The Future of Mortality

Mortality Forecasting by Extrapolation of Deaths Curve Evolution Patterns

- 2nd Ulm Actuarial Day
- March, 29th 2019
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- Joint work with Matthias Börger and Jochen Ruß

Agenda

Motivation

- Estimates for future mortality are important in many areas, e.g. for projections of social security systems or risk management in the private pension and life insurance sector.

On the one hand…

- There is a variety of mortality models, e.g.:
  - the Lee-Carter-Model (LC; Lee and Carter, 1992),
  - the Cairns-Blake-Dowd-Model (CBD; Cairns et al., 2006).
- However, in many cases, the parameters in these models **lack a clear demographic interpretation**. This might lead to mortality forecasts which are not **plausible from a demographic perspective**.

On the other hand…

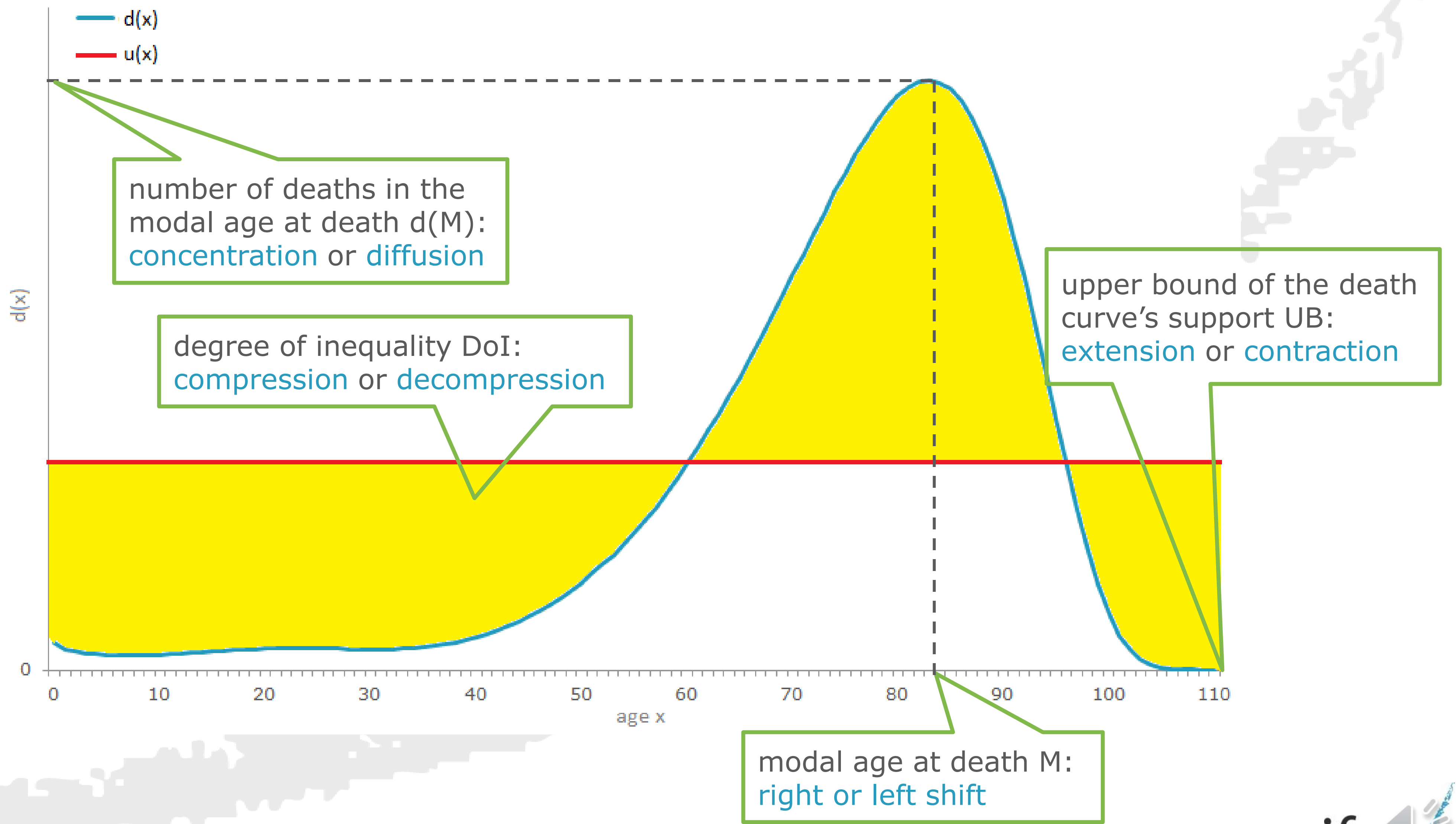
- Also demographers make forecasts for future mortality, e.g.:
  - Oeppen and Vaupel (2002) forecast the world record life expectancy.
  - Dong et al. (2016) "suggest that the maximum lifespan of humans is fixed".
- However, their forecasts often focus on **single aspects of the mortality evolution only** and they typically do not make suggestions for the complete distribution of deaths over all ages in any future year.

- It seems worthwhile to **combine the purely statistical approach of mortality models with the demographic expertise**.
- This is what we do in what follows.

The Classification Framework of Börger et al. (1)

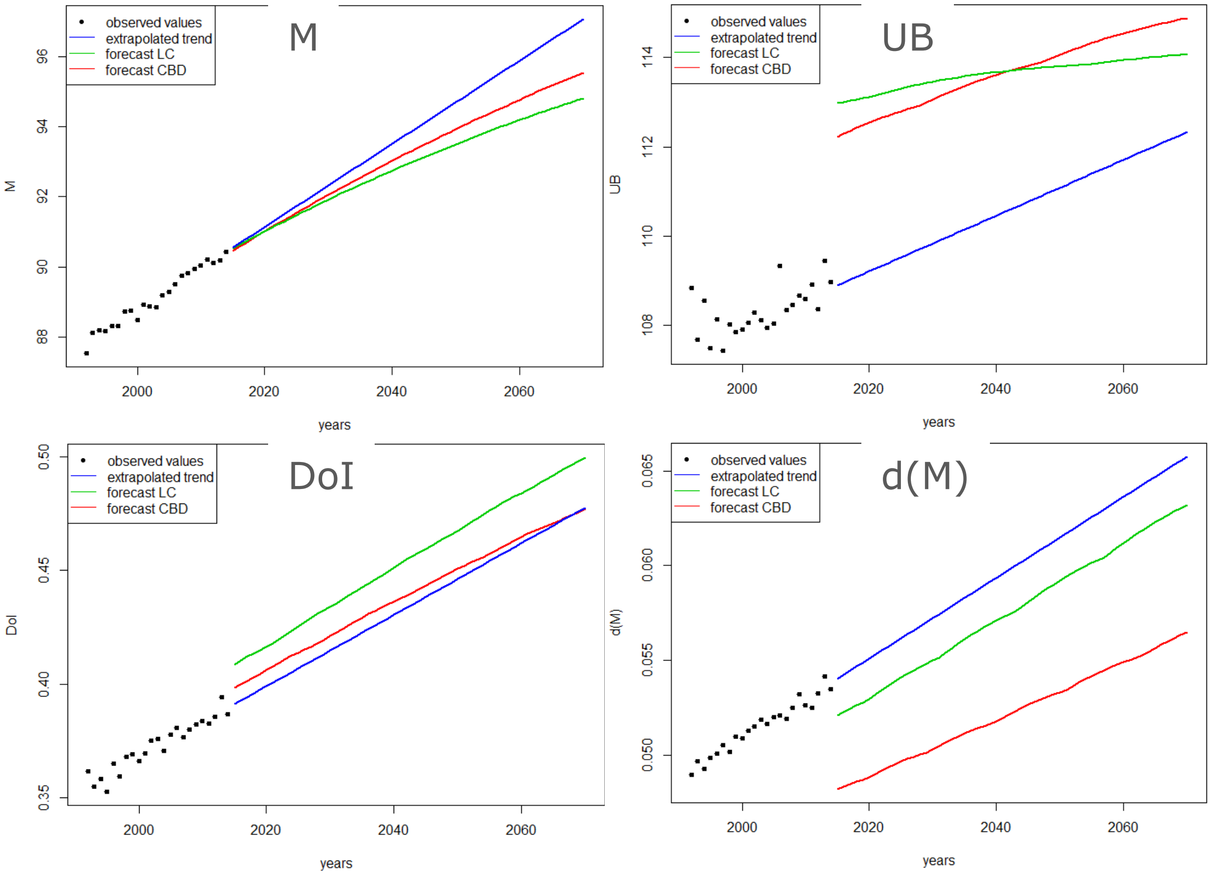
- The deaths curve (i.e. the density function of the distribution of lifetimes) typically changes over time and a relevant question is, **how the deaths curve changes over time**.
- In literature there have been many different approaches for the classification of mortality evolution patterns, and we detected some shortcomings inherent in these approaches (see Börger et al, 2018).
- Therefore, we introduced a **unique classification framework for mortality evolution patterns** (for details see Börger et al., 2018). This framework, …
  - … eliminates the shortcomings of previous approaches, …
  - … builds on four statistics on the deaths curve, and …
  - … gives a comprehensive picture of the deaths curve's evolution in the past.
- With the model we present here, we want to exploit the understanding of the deaths curve's evolution obtained from this framework. Therefore, it is based on a prediction of these four statistics.
- The statistics of the classification framework of Börger et al. (2018) are introduced in what follows.

The Classification Framework of Börger et al. (2)



Consistency Issues in Existing Forecasting Models

- Börger et al. (2018) also suggest a method for **detection of trends** in the respective time series which is based on the assumption of **piecewise linear trends** in each statistic's time series.
- A mortality forecast can be regarded as **demographically reasonable**, if the most recent trends in these four statistics are **smoothly continued**.
- We forecast mortality with the LC and the CBD mortality models. To this end, we use mortality data for Swiss females from 1992 to 2014.
- We determine the respective forecasts for the four statistics:

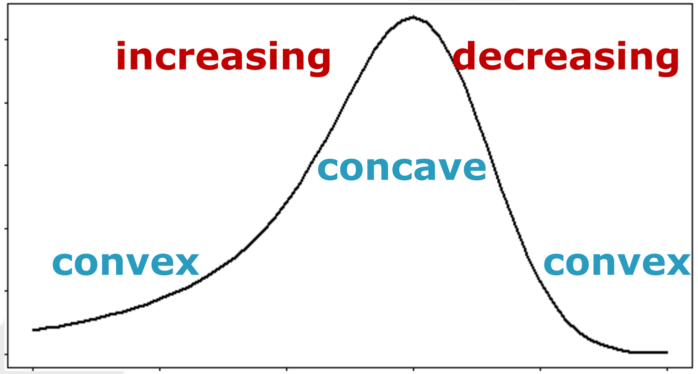


Data obtained from the Human Mortality Database (HMD, 2015).

- Three of the four statistics exhibit significant jumps at the transition from the calibration to the forecasting period in both mortality models.
- This indicates that the extrapolated deaths curves do not exhibit smooth changes at this transition.

A New Best Estimate Mortality Model – Theoretical Concept

- We assume that a prediction $(M_t, UB_t, DoI_t, d(M)_t)$ of the four statistics for any year t of the forecasting period is given.
- As the deaths curve is a (special) density function, we start at the **space of all density functions** on the interval between the starting age x_0 and UB_t. We denote this space by $\mathcal{D}_{t,x_0}$.
- We observed, that the deaths curve has a **typical shape**: For example, a typical deaths curve…
  - … is unimodal for sufficiently high starting age, …
  - … is monotonically increasing left of the mode, …
  - … monotonically decreasing right of the mode, …
  - … convex at the left and right tail, …
  - … concave around the mode.



- Thus in our model, we choose a set of **deaths curves with reasonable shape** from the aforementioned space and we denote this set as D_{t,x_0}.
- For any future year, a potential deaths curve's forecast is a density function from this set, which **fits the forecasts of our four statistics** $(M_t, UB_t, DoI_t, d(M)_t)$. The set of these deaths curves is denoted with $\overline{D}_{t,x_0}$.

A New Best Estimate Mortality Model – Practical Implementation (1)

- In what follows, we will give the most important ideas of our practical implementation of the new mortality model.
- For details we refer to Börger et al. (2019)

A New Best Estimate Mortality Model – Practical Implementation (2)

- For an implementation of this procedure a feasible representation of the deaths curve must be chosen. We use a **B-Spline representation of the deaths curve** with 21 B-splines of fifth degree, such that the deaths curve can be written as

$$\hat{d}_t(x) = \sum_{j=1}^{21} a_t^{(j)} \cdot b_t^{(j)}(x) = B_t(x) * a_t,$$

where $B_t\colon \mathbb{R} \to \mathbb{R}^{21}$ is a vector-valued function and $a_t = \left(a_t^{(1)}, \ldots, a_t^{(21)}\right)^T$ is a vector of spline weights.

- Each spline $b_t^{(j)}(x)$, $j \in \{1, \ldots, 21\}$ is centered at its so-called knot $k_t^{(j)}$ and symmetric around this knot. Furthermore, it is different from zero only on a certain interval around this knot.
- In what follows, we always start in a year t_0, where we are given a deaths curve d_{t_0} and its B-spline representation with $k_{t_0}^{(j)}$, $j \in \{1, \ldots, 21\}$ and a_{t_0}.

To specify a concrete spline representation of a deaths curve for a future year t, we need to determine:

- the **spline knot positions** $k_t^{(j)}$, $j \in \{1, \ldots, 21\}$
- the **spline weights** $a_t = \left(a_t^{(1)}, \ldots, a_t^{(21)}\right)^T$

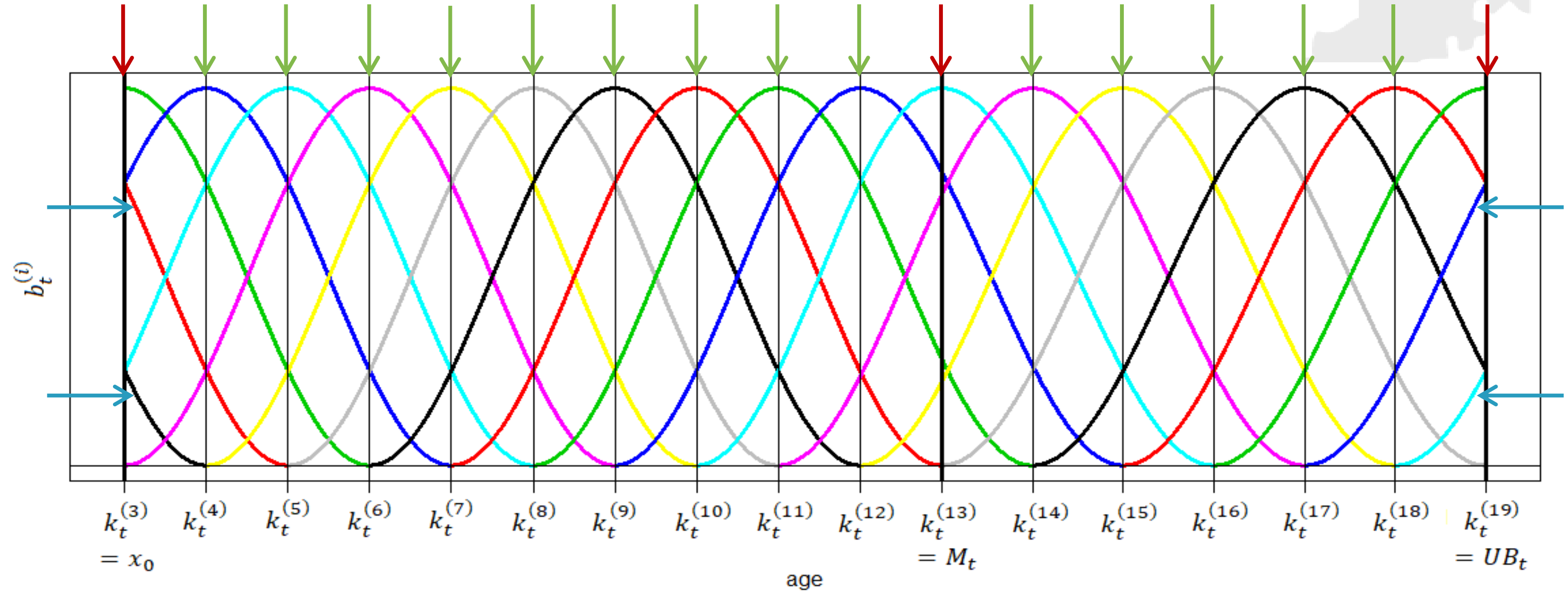
A New Best Estimate Mortality Model - Practical Implementation (3)

For the **positioning of the splines** we proceed as follows:

Step 1: We set one knot at each endpoint of the deaths curve's support, i.e. at x_0 and at UB_t, and at the modal age at death M_t.

Step 2: Between x_0 and M_t we add nine equidistant knots and between M_t and UB_t we add five equidistant knots.

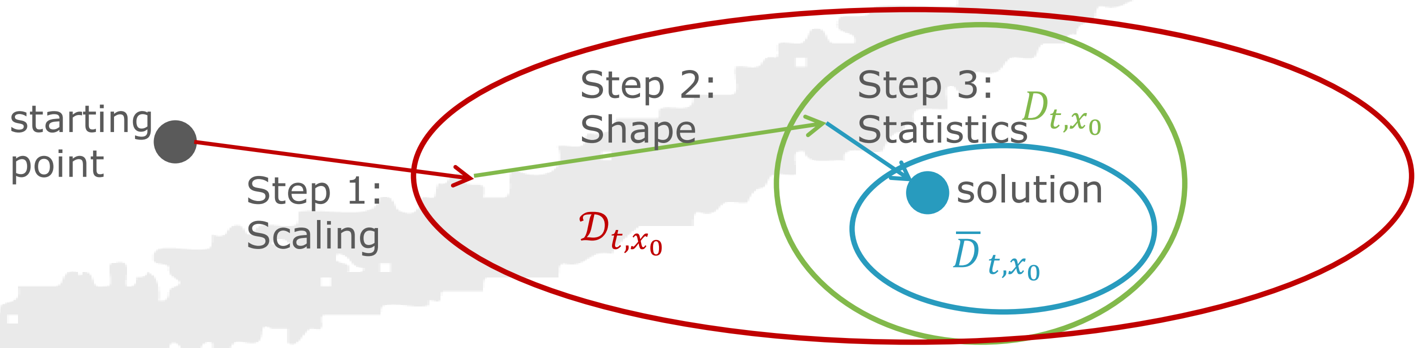
Step 3: For sufficient flexibility at the boundaries of the support we need two additional knots left of x_0 and right of UB_t which are positioned such that all knots left or right of M_t are equidistant.



A New Best Estimate Mortality Model - Practical Implementation (4)

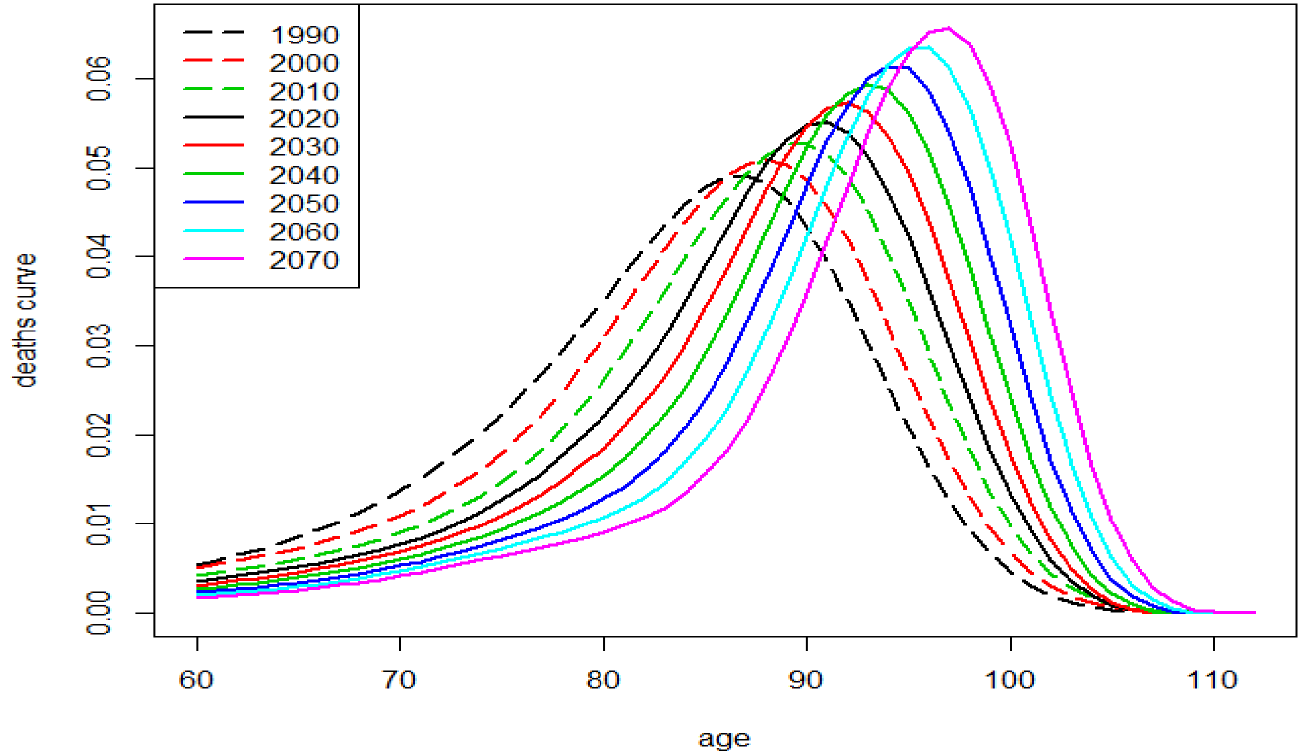
For the **determination of spline weights** we proceed as follows:

- **Step 1:** We determine a constant c such that $\int_{x_0}^{UB_t} \tilde{d}_t(x)dx = \int_{x_0}^{UB_t} B_t(x) * c \cdot a_t dx = \int_{x_0}^{UB_t} B_t(x) * \tilde{a}_t dx = 1$, i.e. $\tilde{d}_t(x) \in \mathcal{D}_{t,x_0}$.
- **Step 2:** We minimize a vector-valued function in the vector of spline weights for the shape requirements with the method of gradient descent to alter the vector $\tilde{a}_t$, such that $\tilde{d}_t(x) = B_t(x) * \tilde{a}_t \in D_{t,x_0}$.
- **Step 3:** We minimize a vector-valued function in the vector of spline weights for the requirements on the four statistics again with the method of gradient descent to alter the vector $\tilde{a}_t$, such that $\tilde{d}_t(x) = B_t(x) * \tilde{a}_t \in \bar{D}_{t,x_0}$.
- After this final modification, the vector $a_t = \tilde{a}_t$ is the solution of this algorithm.



Examples – Extrapolation of the Most Recent Trend (1)

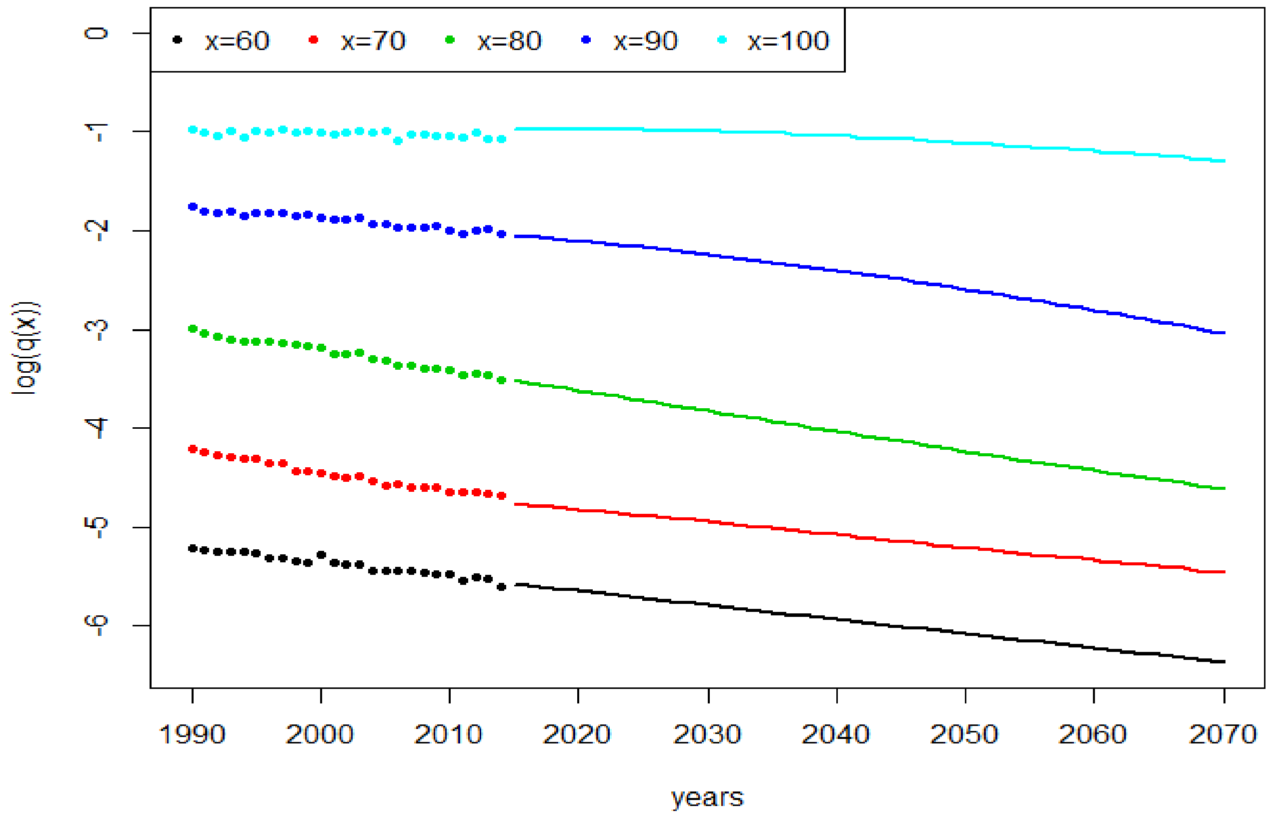
- For the following examples we use mortality data for Swiss females obtained from the HMD (2015).
- In a first example, we **linearly extrapolated the most recent trend for each statistic until 2070** and determined a deaths curve for each calendar year:



We can see, that the deaths curves are extrapolated in a reasonable way.

Examples – Extrapolation of the Most Recent Trend (2)

For these forecasts we also determined $\log(q_t(x))$ for selected ages from 60 to 100:



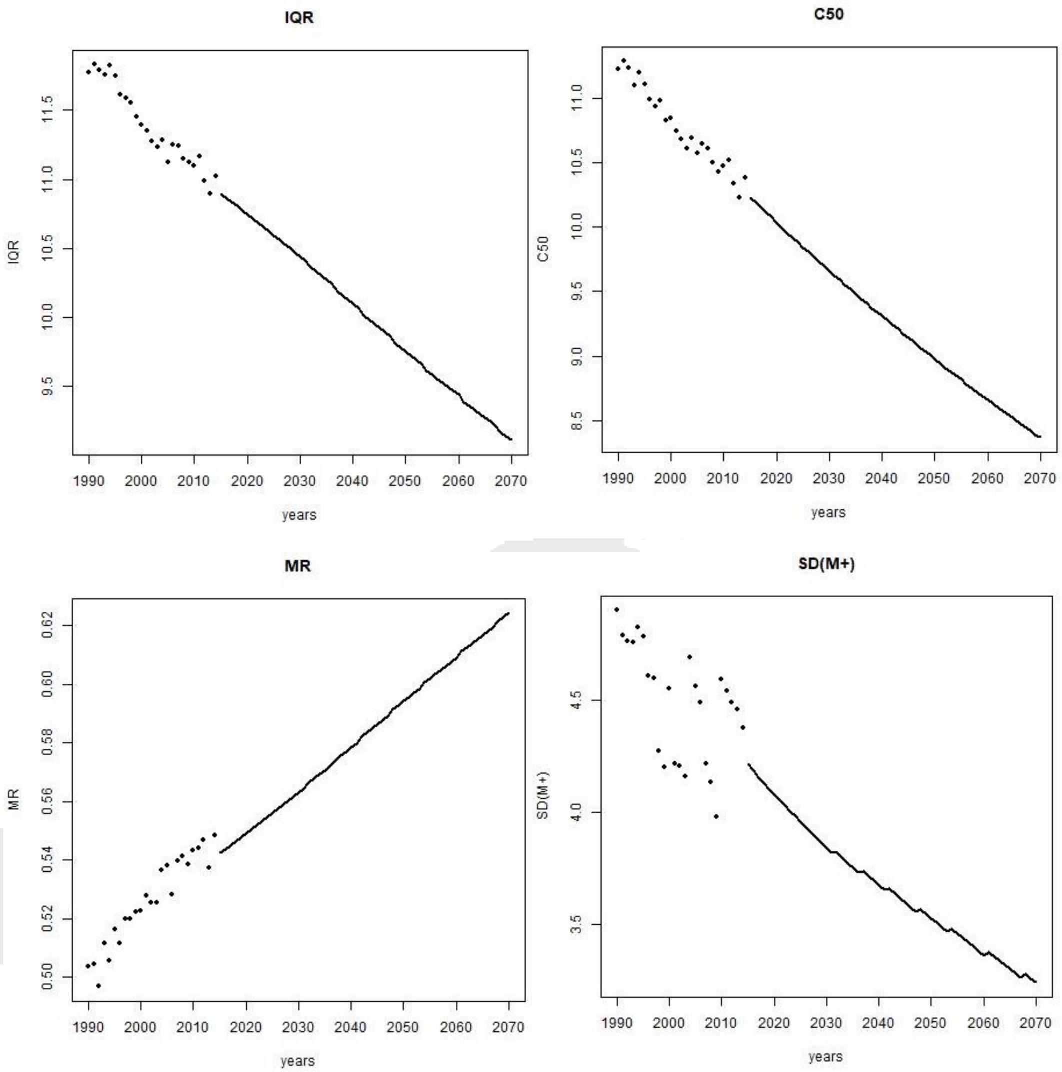
We can see that these are also reasonably extrapolated to the future.

Examples – Extrapolation of the Most Recent Trend (3)

Also for other demographical statistics we find that the most recent trends are reasonably extrapolated until 2070 with our model:

As examples we chose (besides others)…

- …the Inter-Quartile-Range IQR (see Wilmoth and Horiuchi, 1999),
- … the length of the shortest age-intervall in which 50% of the population are dying, $C50$ (see Kannisto, 2000),
- … the Moving Rectangle MR (see Wilmoth and Horiuchi, 1999), and
- … the standard deviation of the age distribution at death above the modal age at death $SD(M+)$ (see e.g. Kannisto, 2001).



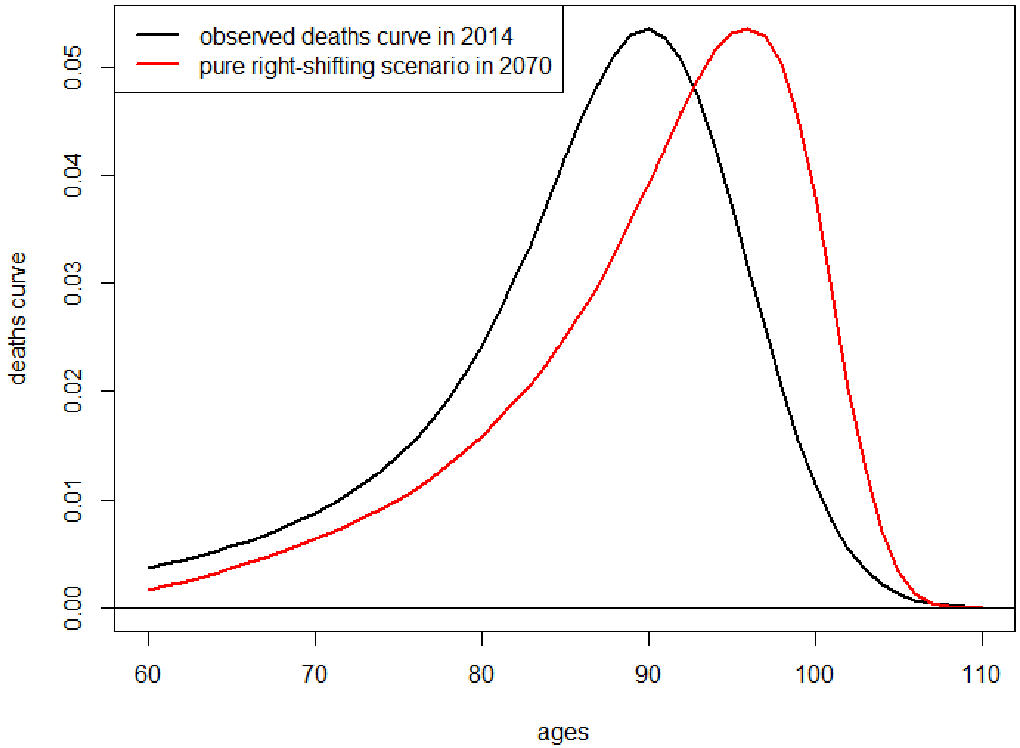
Examples – Special Scenarios (1)

- The examples presented so far are based on an extrapolation of the most recent observed trend of each statistic. Alternatively, the trends in the four statistics can also be altered in our model.
- In our paper we therefore analyze…
  - … pure scenarios, i.e. scenarios where only one component continues its trend while the other three components stay neutral, and
  - … a stress scenario, i.e. a scenario where (in our case) two components intensify their increase.
- These examples illustrate how expert's opinion can be used for mortality forecasts.

Examples – Special Scenarios (2)

Pure Scenarios

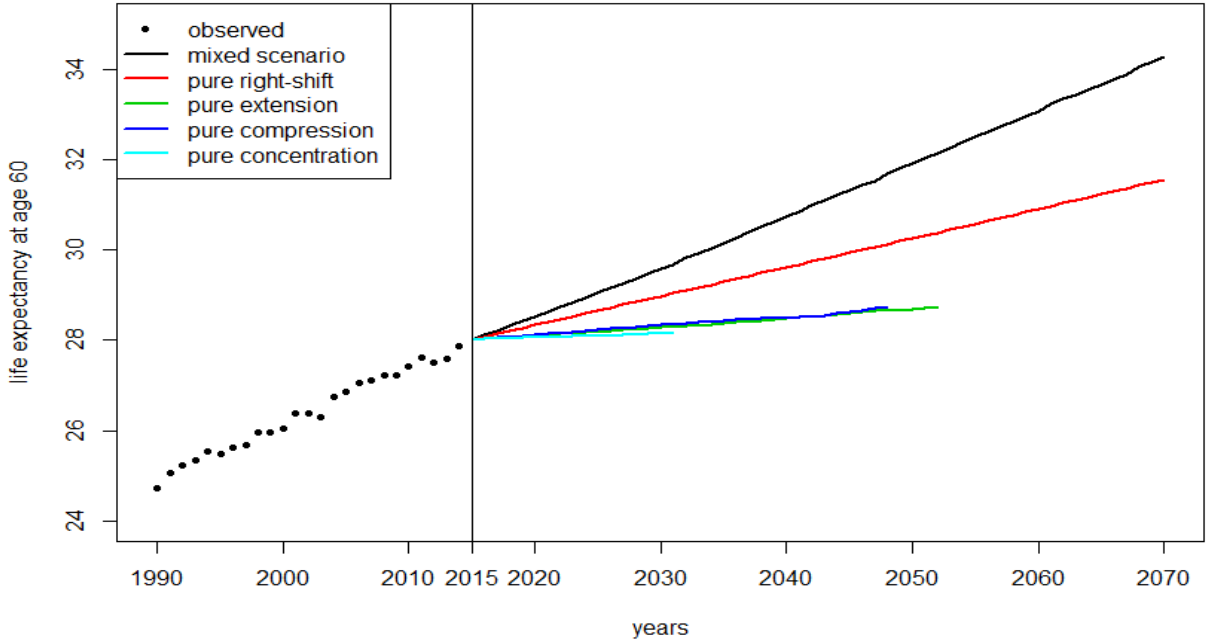
- For each pure scenario we compare the deaths curve in 2014 to the last future year where we still have reasonable deaths curves.
- Here we only show the pure right-shifting mortality scenario.
- For the other pure scenarios we refer to Börger et al. (2019).



Examples – Special Scenarios (3)

Pure scenarios

For each pure scenario as well as for the base scenario (i.e. extrapolation of the most recent trend) we determined the **remaining period life expectancy at the starting age 60** $e(60)$:



- In each pure scenario, the increase of the forecasts of $e(60)$ is slower than in the mixed scenario.
- The increase in $e(60)$ in the pure right-shifting scenario is faster than in any other pure scenario.
- Note that in all pure scenarios (except right-shifting mortality) we did not obtain reasonably shaped deaths curves for each year of the extrapolation horizon (for details see Börger et al., 2019).

Examples – Special Scenarios (4)

Stress scenario

- As an example for a stress scenario, we use the base scenario but **double the intensity of right-shifting mortality and extension in 2015**.
- Demographically this expert scenario means that the position of the deaths curve will change twice as fast in the future than recently observed, while **trends in the deaths curve's shape remain unaltered**.
- In this scenario we obtained reasonably shaped deaths curves until 2070.
- To illustrate the differences between the base scenario and the stress scenario we calculated the **cohort life expectancy of a 60-year old Swiss female in 2015** (i.e. with year of birth 1954).
- This figure increases from 30.4 years in the base scenario to 32.1 years in the stress scenario which is **an increase of more than 5.4%**.

Conclusion

- In the paper we discuss the **plausibility of mortality forecasts from a demographic perspective**:
  - Trends in key demographic figures should be reasonably extrapolated into the future.
  - We illustrate this with two well-known mortality models: the Lee-Carter and the Cairns-Blake-Dowd mortality model.
- We develop a **new deterministic mortality model** which …
  - … builds on four demographic statistics on the deaths curve, …
  - … forecasts future mortality in a demographically reasonable manner, and …
  - … allows for the incorporation of expert’s opinions.
- We discuss issues in the practical implementation of this model.
- We exemplarily illustrate the benefits of the model with different forecast mortality scenarios.

Thank you for your attention!

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References (1)

Presentation is based on:

Börger, M., Genz, M., and Ruß, J. (2019). The Future of Mortality: Mortality Forecasting by Extrapolation of Deaths Curve Evolution Patterns. Working Paper. Retrieved from https://www.ifa-ulm.de/fileadmin/user_upload/download/forschung/2019_ifa_Boerger_-etal_The-Future-of-Mortality-Mortality-Forecasting-by-Extrapolation-of-Deaths-Curve-Evolution-Patterns.pdf

Cited in the slides:

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Technical paper for B-splines:

Schoenberg, I.J. (1946). Contributions to the problem of approximation of equidistant data by analytic functions. *Quarterly of Applied Mathematics 4*(1–2): 45–99, 112–141.