It Takes Two: Why Mortality Trend Modeling is more than modeling one Mortality Trend

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- March 29, 2019
- 2nd Ulm Actuarial Day

Introduction

Highlights of recent SwissRe sigma study on „Mortality Improvements“

- “mortality improvement has slowed unusually in the US, UK, Germany, the Netherlands and Taiwan”
- “… but the recent slowdown is typically not statistically significant.”
- “Extrapolating future mortality trends solely from recent experience can be misleading unless we believe there has been a structural break.”
- “The ability to distinguish between shifts in the underlying mortality trend and short-term variability is crucial because a change in mortality trend is an aggregate risk that cannot be easily diversified away nor perfectly hedged.”

Swiss Re Institute

Nr. 6/2018

sigma

Verbesserung der Sterblichkeit: Vergangenheit verstehen und Zukunft antizipieren

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03 Jüngste Entwicklungen der Sterblichkeit
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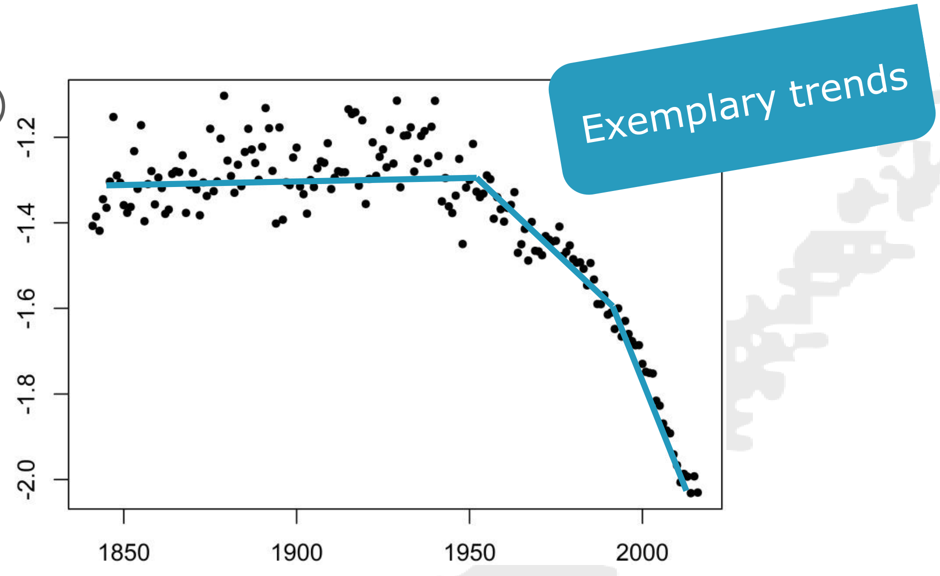
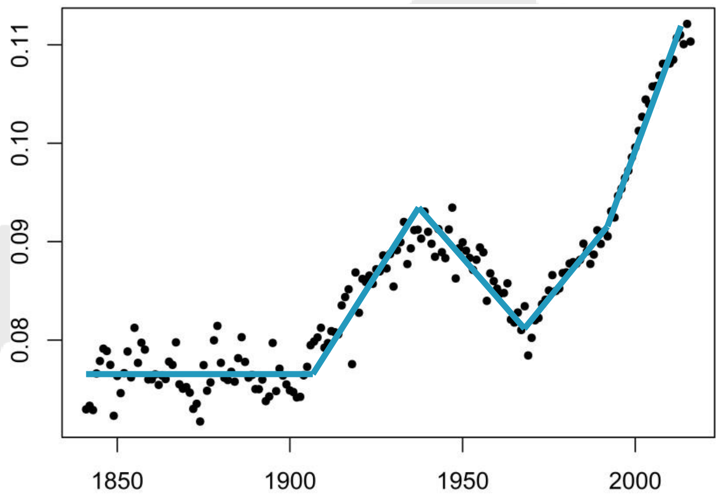
sigma 50 YEARS

Introduction

- Two parameter processes (Cairns et al. (2006))
  - $\log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^1 + \kappa_t^2 \cdot (x - \bar{x})$
  - Parameters calibrated for English and Welsh males older than 60
- Classical simulation approach: Random Walk with drift

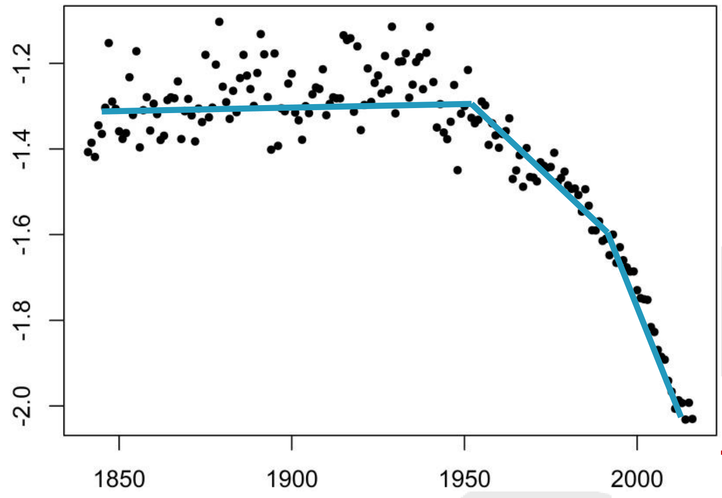


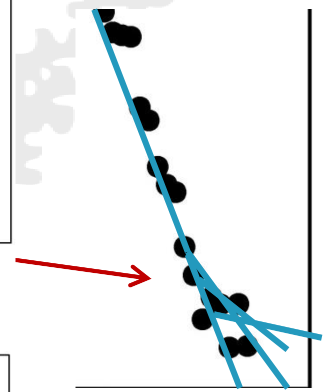
- Historic trend changed once in a while
  - Only a piecewise linear trend with random changes in the trends slope
  - Random fluctuation around the prevailing trend
- In principle, our approach can be applied to any changing mortality trend model



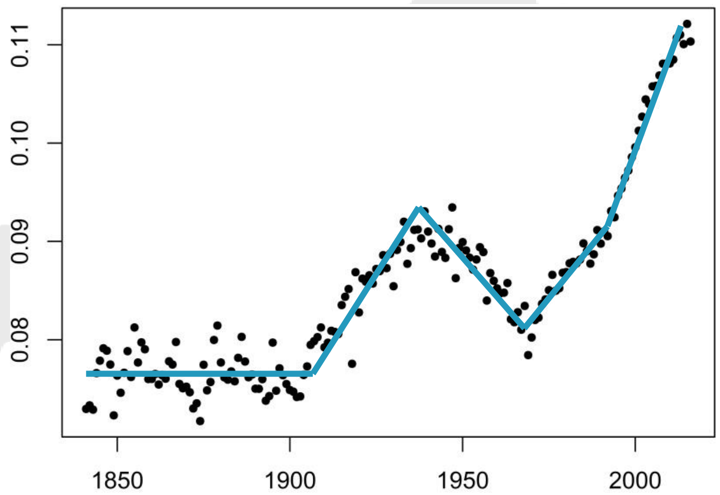
Introduction

- We don’t know the current mortality trend for sure
- But the estimate for the current trend seems a good best estimate for the future evolution
- Possible future changes of the trend in both directions





- One model for the actual mortality trend
- One model for the estimation of the current trend at some point in time, i.e. the estimated mortality trend
- In many situations, both components are necessary



Agenda

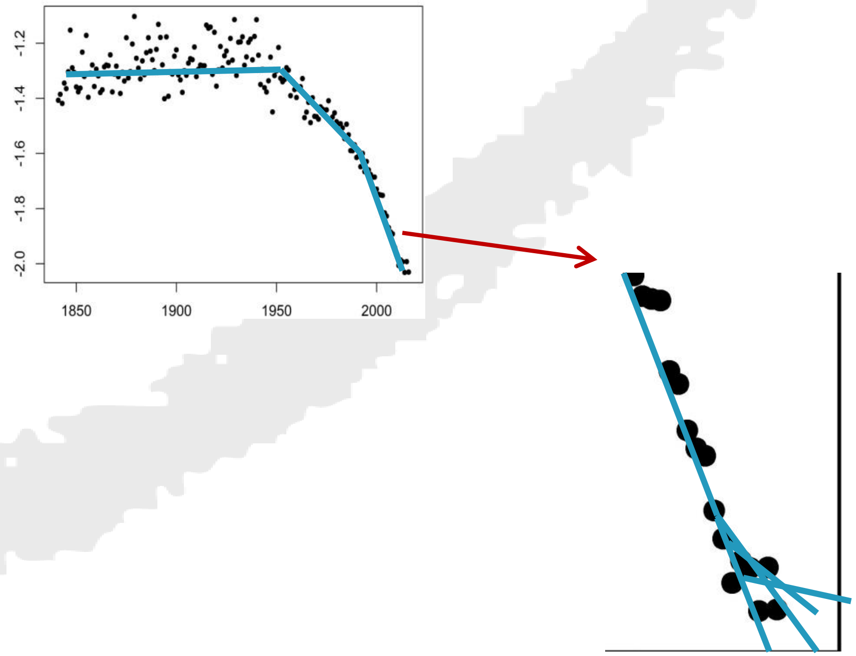
- **Why two mortality trends?**
  - Actual mortality trend (AMT)
  - Estimated mortality trend (EMT)
  - Some examples
- A combined model for AMT & EMT
  - AMT component
  - EMT component
- Applications
- Conclusion

Why two mortality trends?

Actual Mortality Trend (AMT)

- The AMT describes realized mortality trends
  - Core of most existing mortality models
  - Time and magnitude of changes in the AMT and the error structure around the trend process need to be modeled
- We have an idea of the historical AMT but it's not fully observable!
- We can't always distinguish between a recent trend change and "normal" random fluctuation around the prevailing trend → possible undetected trend change in the recent years
- Unknown current value and slope of the AMT

"The ability to distinguish between shifts in the underlying mortality trend and short-term variability is crucial because a change in mortality trend is an aggregate risk that cannot be easily diversified away nor perfectly hedged."



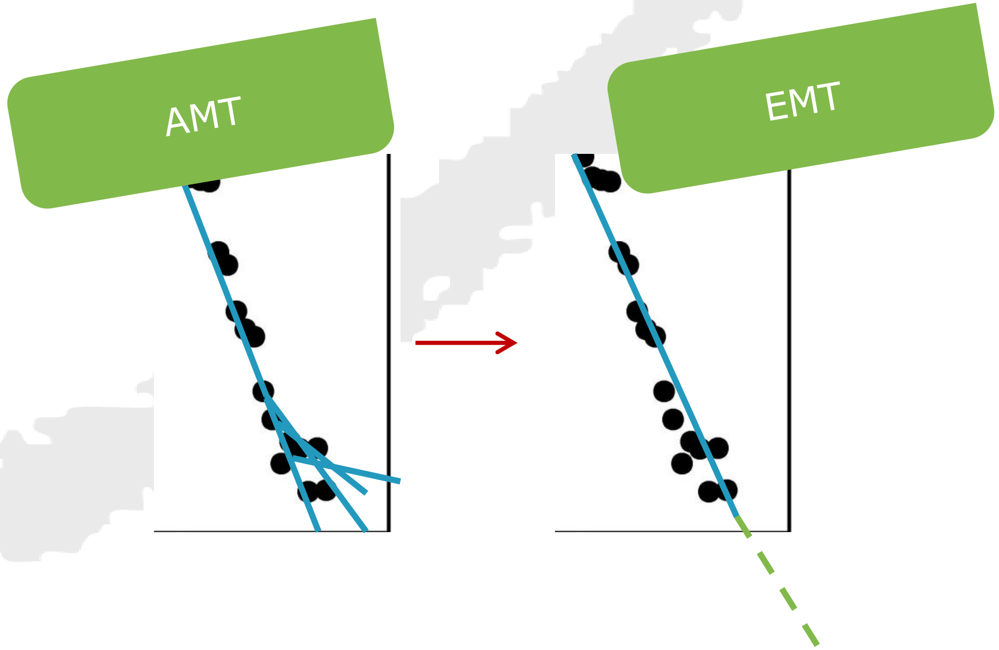
Why two mortality trends?

Estimated Mortality Trend (EMT)

- The EMT describes the expectation of an actuary/demographer about the AMT, i.e. the current slope and value of the mortality trend at some point in time
- Based on most recent historical, observed mortality evolution and updated as soon as new observations become available
- The EMT is the basis for mortality projections, (generational) mortality tables, reserves, etc.

"… but the recent slowdown is typically not statistically significant."

"Extrapolating future mortality trends solely from recent experience can be misleading unless we believe there has been a structural break."



Why two mortality trends?
Some examples

Why another trend?

- Requirement for AMT and/or EMT depends on application:
  - Reserves for a portfolio → EMT today
  - Capital for a portfolio run-off → AMT over the run-off
  - Reserves for a portfolio after 10 years → AMT over the 10 years, EMT after 10 years
  - Payout of a mortality derivative → AMT up to maturity, EMT at maturity
  - Analyse the hedge effectiveness of the previous derivative → EMT at maturity, AMT beyond

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  - EMT component
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A Combined model for AMT/EMT

AMT component

- Continuous piecewise linear trend, with random changes in the slope and random fluctuation around the trend
- AMT model specification:
  - Model the trend process with random noise → $\kappa_t = \hat{\kappa}_t + \epsilon_t;\ \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$
  - Extrapolate the most recent actual mortality trend → $\hat{\kappa}_t = \hat{\kappa}_{t-1} + \hat{d}_t$
  - In every year, there is a possible change in the mortality trend with probability p

    $$\rightarrow \hat{d}_t = \begin{Bmatrix} \hat{d}_{t-1} & with\ probability\ 1-p \\ \hat{d}_{t-1} + \lambda_t & with\ probability\ p \end{Bmatrix}$$
  - In the case of a trend change → $\lambda_t = M_t \cdot S_t$
    - With absolute magnitude of the trend change $M_t \sim \mathcal{LN}(\mu, \sigma^2)$
    - Sign of the trend change S_t bernoulli distributed with values -1, 1 each with probability $\frac{1}{2}$

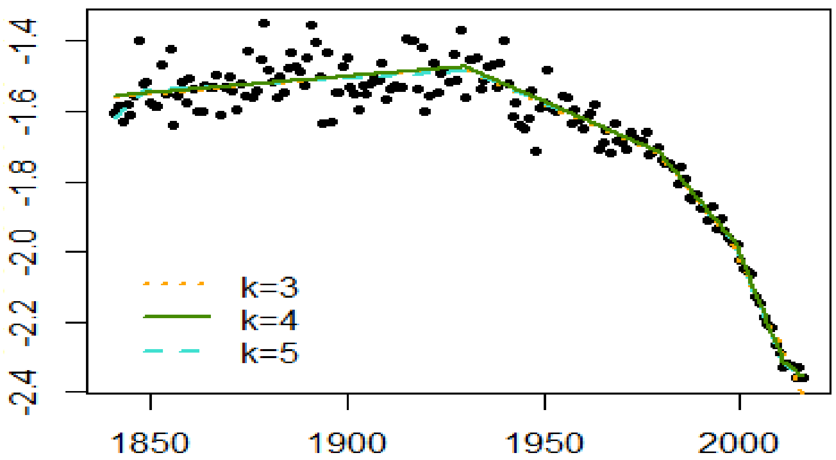
Parameters to be estimated for projections:

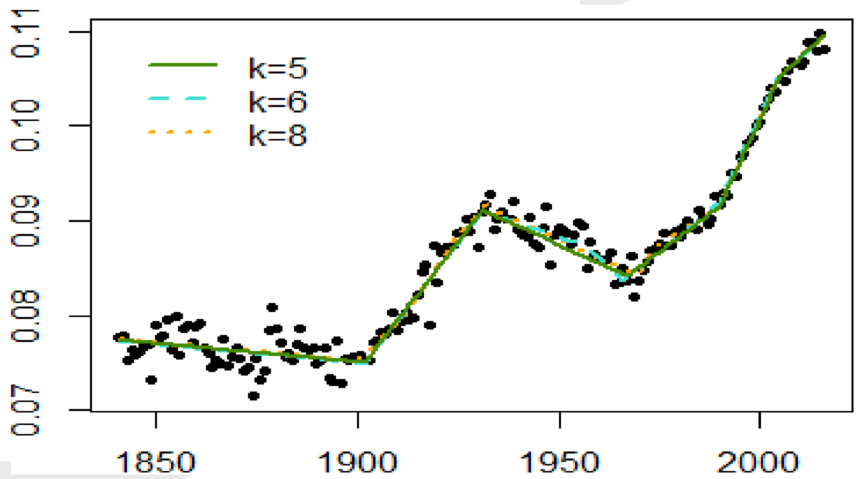
$p, \sigma_\epsilon^2, \mu, \sigma^2, \hat{d}_n, \hat{\kappa}_n$

A Combined model for AMT/EMT

AMT component

Idea: Use historic trends to estimate the parameters $p, \sigma_\epsilon^2, \mu, \sigma^2, \hat{d}_n, \hat{\kappa}_n$



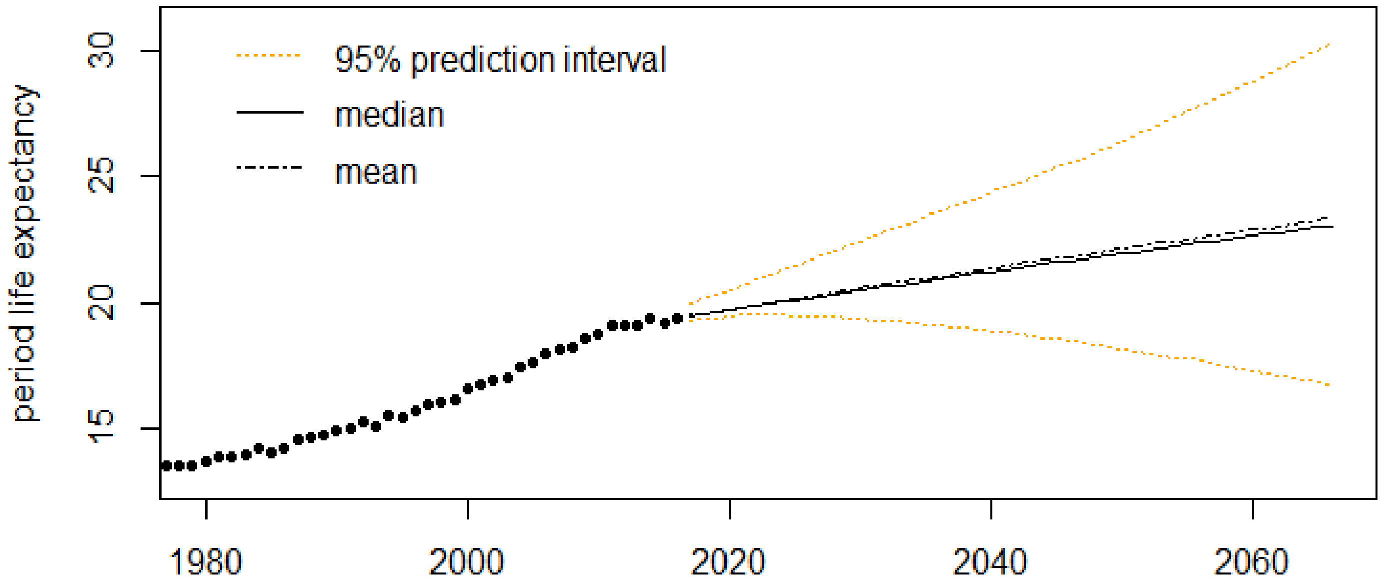


For details on the calibration we refer to Börger and Schupp (2018) and Schupp (2019). Parameter uncertainty included.

A Combined model for AMT/EMT

AMT component

Period life expectancies for 65-year old males



Ongoing of recent improvements and also slowdown of mortality improvements incorporated

A Combined model for AMT/EMT
EMT component

- We don't know today's AMT, but we want a model to estimate it: $\mathbb{E}(\hat{d}_t) = \tilde{d}_t$ and $\mathbb{E}(\hat{\kappa}_t) = \tilde{\kappa}_t$
- symmetric future trend changes in AMT
  - → $\mathbb{E}(\hat{d}_T) = \tilde{d}_t$ and $\mathbb{E}(\hat{\kappa}_T) = \tilde{\kappa}_t + (T-t)\ \tilde{d}_t, T > t$

- Calculation of EMT is complex and not feasible in a simulation
  - path-dependent calculation of the EMT
  - path-dependent recalibration of whole AMT

- Piecewise linear trend process with symmetric changes in the AMT

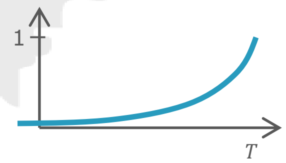
  → Calibrate the EMT with a weighted linear regression on most recent data

A Combined model for AMT/EMT

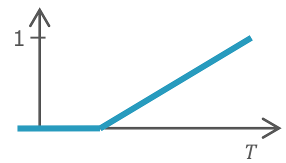
EMT component

- How many years should be included in the regression?
  - Too many → delayed reaction of EMT on trend changes in the AMT
  - Too little → EMT is vulnerable to random noise in the AMT

- Higher influence of most recent data in the estimation of the regression
  - Weighted exponential regression in year T : $w_{exp}(t,T) = \frac{1}{\left(1+{}^{1}/_{h_{exp}}\right)^{T-t}}, t \leq T.$



  - Weighted linear regression in year T : $w_{lin}(t,T) = \max\left(0; 1 - \frac{1}{h_{lin}}(T-t)\right), t \leq T,$



  - Weighted constant regression in year T : $w_{const}(t,T) = \begin{cases} 1 & , if\ T - h_{const} < t \leq T \\ 0 & , if\ t \leq T - h_{const} \end{cases}.$

A Combined model for AMT/EMT

EMT component

Calibration of the weighting parameters

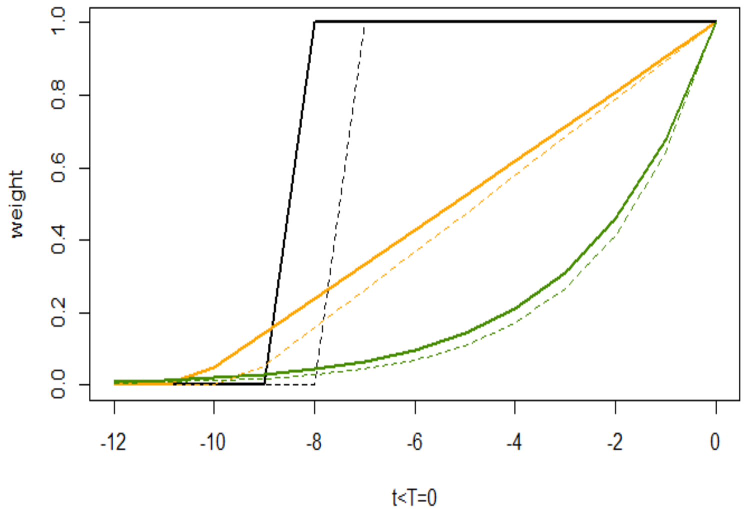
- Calibrate the AMT model
- Simulate the future evolution of the AMT 100.000 times to avoid dependencies on fixed historical trends.

- EMT calibration
  - After T=40 years, calculate the optimal weighting parameters based on two criteria:
    - EMTs $\tilde{d}_T^{(i)}$ are close to AMTs $\hat{d}_T^{(i)}$ (in terms of MSE)
    - EMTs 65-year olds cohort life expectancy $\tilde{e}_{65,T}$ close to AMTs $\hat{e}_{65,t_\omega}$

A Combined model for AMT/EMT

EMT component - results

	weights	constant	linear	exponential
$\tilde{d}_T^{(1)}$	$h_\cdot^{(1)}$	9.0	10.5	2.1
	mse	$1.45 \cdot 10^{-5}$	$1.31 \cdot 10^{-5}$	$1.35 \cdot 10^{-5}$
$\tilde{d}_T^{(2)}$	$h_\cdot^{(2)}$	8.0	9.5	1.8
	mse	$3.19 \cdot 10^{-8}$	$2.88 \cdot 10^{-8}$	$2.95 \cdot 10^{-8}$



weights	constant	linear	exponential	fixed period
$h_\cdot^{(1)}$	10.0	11.0	2.2	30
$h_\cdot^{(2)}$	8.0	10.5	2.1	30
mse	1.59	1.56	1.58	3.04
$P(\tilde{e}_{65,T} < 95\% \cdot \hat{e}_{65,t_\omega})$	**8.7%**	**8.5%**	**8.7%**	**15.0%**
$P(\tilde{e}_{65,T} > 105\% \cdot \hat{e}_{65,t_\omega})$	**7.3%**	**7.1%**	**7.0%**	**11.1%**

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A Combined model for AMT/EMT

Examples

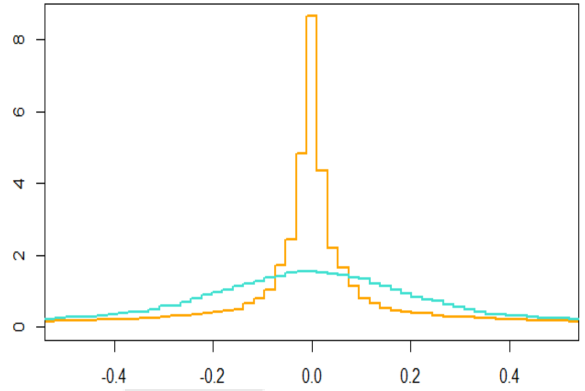
- 1. Hedge Effectiveness of a Value Hedge
- 2. Safety Margins in Annuity Conversion Rates
- 3. SCR for Longevity Risk

- Common assumptions
  - Deterministic and constant interest rate 2%
  - Annuitants'/pensioners' mortality rates are exactly as for males in England and Wales
  - Portfolios are large enough → no unsystematic mortality risk
  - For the EMT's, we use linear weighting based on life expectancy optimization

A Combined model for AMT/EMT

Example 1

Hedge Effectiveness of a Value Hedge



- Pension fund with members aged 45 in t_0
- Hedge provider offers value hedge when they retire at $T = 20 + t_0$
- If necessary, hedge fills up fund's liabilities at expiry.
- Two Risks:
  - AMT changes after T
  - AMT assumption at T is inaccurate
- Unfortunately, the pension fund's trustees do not distinguish between AMT and EMT. They assume, that the current AMT is observable. Thus, they think their remaining risk is
  - $PV_T(pension\ payouts \mid AMT_{t_\omega}) - PV_T(pension\ payouts \mid AMT_T)$ (yellow)
  - Hedge effectiveness: $1 - \frac{Var(Risk\ after\ hedge)}{Var(without\ hedge)} = \mathbf{92,1\%}$
- However, the actual risk is
  - $PV_T(pension\ payouts \mid AMT_{t_\omega}) - PV_T(pension\ payouts \mid EMT_T)$ (blue)
  - True hedge effectiveness: $1 - \frac{Var(Risk\ after\ hedge)}{Var(without\ hedge)} = \mathbf{87,2\%}$

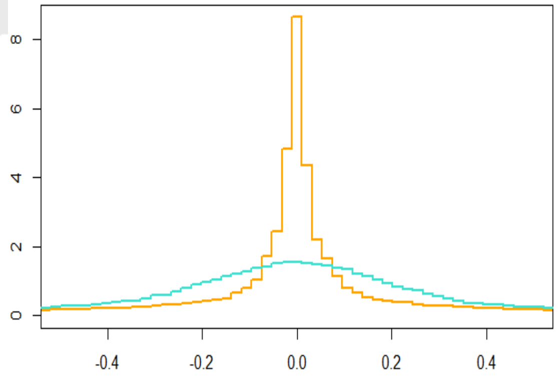
Hedge looks better than it is

A Combined model for AMT/EMT
Example 2

Safety Margins in Annuity Conversion Rates

- Insurer with unit-linked deferred annuities aged 45 in t_0
- Conversion in life-long annuity at $T = 20 + t_0$ with security margin on the fair rate
- Security margin, such that probability for losses from increasing longevity is 1%.
- Again, the insurer does not distinguish between AMT and EMT.
- They assume, that the current AMT is observable. Thus, they think their payout is
  - $PV_T\left(pension\ payouts \mid AMT_{t_\omega}\right) - PV_T(pension\ payouts \mid AMT_T)$,
  - Reduction of fair rate: **7,8%**
- However, the actual payout is
  - $PV_T\left(pension\ payouts \mid AMT_{t_\omega}\right) - PV_T(pension\ payouts \mid EMT_T)$.
  - Required reduction of fair rate: **9,6%**

Underestimation of longevity risk



A Combined model for AMT/EMT

Example 3

SCR for Longevity Risk

- Consider portfolio of 75-year old annuitants at $T = 20 + t_0$ (no costs, no premiums)
- Insurer with internal model calculates the SCR as the 99,5% percentile of (see Börger (2010)) :
- $\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$
  - CF_{T+1} actual cashflow
    - Realized mortality evolution over 1-year horizon
    - →AMT component
  - BEL_t best-estimate of liabilities
    - Influence of the additional one year observation
    - Influence too large→ overestimation of annual changes in EMT
    - Influence too small → underestimation of annual changes in EMT
    - →EMT component with optimal weighting

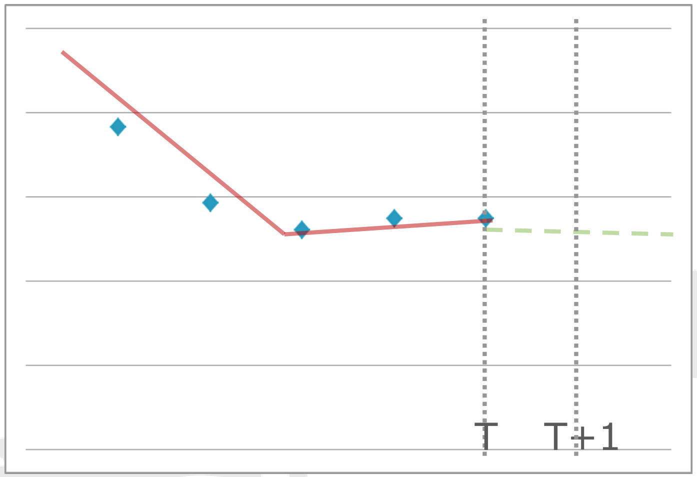
A Combined model for AMT/EMT

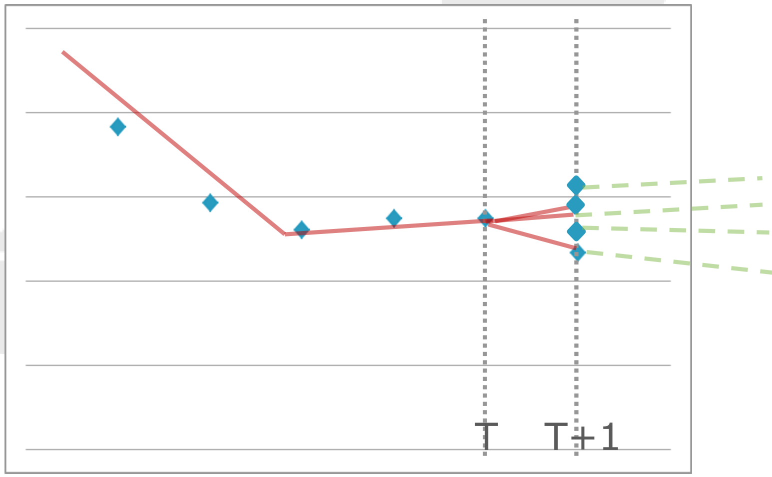
Example 3

SCR for Longevity Risk – continued

- $\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$
- Estimate AMT up to T 10.000 times (outer paths)
  - For each, simulate 10.000 inner 1-year paths
  - Estimate 99.5% precentile of ΔBEL_{T+1} based on the 10.000 inner paths
- Illustration: One outer path

$$\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$$





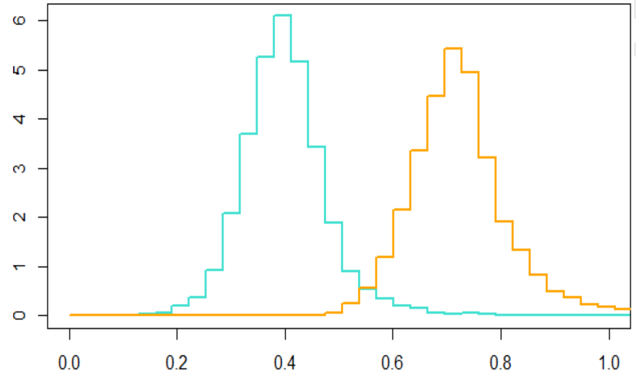
A Combined model for AMT/EMT

Example 3

SCR for Longevity Risk – continued

- If the insurer falsely assumes the AMT to be known, he would calculate the BEL_t based on the AMT.

→The SCR would be on average **0.73** (yellow).



- Instead, if the insurer recognizes the AMT to be unknown,

→the SCR would be on average **0.40** (blue)

- If the AMT is assumed to be known, the longevity risk would be **overestimated** in this example!
- Why?
  - AMT exhibits rather massive trend changes in one year
  - Annual changes in EMT are not that strong as the EMT does not pick up trend changes immediately

Conclusion

- Two trends need to be distinguished and modeled
  - The actual mortality trend (AMT) is the prevailing, unobservable mortality trend
  - The estimated mortality trend (EMT) is the estimate of the AMT
- The trend to consider depends on the question in view
- The AMT is modeled as a continuous and piecewise linear trend with random changes in the trend’s slope
- Choice of EMT approach is crucial in many practical situations
  - A weighted regression approach seems reasonable
  - Optimal regression weights can be determined in a practical setting

- If the AMT is wrongfully assumed observable, risk is significantly misestimated in all our examples – sometimes underestimated, sometimes overestimated

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