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The Risk Adjusted Scenario Set 1
A Tool for Quantitative ERM

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Introduction

- Motivation
- The Raw Math i.e., what is the RASS?
  - Financial Engineering approach
  - Actuarial approach – gets the same answer
  - Sensitivity to input assumptions, Illiquid instrument pricing
  - How do RASS values roll forward in time? Tools for A/LM
- Numerical Example - US Long Term Care, Canadian Term to 100
  - Use of illiquid assets
- Summary and Overview of practical applications
  - Yield curve & implied vol surface extrapolation
  - Using credit risky assets to build an illiquidity premium into a valuation
  - Financial reporting
  - Risk management and A/L M
- Conclusions and further work needed
  - What are "appropriately risk adjusted cash flows"?
- Appendix: simple analytic example - vanilla equity put option

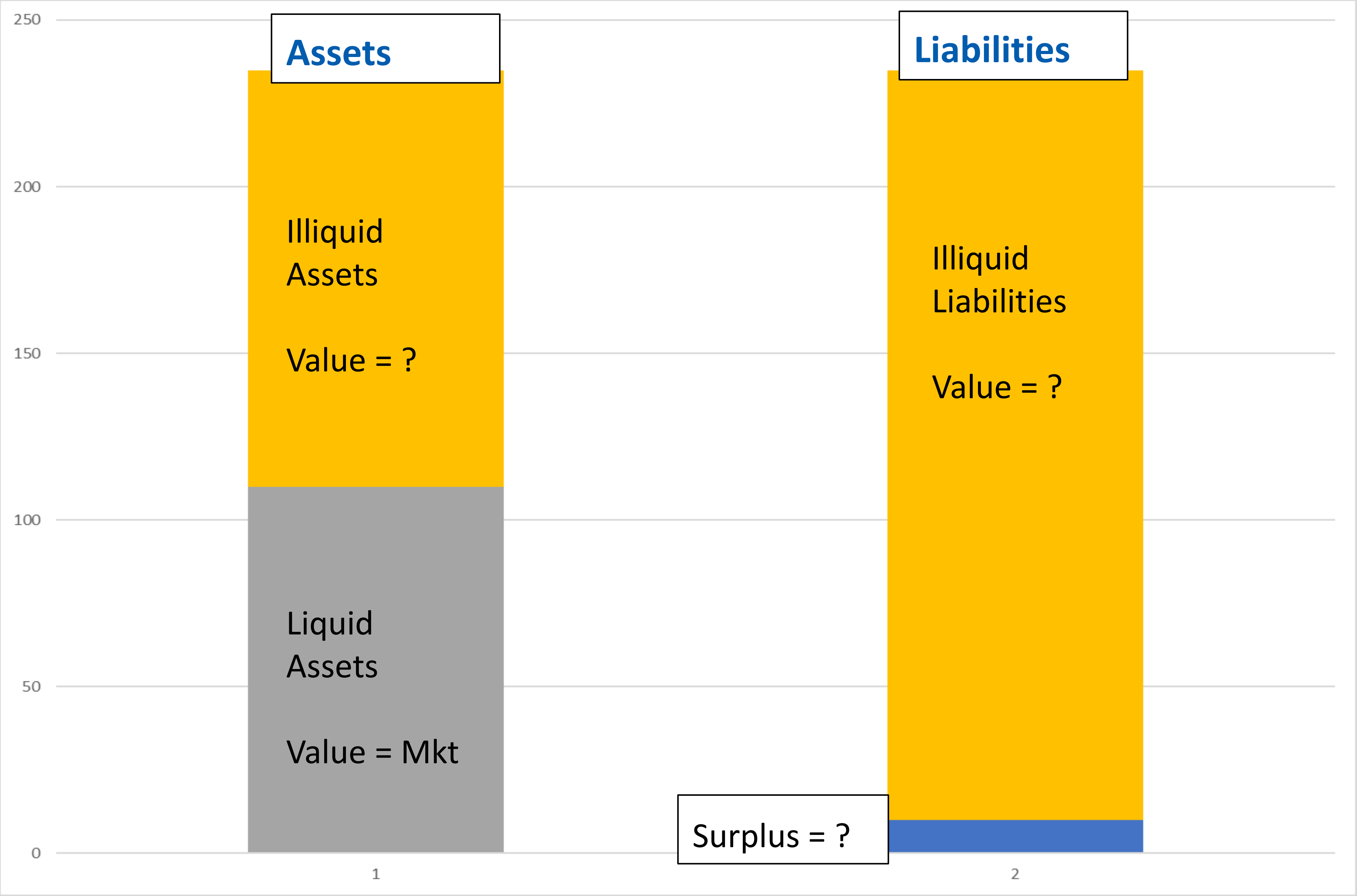
Motivation

- Today's financial actuary looks at the life insurance business through several different "lenses"
  - Regulatory : emphasis on solvency, balance sheet
  - Accounting: emphasis on income measurement
  - Economic: emphasis on risk management, A/L M
- Good News: all moving in a "market consistent" direction
- Bad News: competing priorities: Which one is "real money"?
  - Regulators uncomfortable assuming delta hedging will always work
  - Accounting rules not always consistent with dynamic hedging
  - Life insurance a complex mix of hedgeable and non-hedgeable risks
- Other Issues:
  - Financial engineering inherently prospective, market value oriented
  - traditional actuarial perspective is basically retrospective and book value oriented, e.g., traditional participating (with profits) insurance products
  - Insurance industry relies on the liquidity premium available with many illiquid assets

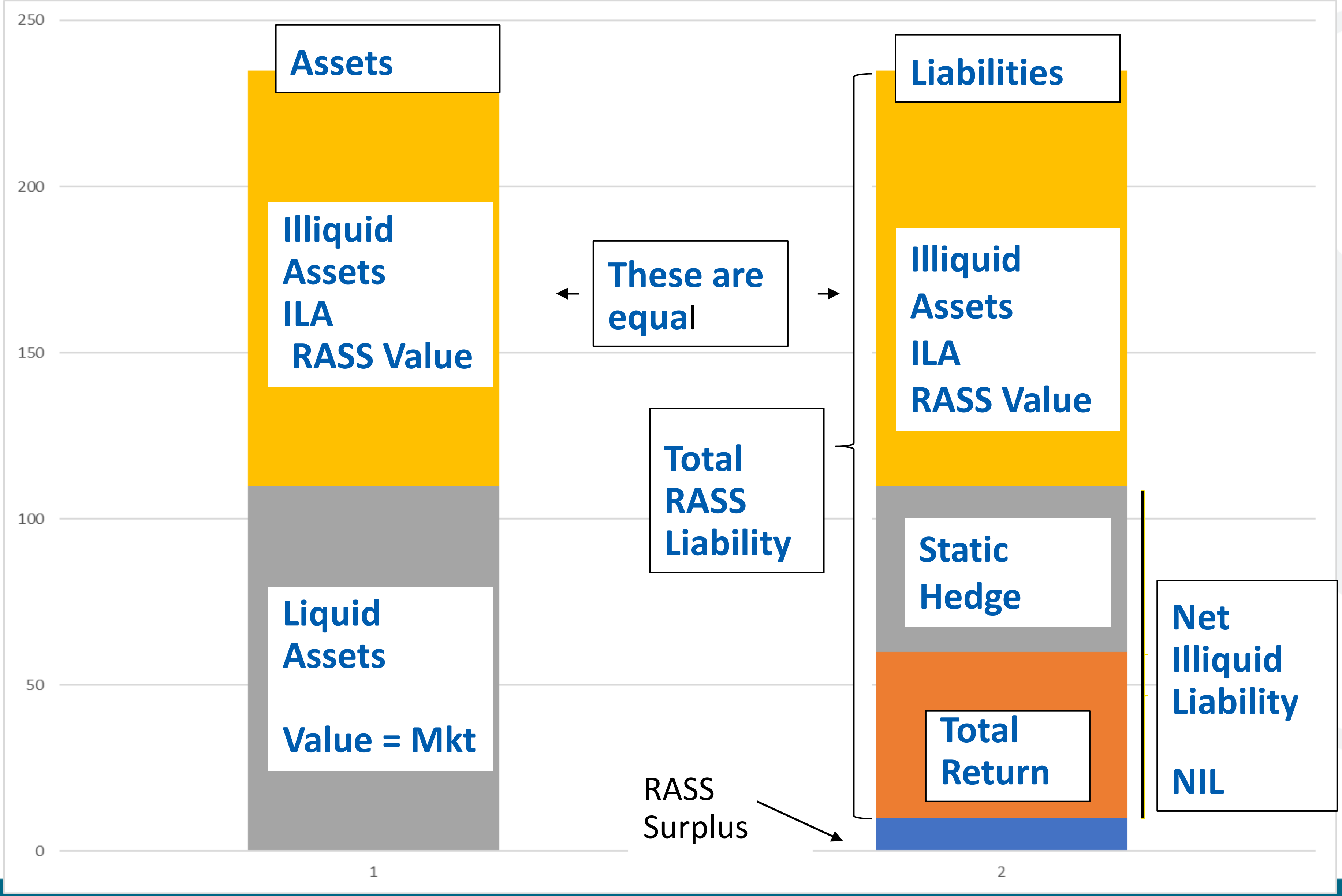
Motivation

- Risk Adjusted Scenario Set (RASS): a tool which has the theoretical power to bridge some of the gaps
  - Can fill “holes” in observable markets (e.g., long yields), long dated options
  - Decomposes a complex life insurance risk into hedgeable and non-hedgeable components ,
  - Can use illiquid and credit risky assets and capture an observed illiquidity premium
  - Can even handle blocks of participating insurance contracts, if you work hard enough
  - Acceptable to all parties? e.g., risk managers, accountants, regulators, financial engineers etc. We’ll see
  - The author’s hope: Each of these professional constituencies could start with the RASS model and then make a small number of adjustments to meet their needs

What is a Market Consistent Balance Sheet ?



Where are we going? The RASS balance sheet



The Raw Math: Starting Point

- Ideas first developed in Canada by CIA around 2001
- Immediate problem: put a value on segregated fund guarantees for Canadian GAAP purposes

Method:

1. Start with a suitably large set of N real world economic scenarios $\mathcal{S}$ (guidelines to prevent "gaming")
2. Project liability "risk adjusted" cash flows (LCF) over each scenario $A \epsilon \mathcal{S}$ and time point $t = 1, \dots, T$ get an array LCF_{tA}
3. Discount liability cash flows using short-term interest rates for each scenario to get a PV vector $L_A = \sum_t v_{tA} LCF_{tA}$
4. Canadian GAAP reserves set at $V = CTE_a[L_A]$ eg. $a = 60\%$
   What is a Conditional Tail Expectation (CTE)? See slide 16
5. Reserves + Capital set at a higher CTE level e.g., $a = 95\%$

- Reasonable first crack at "stochastic modelling" (e.g., simple)
- No assumed risk management → accepted by Canadian regulator
- "quasi closed" model – unlike US regulatory approach
- A disaster from a financial engineering theory viewpoint ☺
- This approach is a very simple example of a RASS

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RASS Model : Financial Engineering Approach

1. Start with a suitably large set of N real world random economic scenarios $\mathcal{S}$, Label them with an index $A = 1, \ldots, N$

2. Choose an "appropriate" set of linearly independent hedge instruments $\mathcal{H}$ such as bonds, swaps, options etc. Hedge instruments need not be on the risk entity's balance sheet
   - Project "appropriately risk adjusted" cash flows for each hedge instrument.
   - Result is an array $HCF^{\alpha}{}_{tA}$ for each $\alpha \epsilon \mathcal{H}$, $t =$ valuation date, $\alpha = 1, \ldots, m$

3. Let Z^{α} be the observed market price of hedge instrument α, at the valuation date

4. Choose an asset to act as numeraire – returns on this asset will be used for discounting. Examples - bank account, stock index, bond fund etc.. Let $v_{tA.} > 0$ be the discount factor from time t to the valuation date on scenario A

5. Choose a CTE level a eg. $a = 60\%$

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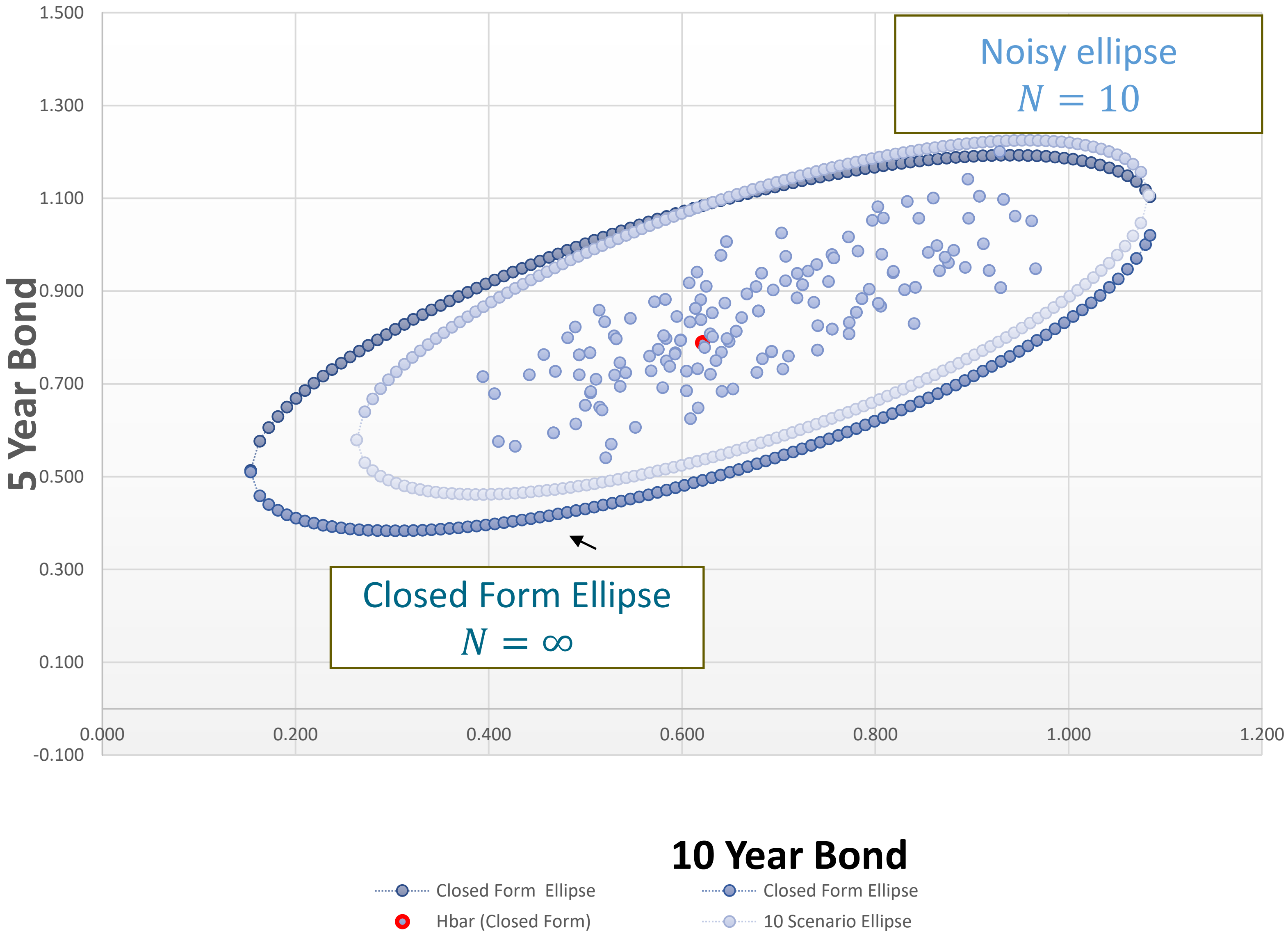
Market Consistency – John M's Approach

- Compute hedge instrument present values $H^{\alpha}{}_{A} = \sum_t v_{tA}\, HCF^{\alpha}{}_{tA}$
- Introduce a set of scenario weights $\lambda^A \; A = 1, \dots, N$
- Subject to linear constraints
  - $\lambda^A \geq 0,$ reasonable and intuitive
  - $\sum_A \lambda^A = 1,$ also intuitive
  - $\sum_A H^{\alpha}{}_{A}\lambda^A = Z^{\alpha},$ intuitive calibration constraints
  - $\lambda^A \leq \frac{1}{N(1-a)}.$ I'll explain this later
- Model is considered feasible if there are scenario weights satisfying the linear constraints
- A necessary condition for feasibility is that the CTE parameter a is large enough
- Can estimate a lower bound a^* for feasibility
- Conclusion: There are either no market consistent scenario weight sets or there are many

Estimating a^*

- Calculate: $\bar{H}^{\alpha} = \frac{\sum_A {H^{\alpha}}_A}{N}$ expected value of hedge instrument α in P measure
- $\Sigma^{\alpha\beta} = \frac{\sum_A ({H^{\alpha}}_A - \bar{H}^{\alpha})({H^{\beta}}_A - \bar{H}^{\beta})}{N}$ covariance matrix
- $\Sigma_{\alpha\beta} = (\Sigma^{\alpha\beta})^{-1}$ if covariance matrix is not invertible the chosen set $\mathcal{H}$ of hedge instruments is not linearly independent, revise the chosen set $\mathcal{H}$
- $\chi^2 = \sum_{\alpha,\beta} (Z^{\alpha} - \bar{H}^{\alpha}) \Sigma_{\alpha\beta} (Z^{\beta} - \bar{H}^{\beta})$ measures deviation of market prices from mean prices
- $\chi^2 < \frac{a}{1-a}$ is a necessary condition for the model to be feasible
- $a > a^* = \chi^2/(1+\chi^2)$ is a practical tool for estimating a lower bound for CTE parameter a
- If model is not feasible may need to bump up the CTE parameter a

Feasibility Ellipse 2 Hedge Instruments, 10 scenario example, $a = 60\%$



What are the interior dots?

- With 10 scenarios and $a = 60\%$ there are $\binom{10}{4} = 210$ “vertex points” where 4 scenarios get a weight of $\lambda = 1/4$ and the remaining 6 scenarios get $\lambda = 0$.
- The Euclidean distance (in λ space) between a vertex point and $\bar{\lambda} = (\frac{1}{N}, \ldots, \frac{1}{N})$ is easy to compute if $Na = an\ integer$.
- $D^2 = Na(\frac{1}{N} - 0)^2 + N(1-a)(\frac{1}{N} - \frac{1}{N(1-a)})^2 = a/[N(1-a)]$
- Applying the linear map defined by $H^{\alpha}{}_{A}$ to the vertex points results in the interior dots
- The feasible region C is the set of Z^{α} points that lie inside the convex hull of the interior dots (for those 10 scenarios)
- As $N \to \infty$ the two ellipses converge, get many more vertex points
- Question: As $N \to \infty$ will the vertex points fill up the ellipse?
- Practical experience suggests the answer is usually yes but there are counter examples, see the appendix

RASS Model : Financial Engineering Version

- The story so far: There is a set F of feasible scenario weights which may be empty or have many possible feasible scenario weight sets
  - Which one do we choose for the RASS?

1. Project "appropriately risk adjusted" liability cash flows (LCF) over each scenario and time point t, get an array LCF_{tA}, $A \in \mathcal{S}, t = 1, \dots, T$
2. Compute present values $L_A = \sum_t v_{tA} LCF_{tA}$
3. Project "appropriately risk adjusted" illiquid asset cash flows $(ILACF)$ over each scenario and time point t, get an array $ILACF_{tA}$, $A \in \mathcal{S}, t = 1, \dots, T$
4. Compute present values $ILA_A = \sum_t v_{tA} ILACF_{tA}$
5. Several Options
   - Option 1: choose the weights to maximize the liability present value
   - $V(\boldsymbol{L}, \boldsymbol{H}, \boldsymbol{Z}, a) = \max_{\boldsymbol{\lambda}} \sum_A L_A \lambda^A$
   - Option 2: choose the weights to maximize the net illiquid liability present value

     $NIL(\boldsymbol{L}, \boldsymbol{ILA}, \boldsymbol{H}, \boldsymbol{Z}, a) = \max_{\boldsymbol{\lambda}} \sum_A (L_A - ILA_A) \lambda^A$ This is my preferred option

When combined with the linear constraints for feasibility both options define linear programming problems. Both can be useful.

RASS Model : Actuarial Version

- No matter which version of the optimization problem we pick the linear inequalities $0 \leq \lambda^A \leq \frac{1}{N(1-a)}$ mean that the optimization process drives most of the weights to either 0 or $\frac{1}{N(1-a)}$
- Very few scenarios end up with weights $0 < \lambda^A < \frac{1}{N(1-a)}$ and, in practice, can often be ignored, they can also be useful
- If this looks like a CTE calculation that's because it is
- Linear Programming Theorem #1
  - Every feasible linear program has a dual version that gives us the same answer
- Option 1 dual: Find a set of unconstrained portfolio weights b_α which minimizes the following

$$V(\boldsymbol{L},\boldsymbol{H},\boldsymbol{Z},a) = \min_{b_\alpha}[\sum_\alpha b_\alpha Z^\alpha + CTE_a(L_A - \sum_\alpha b_\alpha {H^\alpha}_A)]$$

  - First term is the static hedge portfolio, second term is the total return piece
  - Conclude $V = \max_{\boldsymbol{\lambda}} \sum_A L_A \lambda^A = \min_{b_\alpha}[\sum_\alpha b_\alpha Z^\alpha + CTE_a(L_A - \sum_\alpha b_\alpha {H^\alpha}_A)]$

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Theoretical Fine Point

- There are two useful definitions of CTE which are almost, but not quite, the same

- Practitioner's Def'n: $CTE_a(X) = E[(X|X \geq Q_a(X)]$ where $\Pr[X \leq Q_a(X)] = 1 - a$
- Stanislav Uryasev's (2000) Def"n: $CTE_a(X) = \min_{Q}\{Q + E[\max(X - Q, 0)]/(1 - a)\}$
- In practice, the two definitions are not materially different if the number of scenarios N used is appropriately large
- For mathematically precise theoretical work Uryasev's def'n is preferrable, need to use this def'n for the duality result to hold precisely
- For most practical work the first def'n is just fine

- See Uryasev's website for many useful risk management papers
  - In particular "Conditional Value at Risk: Optimization Algorithms and Applications" in the February 2000 edition of *Financial Engineering News*.

RASS Model : Actuarial Version (Option 2)

1. Linear Programming Theorem #1
   - Every feasible linear program has a dual version that gives us the same answer

- Option 2 dual: Find a set of portfolio weights b_α which minimizes the following

$$NIL(\boldsymbol{L}, \boldsymbol{H}, \boldsymbol{Z}, a) = \min_{b_\alpha}[\sum_\alpha b_\alpha Z^\alpha + CTE_a(L_A - ILA_A - \sum_\alpha b_\alpha H^\alpha{}_A)]$$

   - First term is the static hedge portfolio, second term is the total return piece

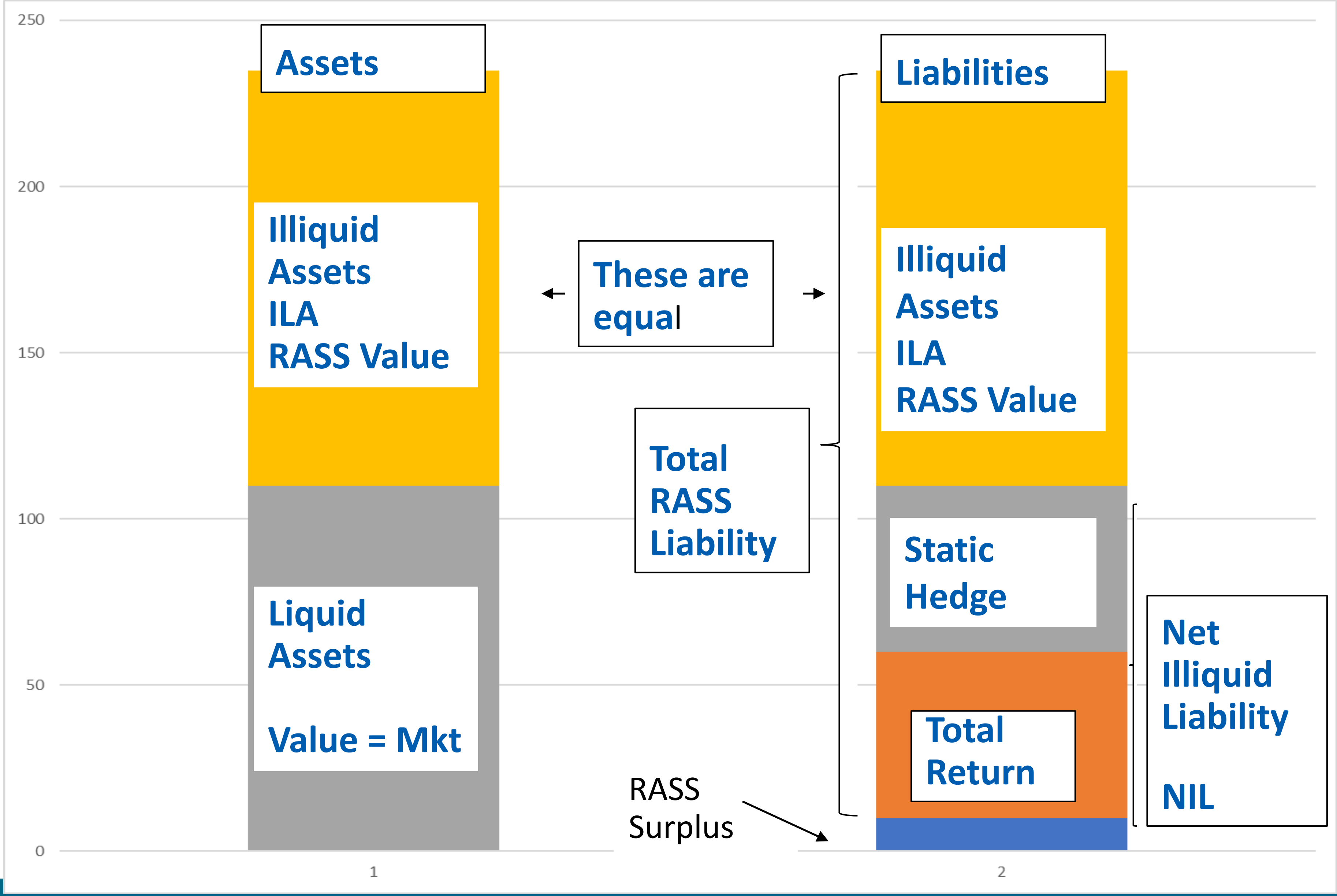
- Conclude $NIL = \max_{\boldsymbol{\lambda}} \sum_A (L_A - ILA_A)\, \lambda^A = \min_{b_\alpha}[\sum_\alpha b_\alpha Z^\alpha + CTE_a(L_A - ILA_A - \sum_\alpha b_\alpha H^\alpha{}_A)]$

- Under option 2 we can then write (using optimal scenario weights and portfolio weights)

- $V = \sum_A L_A \lambda^A = \sum_A ILA_A \lambda^A + \sum_\alpha b_\alpha Z^\alpha + CTE_a[L_A - ILA_A - \sum_\alpha b_\alpha H^\alpha{}_A]$

- This is the result promised back on slide 6, we now have market consistent values for all three provinces of the balance sheet

The RASS balance sheet Option 2



Sensitivities – First Order

- Dual is feasible if the CTE level a is large enough and the hedge instrument input data $H^{\alpha}{}_{A}, Z^{\alpha}$ are internally consistent
- Linear programming literature gives us the following:

- $\frac{\partial NIL}{\partial Z^{\alpha}} = b_{\alpha}$ candidate for a static hedge portfolio, may not be the same as a financial engineer's "greek", more to come
- $\frac{\partial NIL}{\partial L_{A}} = \lambda^{A}$ useful for correcting errors, presenting results and understanding the impact of adding new business or new illiquid assets
- $\frac{\partial NIL}{\partial H^{\alpha}{}_{A}} = -\lambda^{A}\, b_{\alpha}$ need this to understand roll forward in time and reconcile to the financial engineer's concept of a "greek"
- $\frac{\partial NIL}{\partial a} = \frac{(CTE - Q)}{1-a} = \sum_{A} \frac{\max(0, NIL_{A} - \sum_{\alpha} b_{\alpha} H^{\alpha}{}_{A})}{N(1-a)^{2}} \geq 0$

- These results apply when you are at the optimal point and the input shocks are not so large as to render the shocked problem infeasible

RASS Model : Roll Forward

Linear Programming Theorem #2 First order sensitivities

$$\Delta NIL = \sum_\alpha b_\alpha \, \Delta Z^\alpha + \sum_A \lambda^A (\Delta L_A - \Delta ILA_A) - \sum_{\alpha,A} \lambda^A \, b_\alpha \Delta H^\alpha{}_A + \frac{(CTE - Q)}{1-a} \Delta a$$

Dynamic greek $\Delta_\alpha = \frac{\partial NIL}{\partial Z^\alpha} = b_\alpha + \sum_A \lambda^A [\frac{\partial (L_A - ILA_A)}{\partial Z^\alpha} - \sum_\alpha b_\alpha \frac{\partial H^\alpha{}_A}{\partial Z^\alpha}]$

Static hedging and dynamic hedging need not be the same thing but for many simple problems they are

Model need not be self financing as time evolves

$$\Delta L_A - \Delta ILA_A - b_\alpha \Delta H^\alpha{}_A \approx \xi_A (L_A - ILA_A - b_\alpha H^\alpha{}_A) - (LCF_{1A} - ILACF_{1A} - b_\alpha HCF^\alpha{}_{1A})$$

Here ξ_A is the one period interest rate on scenario A so $\sum_A \frac{\lambda^A}{1+\xi_A} = \frac{1}{1+f_1}$

Can then calculate the **total return hurdle rate** $\xi = \frac{\sum_A \xi_A \lambda^A (L_A - ILA_A - b_\alpha H^\alpha{}_A)}{\sum_A \lambda^A (L_A - ILA_A - b_\alpha H^\alpha{}_A)}$

This is an estimate of the minimum rate we need to earn on the numeraire portfolio in order for the model to be self financing over the next time step

RASS Model : Roll Forward #2

One more tool (that you won't find in any text-book)

Need this for second order (convexity) analysis

Property of CTE: $CTE_a(X+Y) \leq CTE_a(X) + CTE_a(Y)$ reflects diversification benefit

But if the perturbation εY is small then

$$CTE_a(X+\varepsilon Y) \approx CTE_a(X) + \varepsilon E[Y|X \geq Q_a(X)] + \frac{1}{2}\varepsilon^2 VAR[Y|X = Q_a(X)]\frac{f_X(Q_a)}{1-a} + \cdots$$

First order term is consistent with the results we got from linear programming text-books

Can use the result above to show that if $\Delta_\alpha = \frac{\partial NIL}{\partial Z^\alpha} = b_\alpha$ then

$\Delta_{\alpha\beta} = \frac{\partial^2 NIL}{\partial Z^\alpha \partial Z^\beta} = -(Q^{\alpha\beta})^{-1}$ where $Q^{\alpha\beta} = COV[(H^\alpha, H^\beta)|NIL = Q_a(NIL)]\frac{f_{NIL}(Q_a)}{1-a}$

This is challenging but doable

Key Point #1: Convexity term comes in with a negative sign (good news for risk mgrs.)

Key Point #2: The RASS model can produce the kind of risk metrics that financial engineers or portfolio managers would want to see for the A/L M process

If $\Delta_\alpha \neq b_\alpha$ then there is more work to do

One Last Theoretical Result

- The RASS knows a lot about the business and the economic environment
- Question: How much does the risk adjusted scenario set (RASS), defined by the λ^A, know about the liability? For example, if someone gives us the λ^A could we reconstruct the Net Illiquid Liability present values NIL_A from that information?
- The general answer is NO (just linear algebra)
- Theorem: If $NIL'_A = NIL_A + \varphi + \sum_\alpha \emptyset_\alpha H_A^\alpha$ where $(\varphi, \emptyset_\alpha)$ are constants then
  - $\lambda'_A = \lambda^A$ and $NIL' = NIL + \varphi + \sum_\alpha \emptyset_\alpha Z^\alpha$
  - $b'_\alpha = b_\alpha + \emptyset_\alpha$
- Implication: two apparently different liabilities can give rise to the same RASS, but with different static hedge strategies
- Example: An equity put option and a call option will have the same RASS as long as the bond and stock are the hedge instrument and numeraire, or the other way around
- Analytic examples in the appendix will validate this claim

Summary of the Raw Math Option 2

Scenario Generation Step
- Real World Scenario Set $\mathcal{S}$
- Hedge Market Values Z^{α}

Illiquid Instrument Projection Step
- Cash Flows LCF_{tA}, $ILACF_{tA}$
- PVs $L_A = \sum_t v_{tA} LCF_{tA}$
- PVs $ILA_A = \sum_t v_{tA} ILACF_{tA}$

Liquid Asset Projection Step
- Numeraire v_{tA}
- Hedge Cash Flows $HCF^{\alpha}{}_{tA}$
- PVs $H^{\alpha}{}_A = \sum_t v_{tA}\, HCF^{\alpha}{}_{tA}$

CTE level $\boldsymbol{a}$

Optimization Step

$$V(\boldsymbol{L},\boldsymbol{H},\boldsymbol{Z},a) = \min_{\boldsymbol{b}}\{\sum_{\alpha} b_{\alpha} Z^{\alpha} + CTE_a[L_A - ILA_A - \sum_{\alpha} b_{\alpha}\, H^{\alpha}{}_A]\}$$
$$= \max_{\boldsymbol{\lambda}} \sum_A (L_A - ILA_A)\,\lambda^A$$

Key Outputs
- Liability Value $V(\boldsymbol{L},\boldsymbol{H},\boldsymbol{Z},a)$
- Static Hedge Portfolio weights b_{α}
- Calibrated Risk Adjusted Scenario weights λ^A, $\sum_A H^{\alpha}{}_A \lambda^A = Z^{\alpha}$
- Discount factors v_{tA}

Applications:
Pricing
A/L M
Financial Rptg.

Summary of the Raw Math – Other Results

- A necessary, but not sufficient, condition for the optimization problem to be feasible is

$$\chi^2 = \sum_{\alpha,\beta}(Z^\alpha - \bar{H}^\alpha)\Sigma_{\alpha\beta}\,(Z^\beta - \bar{H}^\beta) \leq \frac{a}{1-a}$$

or $$a \geq a^* = \chi^2/(1+\chi^2)$$

  - Practical experience suggests this is often good enough to be useful if the number of scenarios N is large enough

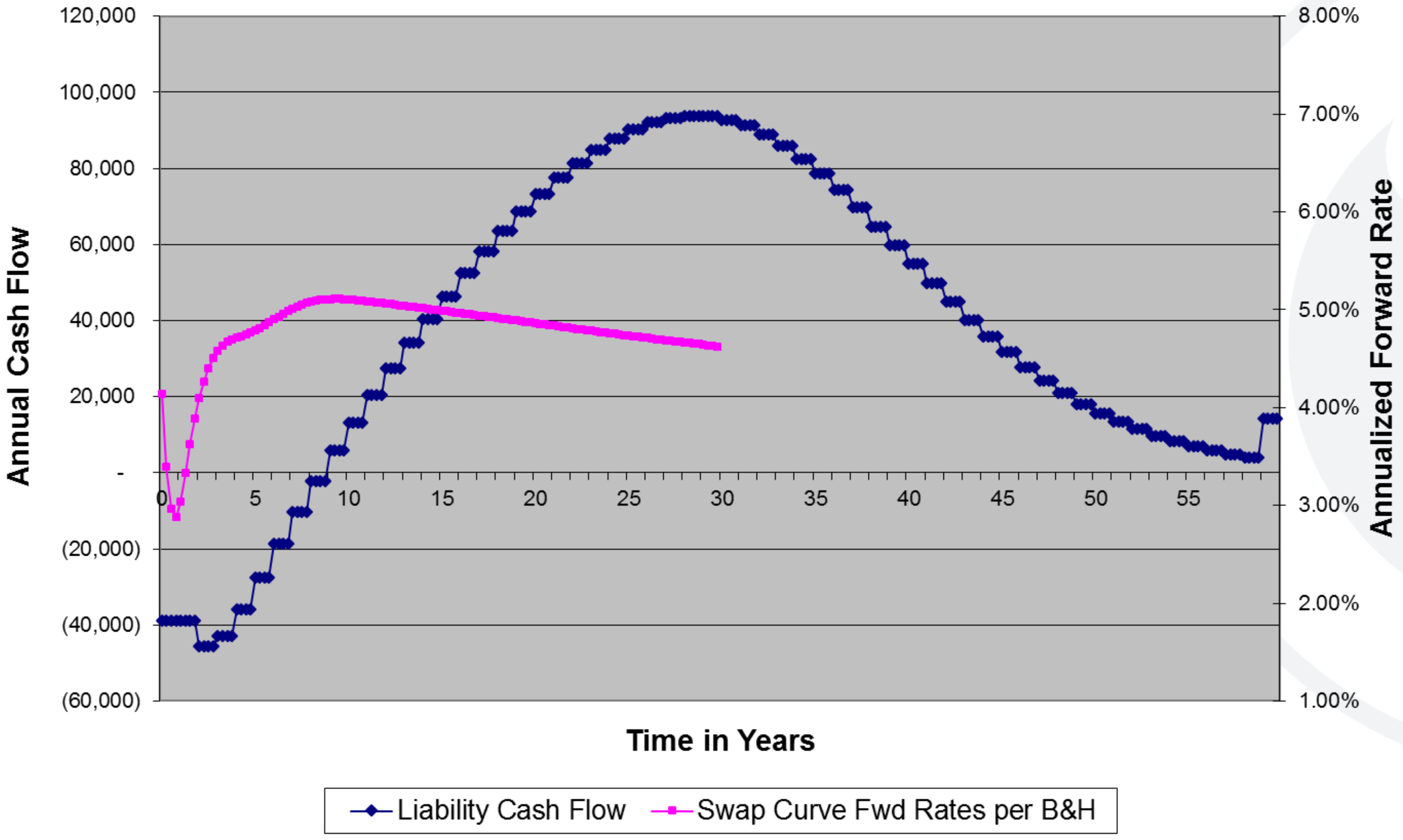
- Two different risks (eg. puts, calls) can give rise to the same risk adjusted scenario set λ^A but will usually have different static hedge strategies b_α
  - Interpretation: a static hedge can put the hedged risk on the cusp between a long and a short position

- First order sensitivities (Option 1):
  - $\Delta V = \sum_\alpha b_\alpha\,\Delta Z^\alpha + \sum_A \lambda^A \Delta L_A - \sum_{\alpha,A} \lambda^A\, b_\alpha \Delta H^\alpha{}_A + \frac{(CTE - Q)}{1-a}\Delta a$

Long Term Care Example: Serious Yield curve extrapolation

- Liability: 60 years of projected liability cash flows on a quarterly time step, most versions of the product offer no cash values hence lapse supported
- Treat cash flows as risk free and deterministic for now
  - More sophisticated models are clearly possible
- Numeraire: Log Normal equity index
  - $dS = S[\mu dt + \sigma dz]$ with $\mu = 8.0\%$, $\sigma = 18.0\%$
  - First period accumulation factor $(1 + \xi_A) = \exp[(\mu - \sigma^2/2)\Delta t + \sigma\sqrt{\Delta t} z_A]$
- Liquid Hedge Instruments: 30 years of zero-coupon bonds with quarterly maturities, assumed risk free for simplicity
- Bond values Z^α based on US swap curve at 9/2008, right in the middle of the financial crisis, (per B&H)
- Illiquid Hedge Instrument: 20 year deferred, 15 year forward starting fixed (4.0%) for equity return swap with various notional amounts
  - over the counter so there would be credit risk issues (ignore for now)
- Scenarios: $N = 25{,}000$
- CTE Level: Base case CTE 60%

Model Inputs: Long Liability at 9/08



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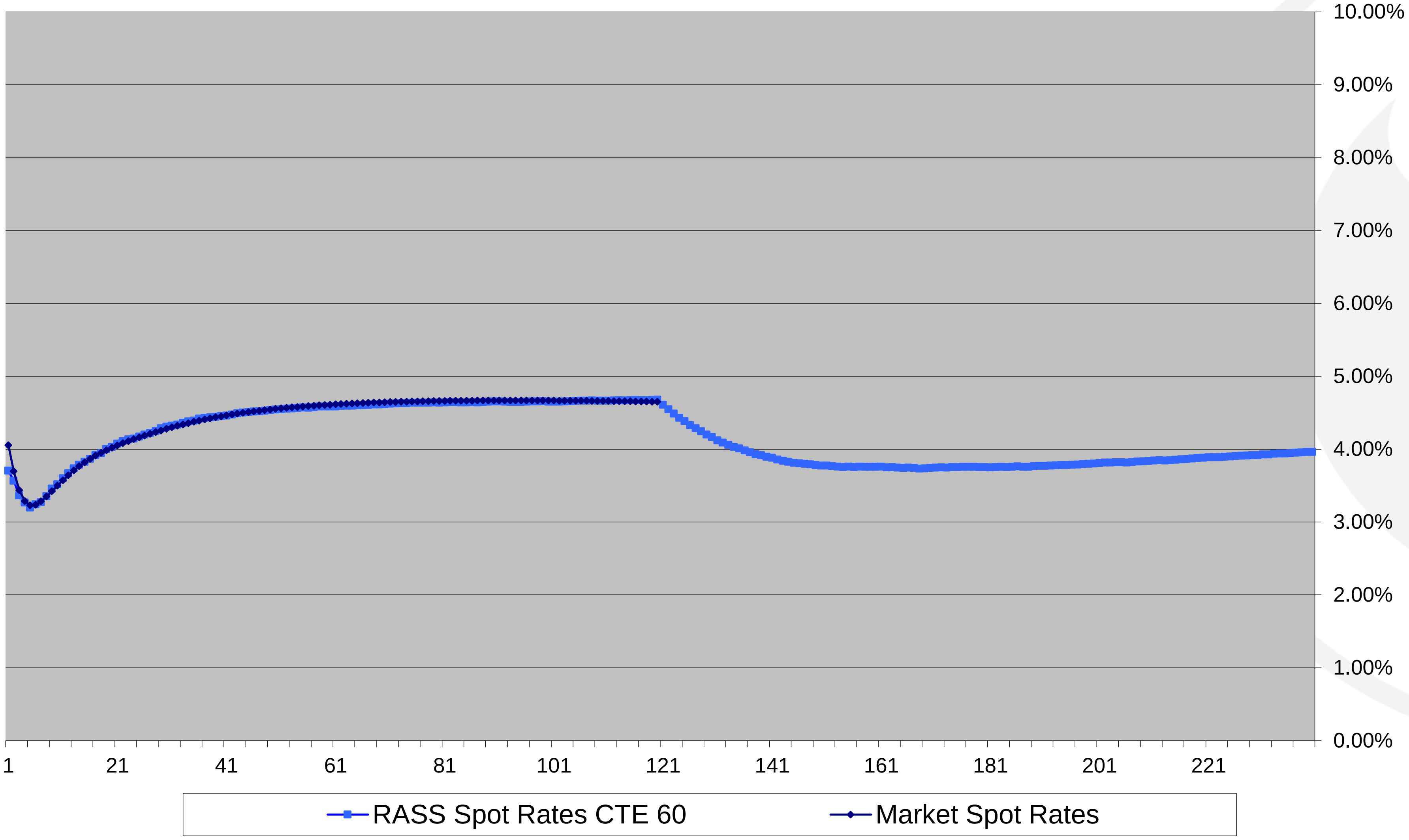
Calculating the optimal RASS

- Option 1 – John M's interior method
  - Iterative method that takes account of special structure
  - Requires less computer memory than commercial linear programming software
  - Two sources of error
    - Iterative method, need a stopping rule
    - Finite scenario set
  - With 25,000 scenarios can take hours to run on an excel platform

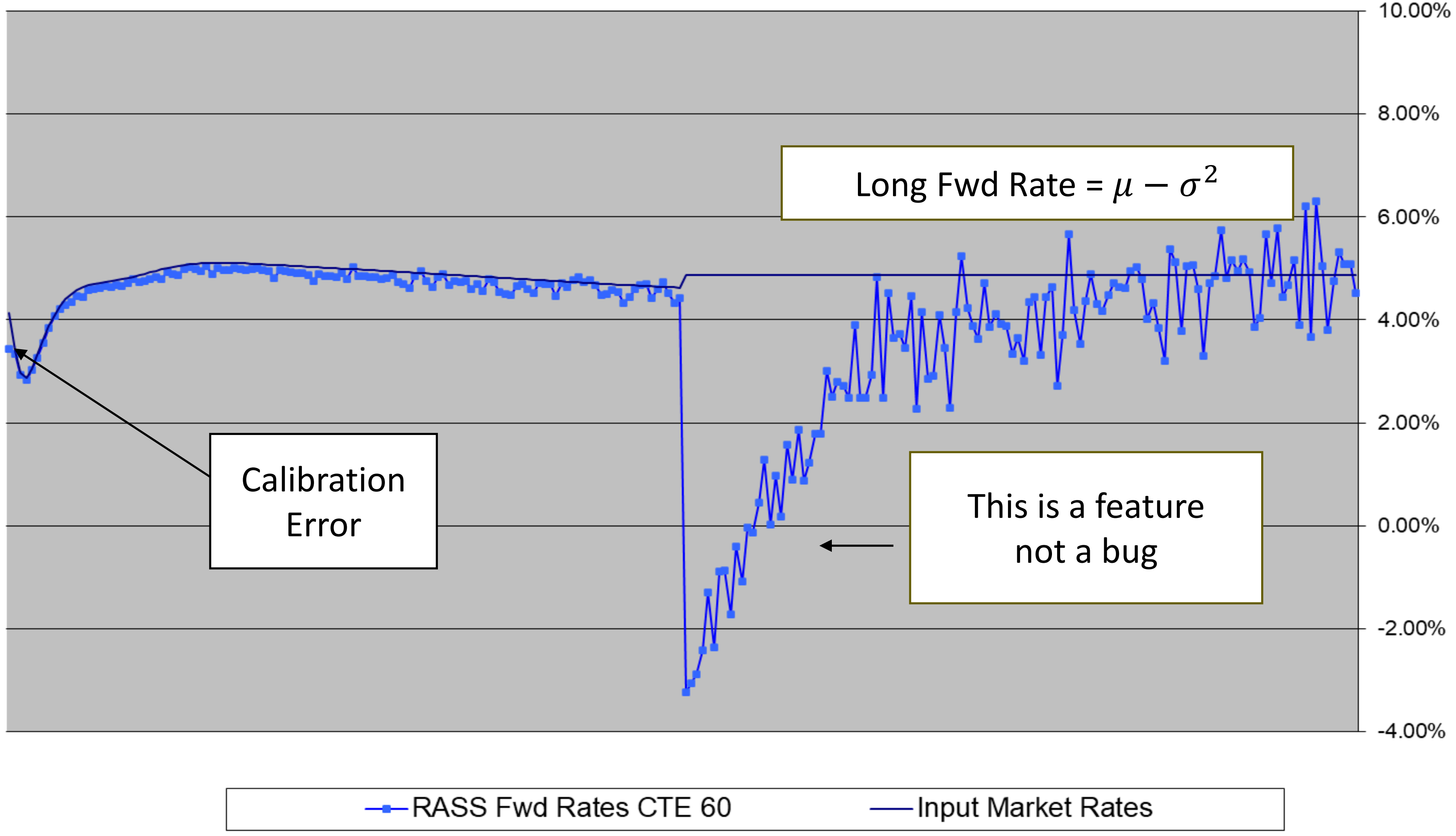
- Option 2 – use commercial linear programming package
  - Exact for the given scenario set
  - Requires a lot of hardware and software resources
  - Need the "industrial strength" version of Solver

			Long Term Care Example @ CTE		60%	**Amounts in $ '000s**			
		Numeraire:		mu=	8%	sigma=	18%	**Short Rate**	4.06%
									Total
Hedge	**Swap**	**Illiquid**	**Static**	**Total**	**Total**	**Sampling**		**Static**	**Return**
Strategy	**Notional**	**Hedge**	**Hedge**	**Return**	**Liability**	**Error**	**a***	**Success%**	**Hurdle**
No Bonds		-	-	3,728	3,728	36	2.46%	87.3%	3.17%
Simple Bonds		-	981	1,572	2,553	16	2.46%	78.1%	3.51%
Regression		-	1,752	345	2,097	6	2.46%	72.5%	5.34%
RASS Optimal		-	1,759	330	2,089	6	2.46%	72.4%	5.65%

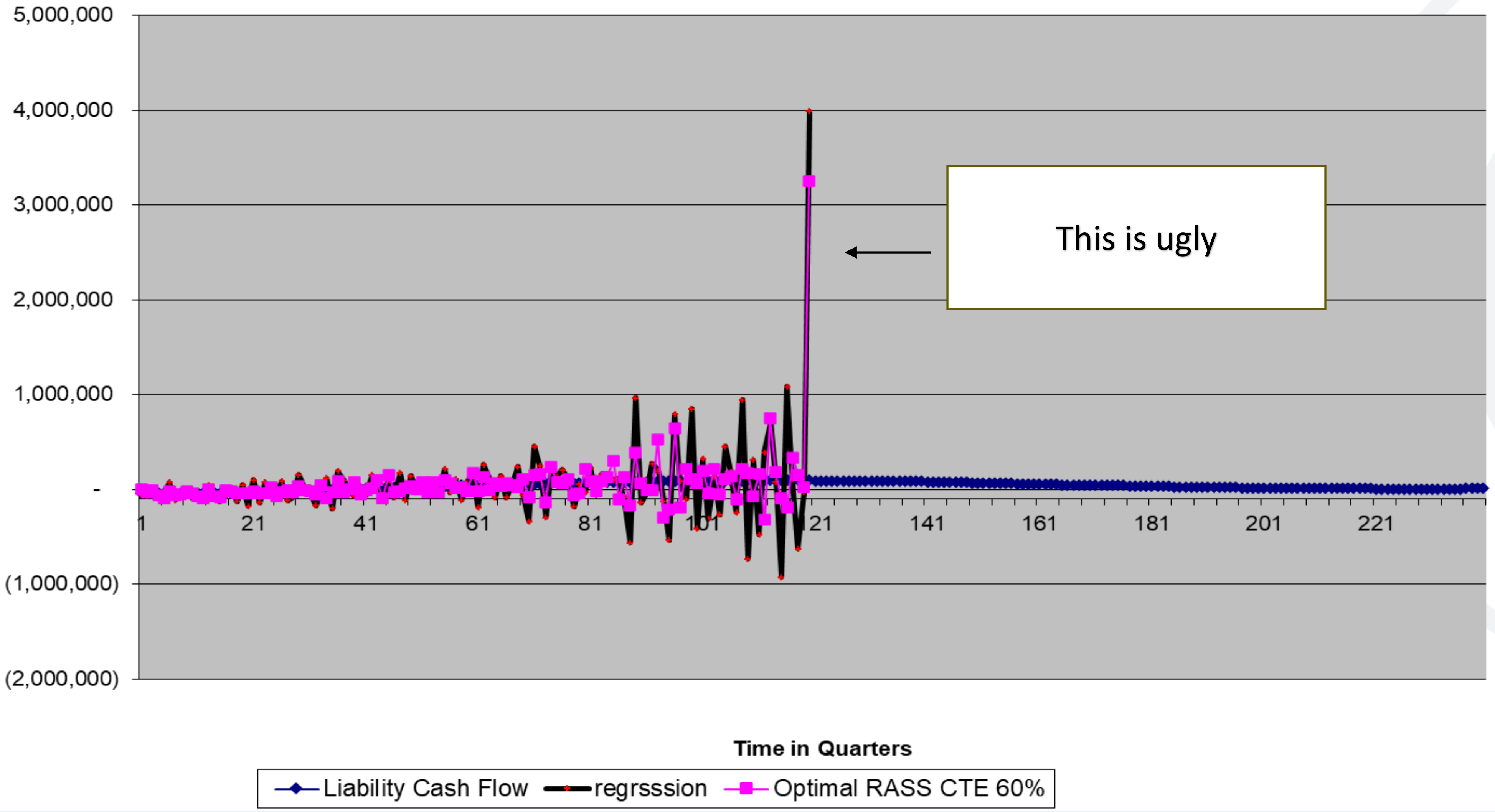
Base Case RASS Spot Yields



Base Case Annual Fwd Rates



Base Case RASS bond flows



Fixing the Problem – We aren't done yet

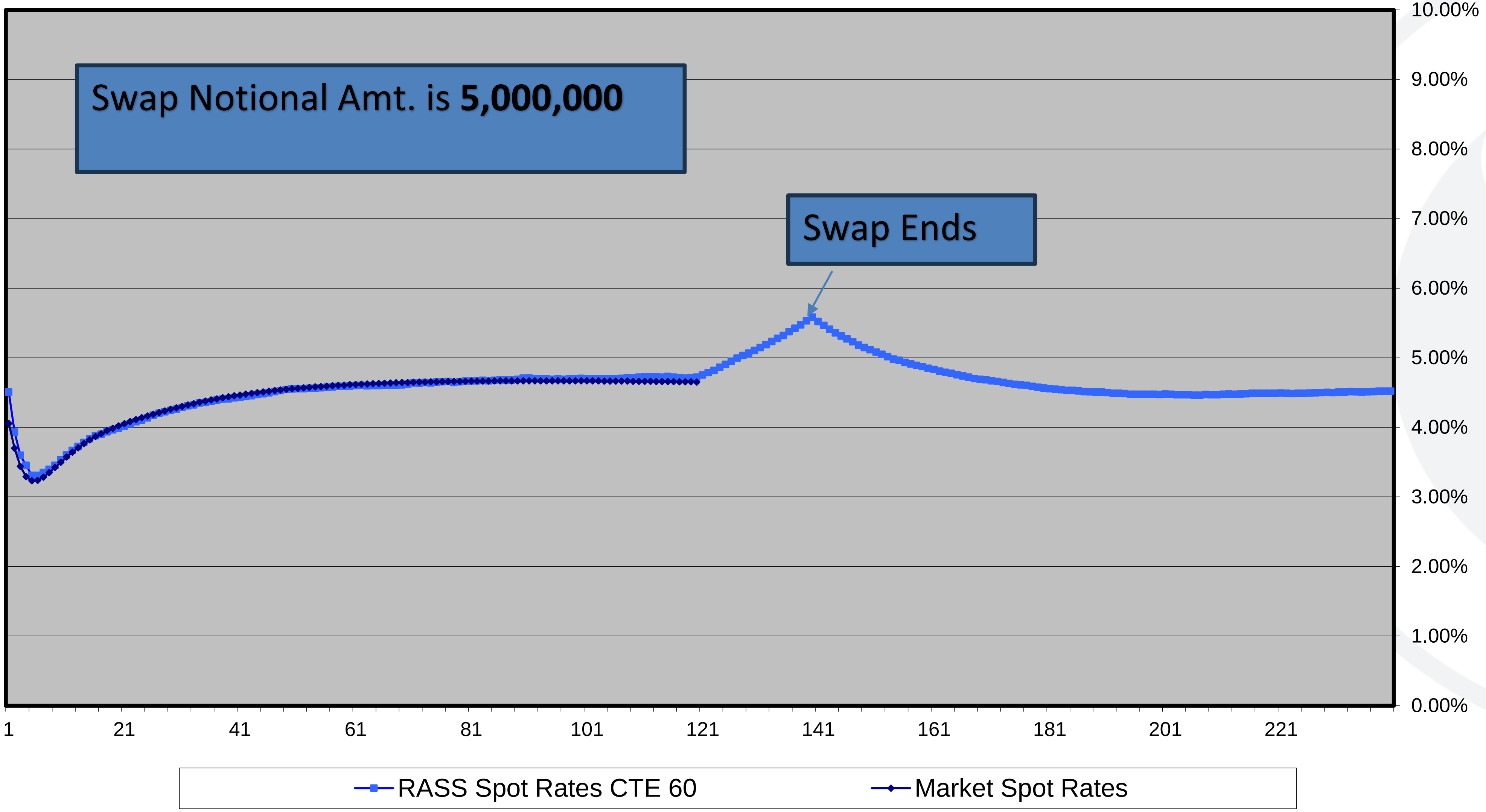
- Example shows that using only liquid bonds as hedge instruments can lead to an impractical hedge strategy. This can happen in standard financial engineering as well
- One Option: Make use of longer illiquid assets already on the balance sheet
- Another Option: Look to Wall Street for some over the counter derivatives that might help
- Today consider a 20-year deferred forward starting fixed for equity swap that runs for 15 years
- Company can price the swap by asking what fixed rate it should receive using the RASS. Answer for this example 3.1%
- Knowing this, we assume the company goes to a Wall Street hedge fund and negotiates a 4.0% fixed rate from the hedge fund
- Next table shows what happens for various notional amounts

Long Term Care Example @ CTE 60% Amounts in $ '000s

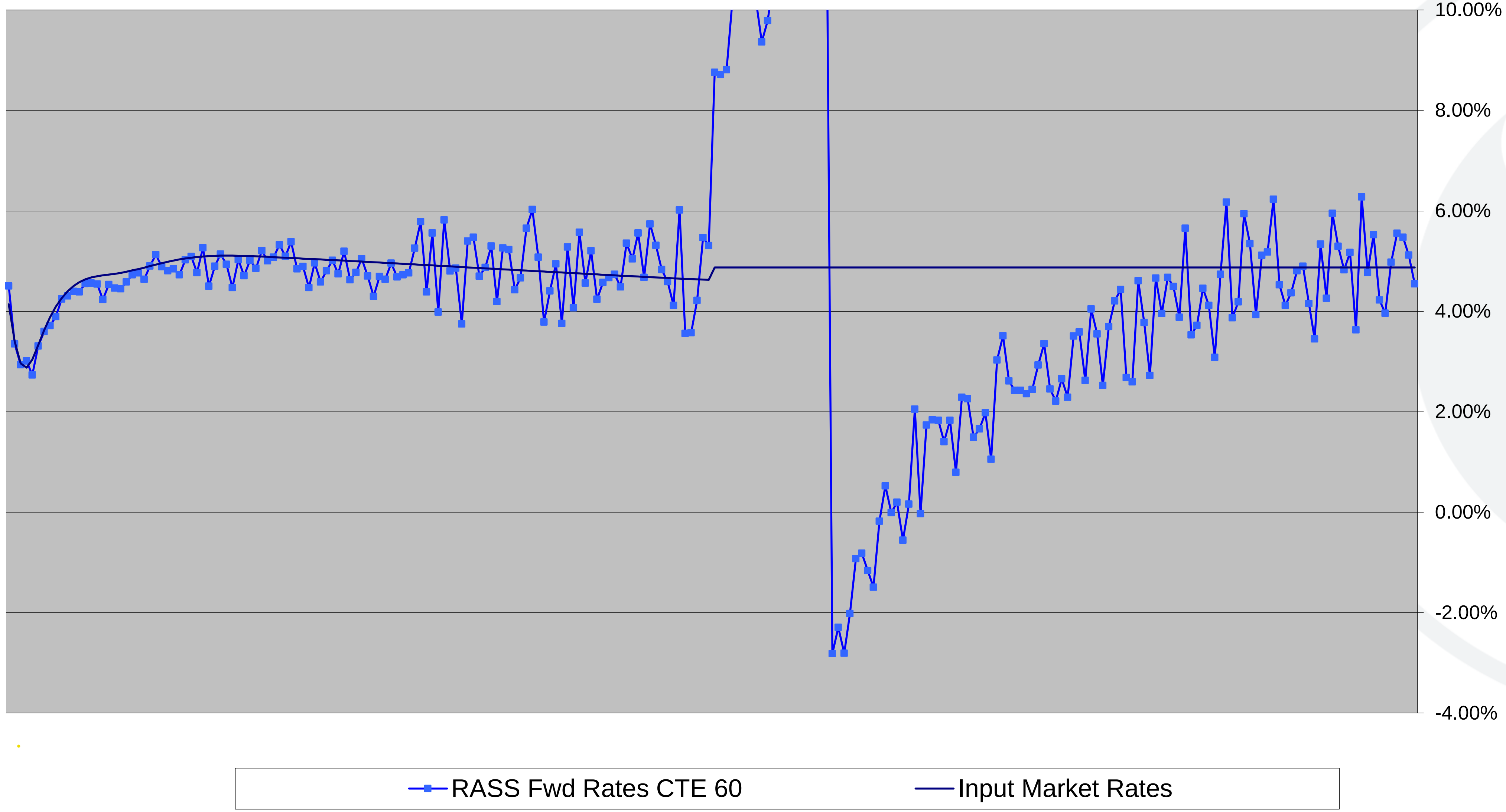
Numeraire: mu= 8% sigma= 18% **Short Rate** 4.06%

Hedge Strategy	Swap Notional	Illiquid Hedge	Static Hedge	Total Return	Total Liability	Sampling Error	a*	Static Success%	Total Return Hurdle
No Bonds		-	-	3,728	3,728	36	2.46%	87.3%	3.17%
Simple Bonds		-	981	1,572	2,553	16	2.46%	78.1%	3.51%
Regression		-	1,752	345	2,097	6	2.46%	72.5%	5.34%
RASS Optimal		-	1,759	330	2,089	6	2.46%	72.4%	5.65%
Values Using a Fiixed 4% for Equity 15 Yr Forward Starting Swap									
RASS Optimal	100	3	1,803	283	2,089	6	2.46%	72.3%	5.92%
RASS Optimal	500	16	1,802	270	2,088	5	2.46%	71.8%	5.82%
RASS Optimal	1,000	(17)	1,808	256	2,047	4	2.46%	71.0%	5.29%
RASS Optimal	5,000	(470)	2,047	145	1,722	2	2.46%	69.9%	4.12%

RASS Spot Yields



Annual Fwd Rates



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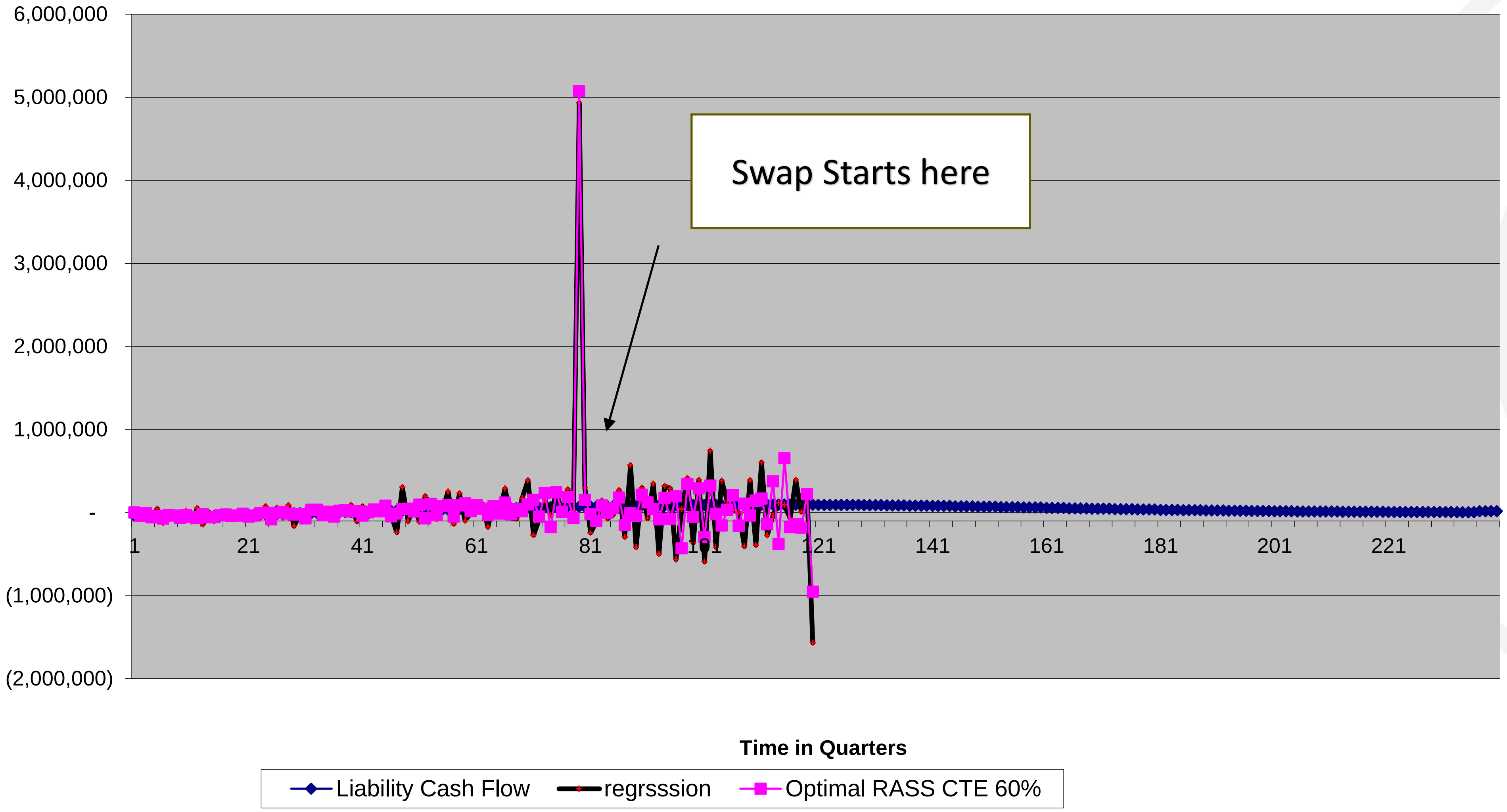
RASS bond flows



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Fixing the Problem – We aren't done yet

- Introducing the swap contract had made the situation better
- Extrapolated yield curve bumped up
  - Total liability reduced
  - Future product pricing more competitive
- Negative fwd. rates pushed out 5 years
- Bond bump at duration 30 has moved to duration 20, when the swap starts
  - Situation not perfect but much more manageable

- Ample scope for more creative thinking e.g., use a ladder of swaps with staggered starting dates

Final Thoughts and Conclusions 1: Risk Managers

- The RASS model solves a number of risk management problems
  - Yield curve extrapolation per the prior example, together with a consistent A/L M strategy
  - If we use credit risky assets as hedge instruments and model cash flows as best estimate + margin for economic capital, then the calibration routine will build any market liquidity premium into the scenario weights
  - Model can accommodate illiquid assets; they are now a perfect match for one component of the RASS liability
  - A/L M problem reduced to managing liquid assets vs Net Illiquid Liability
    - Model can produce "greeks" for the NIL, may need a few new chapters in the financial engineering textbooks
  - RASS is a reasonable starting point for pricing new illiquid instruments and measuring the value created/destroyed by new transactions
  - There is a way to use the RASS model to put a market consistent value on blocks of participating (e.g., with profits) insurance business
- The technology needed to implement the RASS is available today
  - Actuarial projection platforms, industrial strength linear programming tools

Final Thoughts and Conclusions 2: Regulators

- I can't speak for regulators but…
- I hope regulators will like some aspects of the RASS approach
  - does not assume dynamic hedging,
  - with the static hedge in place there is an approximate probability of $(1 + a)/2$ of maturing the obligations by doing nothing in the way of active risk mgmt. going forward.
  - Actual static success % is a model output, must be greater than a
  - No need for a computationally expensive hedge projection analysis to reach that conclusion
- An aspect they may not like
  - If they have to break up a company into pieces, the sum of the parts may not be equal to the whole since RASS values take credit for risk diversification
    - There are economic capital solutions to that problem

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Final Thoughts and Conclusions 3: The Accountants

- I can't speak for the accounting profession but…
- Aspects they should like
  - All illiquid instruments on the balance sheet are valued with respect to a market calibrated RASS
  - Every value reflects the instrument's marginal contribution to the total risk
  - No need to value some assets at book while others are at market
- Aspects they may not like
  - two different insurers could put different values on the same illiquid instrument, values depend on current market and insurer's risk structure
  - the recognition of gains/losses at issue or purchase
  - recognizing the impact of assumption changes in current income
- These are issues that the Canadian Actuarial Profession came to terms with back in 1992 with the introduction of Canadian GAAP
- One solution is to add a CSM (Contractual Service Margin) to both sides of the balance sheet like IFRS

Final Thoughts and Conclusions 4: Financial Engineers

- I can't speak for financial engineers but …

- Aspects they should like
  - The Illiquid assets are now a perfect match for one component of the liability
  - Allows them to focus on managing the liquid assets, their forte
- Aspects they may not like
  - We will need to add a few new chapters to the financial engineering text-books to understand the greeks associated with the Net Illiquid Liability
  - This is the subject matter of the second installment in this series "The Risk Adjusted Scenario Set 2"

Final Thoughts and Conclusions 5: Further Work

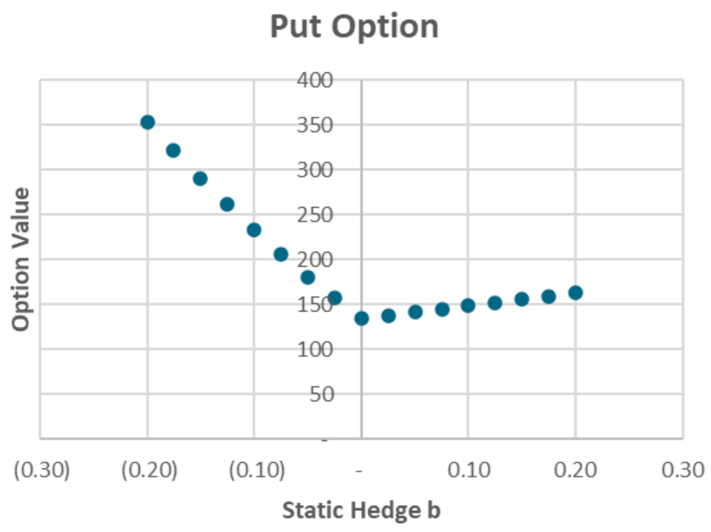
- What are "appropriately risk adjusted cash flows"? This is the subject of installment 3 in this series
- Author presented a paper on this topic at the SOA's 2014 ERM Symposium in Chicago
- Basic idea: every assumption should have three components
  1. A best estimate
  2. A static margin for short term risk such as a contagion event
  3. A dynamic margin for longer term risk (assumption changes)
- Paper shows how to engineer these margins, so the margin release is consistent with the cost of holding economic capital. This means the surplus on the balance sheet is a reasonable estimate of the value of the in-force business
- Title: "Down but not Out, A Cost of Capital Approach to Fair Value Risk Margins"
- No doubt other risk managers will have different views
- Implementing any approach to risk margins requires a good degree of professionalism

Appendix: The Black Scholes Problem (Skip on first reading)

- Apply the RASS model to the classical Black Scholes equity option
- Parameter Assumptions
  - Lognormal Equity:
  - $dS = S[\mu dt + \sigma dz]$ with $\mu = 8.0\%$, $\sigma = 18.0\%$
  - if $T > t$ then $S(T) = S(t)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)(T-t) + \sigma\sqrt{T-t}z\right]$ where $z \sim N(0,1)$.
  - The numeraire used for discounting is a constant interest rate zero coupon bond
  - Interest rate $r = 3.0\%$ bond value $Z(t,T) = e^{-r(T-t)}$
- Liability: Simple Put Option with maturity at time $T - t = 10$ with strike price $K = kS(t)$ and $k = 125\%$.
  - $K = S(T) \rightarrow z = d = \left[\ln\left(\frac{K}{S(t)}\right) - \left(\mu - \frac{\sigma^2}{2}\right)(T-t)\right] / \sigma\sqrt{T-t}$
- $V = \min_b \{ bS + \frac{Z}{1-a}\int_{W(b)} \frac{e^{-z^2/2}}{\sqrt{2\pi}} \frac{\max[0, K - S(T)] - b}{N(T)} dz \}$
  - $W(b)$ is the CTE window, must satisfy $\frac{1}{1-a}\int_{W(b)} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz = 1$
  - We must solve for both b and $W(b)$

What is the CTE Window?

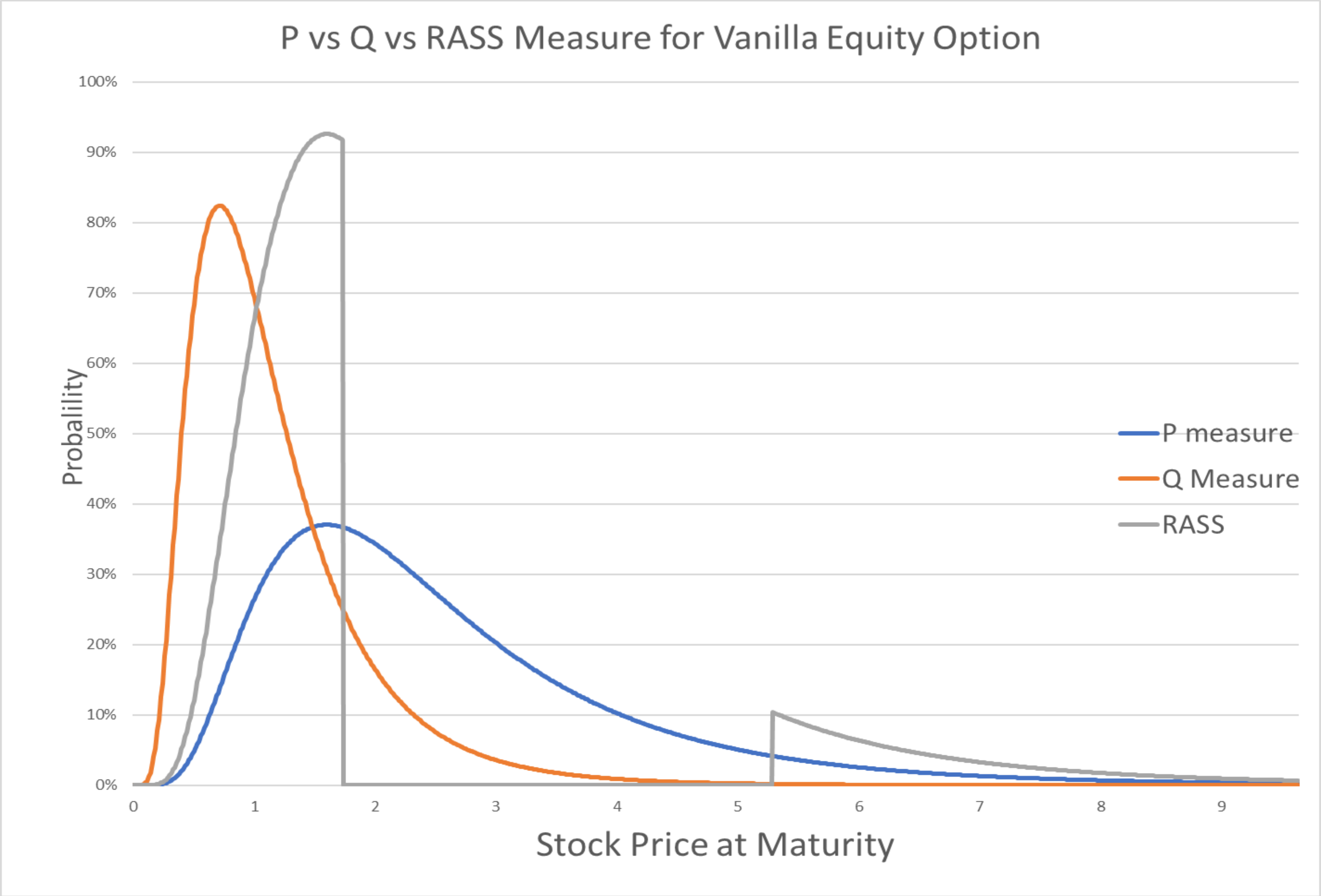
- Start with a quick Monte Carlo study



- We can explain this pattern by assuming the CTE window has the form
  - $W(b) = (-\infty, x) \cup (y, \infty)$ with $\Phi(x) + \Phi(-y) = 1 - a, y > x$
  - The transition is quite abrupt, well-defined minimum
  - This also follows from the comments on slide 21

Analytic Details: the CTE Window

- $V = \min_{\boldsymbol{b}} \left(bS + \frac{Z(t,T)}{1-a} \int_{W(b)} \frac{e^{-z^2/2}}{\sqrt{2\pi}} \{\max[0, K - S(T)] - bS(T)\} dz \right)$
- $V = \min_{\boldsymbol{b}} \left(bS + \frac{Z(t,T)}{1-a} \{\int_{-\infty}^{x} + \int_{y}^{\infty}\} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} \{\max[0, K - S(T)] - bS(T)\} dz \right)$
- With $\Phi(x) + \Phi(-y) = 1 - a$ this is now a standard calculus problem
- $\frac{\partial V}{\partial b} = 0 \rightarrow S = \frac{Z(t,T)}{1-a} \{\int_{-\infty}^{x} + \int_{y}^{\infty}\} \frac{e^{-z^2/2}}{\sqrt{2\pi}} S \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)(T-t) + \sigma\sqrt{T-t}z\right] dz$
    - Interpretation: model reprices the hedge instrument (Tasche)
    - $S(t) = Z(t,T)S(t)\frac{e^{+[\mu(T-t)]}}{1-a}\left[\Phi\left(x - \sigma\sqrt{T-t}\right) + 1 - \Phi\left(y - \sigma\sqrt{T-t}\right)\right]$
    - $Z(t,T) = \frac{e^{-[\mu(T-t)]}(1-a)}{\left[\Phi\left(x-\sigma\sqrt{T-t}\right)+1-\Phi\left(y-\sigma\sqrt{T-t}\right)\right]}$, the value of $S(t)$ drops out
    - A non-linear equation that must be solved numerically for x
    - This puts bounds on the bond values that can be used


P vs Q vs RASS Measure for Vanilla Equity Option
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Probalility
0
1
2
3
4
5
6
7
8
9
Stock Price at Maturity
P measure
Q Measure
RASS

What is the static hedge parameter b?

- Static hedge parameter b given by demanding left and right quantiles are equal
- $Q_L = \max\left[0, Ke^{-\left((\mu-\sigma^2/2)\right)(T-t)-\sigma\sqrt{T-t}x} - S(t)\right] - bZSe^{+\left((\mu-\sigma^2/2)\right)(T-t)+\sigma\sqrt{T-t}x}$
- $Q_R = \max\left[0, Ke^{-\left((\mu-\sigma^2/2)\right)(T-t)-\sigma\sqrt{T-t}y)} - S(t)\right] - bZSe^{+\left((\mu-\sigma^2/2)\right)(T-t)+\sigma\sqrt{T-t}y}$
- $Q_L = Q_R$ implies the static hedge parameter must be

$$b = \frac{\max\left[0, K - S(t)e^{+\left((\mu-\sigma^2/2)\right)(T-t)+\sigma\sqrt{T-t}x)}\right] - \max\left[0, K - S(t)e^{+\left((\mu-\sigma^2/2)\right)(T-t)+\sigma\sqrt{T-t}y}\right]}{S(t)e^{+((\mu-\sigma^2/2))(T-t)+\sigma\sqrt{T-t}x} - S(t)e^{+((\mu-\sigma^2/2))(T-t)+\sigma\sqrt{T-t}y}}$$

- Total option value given by
  - $V = \frac{Z(t,T)}{1-a}\{\int_{-\infty}^{x} + \int_{y}^{\infty}\}\frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\max[0, K - S(T)]\,dz$
  - $= l_L(x) + l_R(y)$(dual form)
  - $V = bS(t) + (V - bS(t))$ static hedge + numeraire part

Summary of Analytic Results for Point in Time valuation

- The RASS (λ^A) is defined by those scenarios that pass through $W = (-\infty, x) \cup (y, \infty)$ at the maturity date T
- (x, y) determined by solving the pair of equations
  - $\Phi(x) + \Phi(-y) = 1 - a, y > x$
  - $S(t) = Z(t,T)S(t)\frac{e^{+[\mu(T-t)]}}{1-a}\left[\Phi\left(x - \sigma\sqrt{T-t}\right) + 1 - \Phi\left(y - \sigma\sqrt{T-t}\right)\right]$

  or $\quad Z(t,T) = \frac{e^{-[\mu(T-t)]}(1-a)}{\left[\Phi\left(x-\sigma\sqrt{T-t}\right)+1-\Phi\left(y-\sigma\sqrt{T-t}\right)\right]}$, must solve numerically for x
- Static hedge parameter b given by demanding left and right quantiles are equal
- $b = \frac{\max\left[0, K - S(t)e^{+\left(\left(\mu-\sigma^2/2\right)\right)(T-t)+\sigma\sqrt{T-t}x}\right] - \max\left[0, K - S(t)e^{+\left(\left(\mu-\sigma^2/2\right)\right)(T-t)+\sigma\sqrt{T-t}y}\right]}{S(t)e^{+\left(\left(\mu-\sigma^2/2\right)\right)(T-t)+\sigma\sqrt{T-t}x} - S(t)e^{+\left(\left(\mu-\sigma^2/2\right)\right)(T-t)+\sigma\sqrt{T-t}y}}$
- Total option value given by
  - $V = \frac{Z(t,T)}{1-a}\left\{\int_{-\infty}^{x} + \int_{y}^{\infty}\right\}\frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\max[0, K - S(T)]\,dz$
  - $= l_L(x) + l_R(y)$(dual form)
  - $V = bS(t) + (V - bS(t))$ primal presentation

Summary of Analytic Results: Black Scholes Presentation

- We can rewrite the final result in a form that is directly comparable to the famous Black-Scholes result
- $V = l_L(x) + l_R(y)$ with $d = \frac{\ln\left(\frac{K}{S(t)}\right) - ((\mu - \sigma^2/2))(T-t)}{\sigma\sqrt{T-t}}$
- $= \frac{KZ}{1-a}[\Phi(\min(d,x)] - \frac{S(t)Ze^{\mu(T-t)}\Phi[(\min(d,x) - \sigma\sqrt{T-t})}{1-a}$

$+\frac{K}{1-a}[\Phi(max(d,y) - \Phi(\mathrm{y})))] - \frac{S(t)Ze^{\mu(T-t)}}{1-a}[\Phi(\max(d,y) - \sigma\sqrt{T-t}) - \Phi\big(y - \sigma\sqrt{T-t}\big)]$

- $= \frac{KZ}{1-a}\{\Phi\big(\min(d,x)\big) + \Phi(y) - \Phi(\max(d,y)\}$- $\frac{S(t)Ze^{\mu(T-t)}}{1-a}\{\Phi[\min(d,x) - \sigma\sqrt{T-t}] + \Phi[\max(d,y) - \sigma\sqrt{T-t}] - \Phi[(y) - \sigma\sqrt{T-t}]\}$
- Compare to Black Scholes
  - Set $d_1 = \left[\ln\left(\frac{K}{S(t)}\right) - \left(r - \frac{\sigma^2}{2}\right)(T-t)\right] / \sigma\sqrt{T-t}$ then
  - $V_{BS} = e^{-r(T-t)}\int_{-\infty}^{d_1}[K - S(t)\exp[\left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma\sqrt{T-t}z]\frac{\exp[-z^2]}{\sqrt{2\pi}}dz$
  - $= Ke^{-r(T-t)}\Phi(d_1) - S\Phi\big(d_1 - \sigma\sqrt{T-t}\big)$

Vanilla Put Option: Input Assumptions

Static Hedge Closed Form Example				*Put Option*						
					Bond Numeraire Stock Hedge Instrument					
Discounting Parameters					$\bar{H}$	1,649		Z_{max}	Z_{min}	
	μ	8.00%			Σ^2	1,040,143		1.854	0.256	Elliptical
	σ	18.00%			Z	0.741		0.875	0.288	Exact
	Interest r	3.00%			$\chi 2$	0.405		1.33%	12.45%	
	CTE Level α	60%			α^*	28.8%		CTE Window		
	Maturity T-t	10			μ - σ^2	4.76%		Φ(x)	Φ(-y)	
					r + σ^2	7.24%		38.8%	1.2%	
								Feasible?	TRUE	
Liability Parameters								$I_L(x)$	$I_R(y)$	
	Strike % k	125%			b^*	-		139.2	-	
	$S(t)$	1,000			d	(0.729)	Φ(d)	23.3%	-	
	Strike Price K	1,250								

Vanilla Put Option: Results 1 (Point in Time)

								Static
	Static Hedge 2		*Primal Presentation*				Implied	Success
			Debt	*Equity*	Total	FSE	*Vol %*	%
		Monte Carlo	134.8	-	134.8	3.1	14%	84%
		Analytic	139.2	-	139.2	0.0	14%	84%
			Dual Presentatioin					
			I_L	I_R	Total			
			139.2	-	139.2		14%	
			Black Scholes Presentation					
			$K*Z*\Phi$	$S*\Phi'$				
			539.53	(400.4)	139.2		14%	39%
	Black Scholes		$K*Z*\Phi$	$S*\Phi'$	Total			
	d_1	0.150	518.1	(337.4)	180.7		18%	49%
2001 Canadian			$K*Z*\Phi$	$S*\Phi'$	Total			
			539.5	(289.6)	249.96		24%	64%

Static Hedge bS
Dynamic Hedge ΔS

Big
Difference

Analytic Results: Roll Forward Analysis

- We have an almost closed form expression for the option price
- $V = \frac{KZ}{1-a}\{\Phi\left(\min(d,x)\right) + \Phi(y) - \Phi(\max(d,y)\} - \frac{S(t)Ze^{\mu(T-t)}}{1-a}\{\Phi[\min(d,x) - \sigma\sqrt{T-t}] + \Phi[\max(d,y) - \sigma\sqrt{T-t}] - \Phi[(y) - \sigma\sqrt{T-t}]\}$
- $d = \frac{\ln\left(\frac{K}{S(t)}\right) - ((\mu - \sigma^2/2))(T-t)}{\sigma\sqrt{T-t}}$
- If $d < x$ then $V = \frac{KZ}{1-a}\Phi(d) - \frac{S(t)Ze^{\mu(T-t)}}{1-a}\Phi[d - \sigma\sqrt{T-t}]$ and $b = 0$ this is the situation in the current example
- Use Ito's lemma to calculate $dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S}[\mu S dt + \sigma S dz] + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 dt$
- $\frac{\partial V}{\partial S} = \Delta = -\frac{Ze^{\mu(T-t)}}{1-a}\Phi[d - \sigma\sqrt{T-t}], \frac{\partial^2 V}{\partial S^2} = \frac{Ze^{\mu(T-t)}}{1-a}\frac{\varphi(d-\sigma\sqrt{T-t})}{S\sigma\sqrt{T-t)}}$
- $\frac{\partial V}{\partial t} = rV + \frac{KZ}{1-a}\frac{\varphi(d)\sigma}{2\sqrt{T-t}} - \mu\Delta S$
- Conclude $dV = \left[rV + \frac{KZ}{1-a}\frac{\varphi(d)\sigma}{2\sqrt{T-t}}\right]dt + \Delta S\sigma dz$, if $d < x$

Analytic Results: Roll Forward Analysis

- Conclude $dV = \left[rV + \frac{KZ}{1-a}\frac{\varphi(d)\sigma}{2\sqrt{T-t}}\right]dt + \Delta S\sigma dz$, if $d < x$
- So, what is an appropriate Asset strategy?
- Option 1: Static Hedge $dA = rVdt$ since $b = 0$
  - $d(A-V) = \left[-\frac{KZ}{1-a}\frac{\varphi(d)\sigma}{2\sqrt{T-t}}\right]dt - \Delta S\sigma dz$
  - We have a long equity risk and a negative expected return
  - Does not make short term business sense, even though it makes long term business sense due to high static success %

- Option 2: Dynamic Hedging $dA = r(V - \Delta S)dt + \Delta S(\mu dt + \sigma dz)$
  - $d(A-V) = S[-r\Delta - \frac{KZ}{1-a}\frac{\varphi(d)\sigma}{2\sqrt{T-t}}]dt$
  - Makes short term business sense as long *as*
  - $-r\Delta - \frac{KZ}{1-a}\frac{\varphi(d)\sigma}{2\sqrt{T-t}} \geq 0$
  - Makes long term business sense only if the apparent premium above exceeds the long-term costs of dynamic hedging

Roll Forward Analysis: Conclusion

- No point taking the analysis of this example into further detail at this time
- Similar analysis can be done if $x < d < y$ or $y < d$ but that is not the point
- Deciding the best practical A/L M strategy will depend on a broader range of issues than those presented here
- Deciding between the static hedge (minimizing long term risk) vs dynamic hedging (minimizing short term risk) will depend on the circumstances
- Perhaps only a regulator could live with the short-term fluctuations associated with the static hedge approach in this example
- An argument to advance to regulators: we always have the option of locking in the static hedge and walking away
- There are two regulatory scenarios, in theory
  - A) the regulator takes over the business and runs it himself
  - B) the regulator splits the business into blocks and sells them off to otherwise healthy companies
- The RASS model is more consistent with (A) than (B) unless the risk margins built into the "appropriately risk adjusted cash flows" are truly appropriate. That requires actuarial professionalism.

Roll Forward Analysis: To be Continued…

- How can we estimate the relevant roll forward risk metrics when we do not have an analytic model, but only results from Monte Carlo simulations? More to come.
- There are other examples that can be solved in closed form e.g., the same Black-Scholes problem but with an equity numeraire and a bond hedge instrument. The results are not the same. The choice of numeraire matters, unlike traditional financial engineering.
- Analytic examples are useful for developing ideas, but it should be remembered that the RASS model is fundamentally a bulk method. In practice, the value of an option also depends on how it interacts with other instruments on the balance sheet.

THANK YOU

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