing level of *equity volatility*

- The third and fourth multipliers capture the rebasing due to changing level of *risk-free rates*

Reweighting approach (1/2)

Scenarios do not have to be equally weighted

- Say, N scenarios can be assigned different weights w_i
  - *Lots of degrees of freedom*

- But, there are *lots of calibration constraints…*
  - Match prices of derivatives (market consistency)
  - Pass martingale tests (no arbitrage)
- …and an *entropy constraint*
  - Do not deviate "too far" from uniform weights $\frac{1}{N}$
    - To be defined: what "(not) too far" means
- …and a common sense constraint: sum of weights = 1

Sources:
- M. Avellaneda et al. "Weighted Monte Carlo: A new technique for calibrating asset pricing models (2001)
- M. Hoerig, F. Wechsung "Scenario reweighting techniques for the quick recalibration of pricing scenarios" (2013)

Reweighting approach (2/2)

Reweighting: An optimization problem

Minimize the following function:

$$H = S(\{w_i\}) + \mu\left(\sum_{i=1}^{N} w_i - 1\right) + \sum_{m=1}^{M} \lambda_m \circ \left(\sum_{m=1}^{N} \frac{w_i \circ \overline{PV}_{m,i}}{\overline{P}_m} - 1\right)^2$$

where

- i runs through scenarios 1,…,N
- m runs through calibration targets (swaptions, options,…)
- λ_m denotes the importance of target m
- S is a measure of how far weights deviate from uniformity
- μ is a Lagrangian multiplier, so that weights add up to 1
- $\overline{P}_m$ is the price of target asset m after classical rebasing
- $\overline{PV}_{m,i}$ is the discounted PV of that asset in scenario i

Reweighting: calibration constraints

Market consistency & no-arbitrage: lots of constraints

Market Consistency

- Weighted Monte Carlo to approximate option prices for various terms
- Weighted Monte Carlo to approximate swaption prices for various term-tenor combinations

No Arbitrage

- Martingale tests to be passed for all equity-like indices over various horizons
- Martingale tests to be passed for bonds of various terms

Remark: To spice up this list, think of multi-currency ESG files

Reweighting: entropy constraint (1/2)

Define distance from a weight set to uniform weights

Popular measure for *weight set entropy* due to Kullback-Leibler ("negative entropy")

$$S(\{w_i\}) = \sum_{i=1}^{N} w_i \ln w_i$$

- This function attains its minimum $S_{\min} = -\ln N$ for the uniform weight distribution
- Its maximum is attained when all the weight is put upon one scenario (not a great reweighting outcome)

Reweighting: entropy constraint (2/2)

Effective number of scenarios

Question: Which number of uniformly distributed weights would have the same entropy $S(\{w_i\})$?

Answer: For this number N_{eff}, we obtain:

$$S(\{w_i\}) = -\ln N_{eff}$$

Hence, we conclude that

$$N_{eff} = e^{-S(\{w_i\})}$$

Remark: A small effective number of scenarios would typically indicate problems to hit the optimization targets.

FI volatilities and ESG rebasing (1/2)

FI Volatilities and Current Low Yield Environment

- In current market conditions, Black volatilities do not even exist for some term/tenor combinations
  - Even the generation of a univariate downward yield shift is a challenge then
    - Volatilities should not change in a univariate yield shock
- Normal volatilities should be used instead, including the ESG Rebasing work

Remark: It is sensible to *define* Internal Model FI volatility stresses in Normal volatility terms

FI volatilities and ESG rebasing (2/2)

- Relative Delta between Realised vs. Target volatilities:

	24	36	48	60	84	120	144	180	240	300	360
24	0%	0%	0%	1%	1%	1%	-3%	-3%	-1%	0%	-4%
36	0%	1%	1%	1%	1%	1%	-2%	-2%	-1%	-1%	-4%
48	0%	1%	1%	1%	1%	1%	-1%	-2%	-1%	-1%	-4%
60	1%	1%	1%	1%	1%	1%	-1%	-1%	-1%	-1%	-3%
84	1%	1%	1%	1%	1%	1%	0%	-1%	-1%	-1%	-3%
120	1%	1%	1%	1%	1%	1%	0%	0%	-1%	-1%	-3%
144	1%	1%	1%	1%	2%	1%	1%	0%	0%	-1%	-2%
180	2%	2%	2%	2%	2%	1%	1%	0%	0%	-1%	-2%
240	2%	1%	1%	1%	1%	1%	1%	0%	0%	-1%	-2%
300	1%	1%	0%	0%	0%	0%	0%	-1%	-1%	1%	-1%
360	0%	0%	0%	0%	0%	0%	-1%	-2%	-1%	-3%	-1%

ESG rebasing for risk & capital modelling

- Traditional stress scenario testing
  - Automated process, can reduce manual intensive bottlenecks in the results production process
  - Reduced operational risk
  - Ability to focus skilled resource on more value adding work

- Supports advanced capital modelling techniques, e.g. Least Squares Monte Carlo (LSMC)