Peer-to-Peer Risk Sharing with an Application to Flood Risk Pooling

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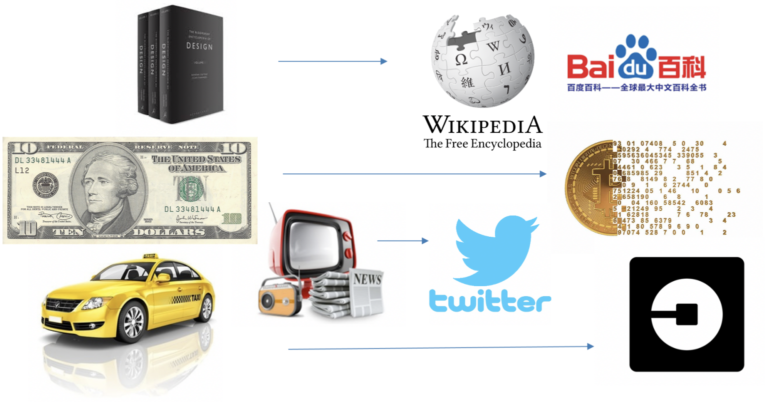
Historic Background

Decentralization / disintermediation



Sharing Economy

Product Sharing

craigslist

ebay

Service Sharing

meal sharing

Care.com
There for you

freelancer

crowdspring

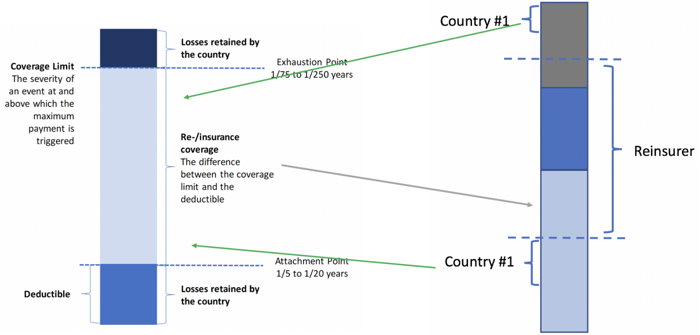
deliv

airbnb

Risk Sharing

CAT Risk Pooling

CAT Risk Pooling



Left panel is drawn from Bollman & Wang (2019)

Risk Sharing

- Main benefit resides that an individual can mitigate very large potential losses by risk sharing.
- Pre-exchange risk $\tilde{X}_i$ and post-exchange risk X_i for $i = 1, \cdots, n$.
- Risk sharing pool should satisfy:
  - Self sufficient:
$$S = \sum_{i=1}^{n} \tilde{X}_i = \sum_{i=1}^{n} X_i.$$
  - Individual's post-agreement loss should be preferable to the original, for example
$$\mathrm{Var}(X_i) \leq \mathrm{Var}(\tilde{X}_i).$$

Non Olet Risk Sharing

This pool framework requires that all agents hand their individual losses over to a pool and agree on how the total pooled loss are shared between agents.

- Pecunia non olet

Non Olet Risk Sharing

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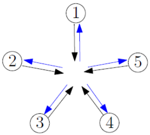
$$X_i = h_i(S)$$

where h_i is some functions depending only on individual's risk profile.

Non Olet Risk Sharing



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$$X_i = h_i(S).$$

Examples:

- Proportional: $h_i(S) = \frac{\mathbb{E}[X_i]}{\mathbb{E}[S]} S$; (CAT risk pooling)
- Conditional mean risk sharing: $h_i(S) = \mathbb{E}[X_i|S]$. (c.f. Denuit and Robert (2021))

Ideal of Decentralized Insurance

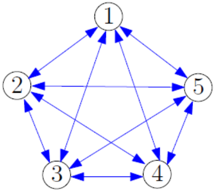
Our research team at the University of Illinois works to design risk sharing mechanisms that meet the following criteria.

- Disintermediation;
- Universal inclusiveness;
- Voluntary participation;
- Cost effectiveness;
- Actuarial fairness.

P2P Risk Sharing

There is no initial transactions take place and a claim by a single member is reimbursed by all other pool agents directly without the intervention of a centralized processing entity.

P2P Risk Sharing



Two-Agent Risk Sharing

Let X_1 and X_2 be arbitrary risks. (Universal inclusiveness)
The allocation ratio matrix is

$$\mathbf{A} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix},$$

where α_{ij} represent the proportion of the j-th agent's loss to be covered by the i-th agent.

How do you devise an allocation mechanism that is appealing to all participants?

It is sensible to consider the P2P risk sharing mechanism that satisfies two properties:

- *(Fairness)* Expected net return must be zero for all.
- *(Pareto optimality)* Post-exchange variance must be minimized as far as possible for all.

Assumptions:

- Zero-balance Conservation:

$$\alpha_{11} + \alpha_{21} = 1, \qquad \alpha_{12} + \alpha_{22} = 1.$$

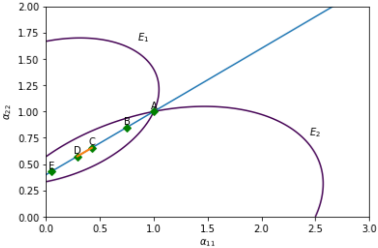
- Actuarial Fairness: $\mathbb{E}[\tilde{X}_i] = \mathbb{E}[X_i]$.

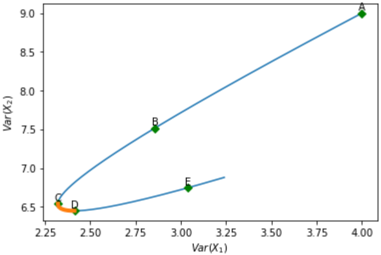
$$\alpha_{11}\mu_1 + (1 - \alpha_{22})\mu_2 = \mu_1.$$

The Pareto optimal problem

$$\min_{\mathbf{A}} w_1 \mathrm{Var}(X_1) + w_2 \mathrm{Var}(X_2)$$

where some weights $w_1, w_2 \geq 0$ and $\mathbf{X} = \mathbf{A}\tilde{\mathbf{X}}$.





Multi-Agent Risk Sharing

The main optimization problem becomes:

$$\hat{\mathbf{A}} = \underset{\mathbf{A}}{\operatorname{amin}} \sum_{i=1}^{n} \operatorname{Var}(X_i) = \underset{\mathbf{A}}{\operatorname{amin}} \operatorname{tr}(\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$$

due to the introduction of a fairness constraint

$$\mathbf{A}\boldsymbol{\mu} = \boldsymbol{\mu}, \qquad \mathbf{e}^\top\mathbf{A} = \mathbf{e}^\top$$

where $\tilde{\mathbf{X}} = (\tilde{X}_1, \ldots, \tilde{X}_n)$ denote all agents' pre-exchange losses whose joint probability distribution has mean $\mathbb{E}(\tilde{\mathbf{X}}) = \boldsymbol{\mu}$ and positive definite covariance matrix $\operatorname{Cov}(\tilde{\mathbf{X}}) = \boldsymbol{\Sigma}$.

The optimal allocation matrix is

$$\hat{\mathbf{A}} = \frac{1}{n}\mathbf{e}\mathbf{e}^\top + k\left(\mathbf{I} - \frac{1}{n}\mathbf{e}\mathbf{e}^\top\right)\boldsymbol{\mu}\boldsymbol{\mu}^\top\boldsymbol{\Sigma}^{-1} \quad \text{where} \quad k^{-1} = \boldsymbol{\mu}^\top\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}.$$

Special cases:

1. Common mean: $\mu_1 = \cdots = \mu_n$, then $\hat{\mathbf{A}} = (1/n)\mathbf{e}\mathbf{e}^\top$.
2. Common mean and variance: $\mu_1 = \cdots = \mu_n$ and $\sigma_1 = \cdots = \sigma_n$, each agent can observe variance reduction,

$$\mathrm{Var}(X_i) \leq \mathrm{Var}(\tilde{X}_i).$$

Variance Reduction Constraint

We further require

- The principle of indemnity: $0 \leq \alpha_{ij} \leq 1$
- Variance reduction: (Voluntary participation)
  $\mathrm{Var}(X_i) \leq \mathrm{Var}(\tilde{X}_i)$.

The optimization problem becomes

$$\underset{\mathbf{A}}{\operatorname{amin}} \sum_{i=1}^{n} \mathrm{Var}\left(X_i\right) = \underset{\mathbf{A}}{\operatorname{amin}} \ \mathrm{tr}(\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$$

with constraints

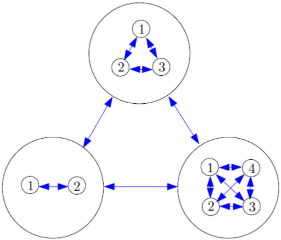
$$\mathbf{A}\boldsymbol{\mu} = \boldsymbol{\mu}, \qquad \mathbf{e}^\top\mathbf{A} = \mathbf{e}^\top, \qquad 0 \leq \mathbf{A} \leq 1, \qquad (\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)\circ\mathbf{I} \leq \boldsymbol{\Sigma}\circ\mathbf{I}$$

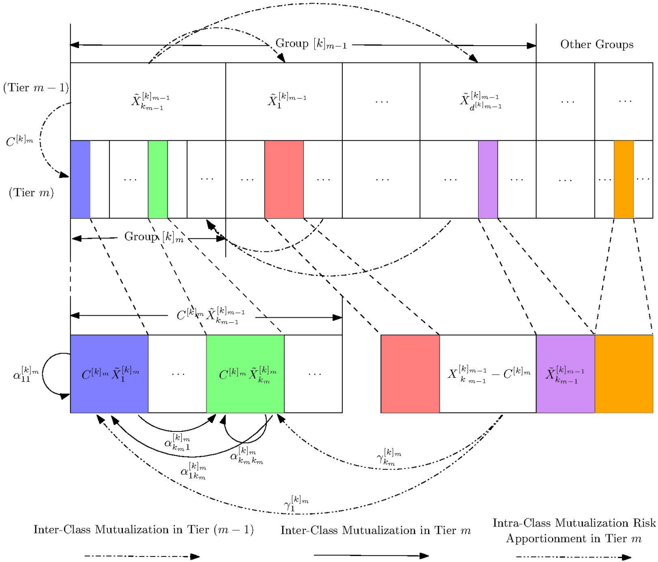
where the symbol $\circ$ denotes the Hadamard product.

Hierarchical P2P Risk Sharing

In a very large risk sharing pool, it may be impractical to execute direct transactions between all agents and a single claimant. Therefore we introduce hierarchical P2P risk sharing. Agents are grouped together and transactions occur between members of a group or between the groups themselves.

Hierarchical P2P Risk Sharing




Group $[k]_{m-1}$
Other Groups
(Tier $m-1$)
$\tilde{X}^{[k]_{m-1}}_{k_{m-1}}$
$\tilde{X}^{[k]_{m-1}}_{1}$
$\tilde{X}^{[k]_{m-1}}_{d^{[k]_{m-1}}}$
$C^{[k]_m}$
(Tier m)
Group $[k]_m$
$C^{[k]_m}\tilde{X}^{[k]_{m-1}}_{k_{m-1}}$
$\alpha^{[k]_m}_{11}$
$C^{[k]_m}\tilde{X}^{[k]_m}_{1}$
$C^{[k]_m}\tilde{X}^{[k]_m}_{k_m}$
$X^{[k]_{m-1}}_{k_{m-1}} - C^{[k]_m}$
$\tilde{X}^{[k]_{m-1}}_{k_{m-1}}$
$\alpha^{[k]_m}_{k_m 1}$
$\alpha^{[k]_m}_{k_m k_m}$
$\alpha^{[k]_m}_{1 k_m}$
$\gamma^{[k]_m}_{k_m}$
$\gamma^{[k]_m}_{1}$
Inter-Class Mutualization in Tier $(m-1)$
Inter-Class Mutualization in Tier m
Intra-Class Mutualization Risk Apportionment in Tier m

- Flood risk is known to be difficult to insure.
- Only 5% of single-family homeowners in the US have flood insurance.
- Property owners are required to purchase flood insurance only if their properties are in Special Flood Hazard Areas (SFHAs), their communities participate in the NFIP and they have federally backed mortgages.
- Property owners are unaware of or underestimating the risk they face because they are not identified as being within the SFHA zone

- The data is from the National Flood Insurance Program. It consists of approximately two million individual flood related claims over multiple decades across the United States.
- Restrict the dataset by only considering claims for single floor properties that occurred during or after the year 2000. And construct quarterly claims time series for each state.

Consider a two-tiered hierarchical example, where 50 states are grouped into nine regions.

	Total Variance	# of parameters	Time(s)
Without P2P	20752.03	–	–
Optimal P2P	2483.81	2500	26.06
Hierarchical P2P	2756.26	435	0.46

Table: Comparison of different models

50 States Flood Risk Sharing

State	Mean	Reduction (%) Optimal P2P	Reduction (%) Hierarchical P2P	State	Mean	Reduction (%) Optimal P2P	Reduction (%) Hierarchical P2P
New England		90.46	93.81	West South Central		81.91	83.10
CT	6.08	87.74	92.23	DE	5.26	82.15	91.47
MA	9.02	91.11	92.59	FL	16.3	76.03	77.02
ME	2.54	86.02	93.61	GA	14.45	79.47	82.01
NH	2.65	86.70	93.05	MD	8.29	84.95	91.57
RI	2.86	91.72	96.20	NC	18.01	80.17	80.81
VT	3.12	93.90	96.98	SC	13.64	85.39	87.87
Mid-Atlantic		87.94	81.97	VA	15.3	80.57	72.84
NJ	14.23	89.01	90.16	WV	12.84	85.68	87.11
NY	21.02	84.72	83.59	East South Central		87.83	84.74
PA	20.04	89.57	75.55	AL	24.01	92.96	91.35
East North Central		87.82	87.15	KY	14.96	84.71	82.86
IL	17.57	79.05	75.01	MS	29.93	79.88	74.33
IN	15.47	79.20	81.35	TN	18.94	86.62	81.99
MI	13.05	87.00	88.07	Mountain		86.79	89.44
OH	19.99	88.39	84.53	AZ	14.71	87.43	87.83
WI	12.21	94.20	95.73	CO	4.99	90.76	95.47
West North Central		92.52	93.35	ID	2.63	89.84	95.79
IA	13.72	93.67	94.53	MT	1.31	82.4	79.94
KS	11.61	92.53	94.49	NM	7.1	77.73	79.73
MN	4.36	85.94	89.04	NV	5.02	85.55	93.29
MO	17.98	86.24	82.30	UT	1.47	83.55	84.37
ND	5.19	96.62	98.30	WY	1.38	89.37	88.88
NE	7.98	93.77	95.31	Pacific		92.27	92.67
SD	7.4	92.84	95.25	AK	9.05	95.99	97.20
West South Central		82.67	75.89	CA	17.13	82.76	82.06
AR	21.88	86.81	83.66	HI	12.74	89.75	90.60
LA	28.41	76.64	66.40	OR	5.08	85.75	84.74
OK	21.34	87.49	84.58	WA	14.35	94.68	94.79
TX	33.68	79.13	68.30				

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Thank you for listening!