ulm university universität ulm

Compression of Life Insurance Portfolios

Uetermeier, Peter | Feb. 20, 2020 |
Institute of Insurance Science

Table of contents

Actuarial projections in life insurance

- project a life insurance company's assets and liabilities into the future
- over a specified time horizon and a set of scenarios
- in order to assess the company's future corporate performance and
- to identify potential risks
- required e.g. for ALM, Solvency II, a company's internal planning process

Complexity of actuarial projections

- long time horizon and many time steps
- large number of scenarios, e.g. financial market
- simultaneous projection and adjustment of assets and liabilities
- dynamic management rules
- nested simulations for valuing embedded options and guarantees in insurance contracts
- for a liability portfolio of millions of policies

$\Rightarrow$ causes an enormous computational complexity requiring massive runtime and storage requirements

Possible solutions

- increase computational power of hardware and software
- estimate central quantities with proxy functions w.r.t underlying risk factors
- reduce number of scenarios
- replace liabilities by a replicating portfolio of standard assets
- use replicated stratified sampling for sensitivity analysis [7]
- compress the liabilities, i.e. select a small number of model points replacing the original liability portfolio

Mathematical model

- insurance contract $C_i = \{c_j^{(i)}, j = 1, 2, \dots\}$
- portfolio of feasible contracts $P = \{C_1, \dots, C_n\} \in \mathcal{P}$, $n \in \mathbb{N}$
- set of quantities $\mathcal{Q}$
- time horizon $\mathcal{T}$
- set of scenarios $\mathcal{S} = \{s : \mathcal{T} \to \Omega\}$
- actuarial projection $f_P : \mathcal{Q} \times \mathcal{T} \times \mathcal{S} \to \mathbb{R}$, $f_P(j, t, s) = y_{jts}^{(P)}$
- compression $c : \mathcal{P} \to \mathcal{P}, P \mapsto P^{MP}$

Two main requirements towards compressed portfolio:

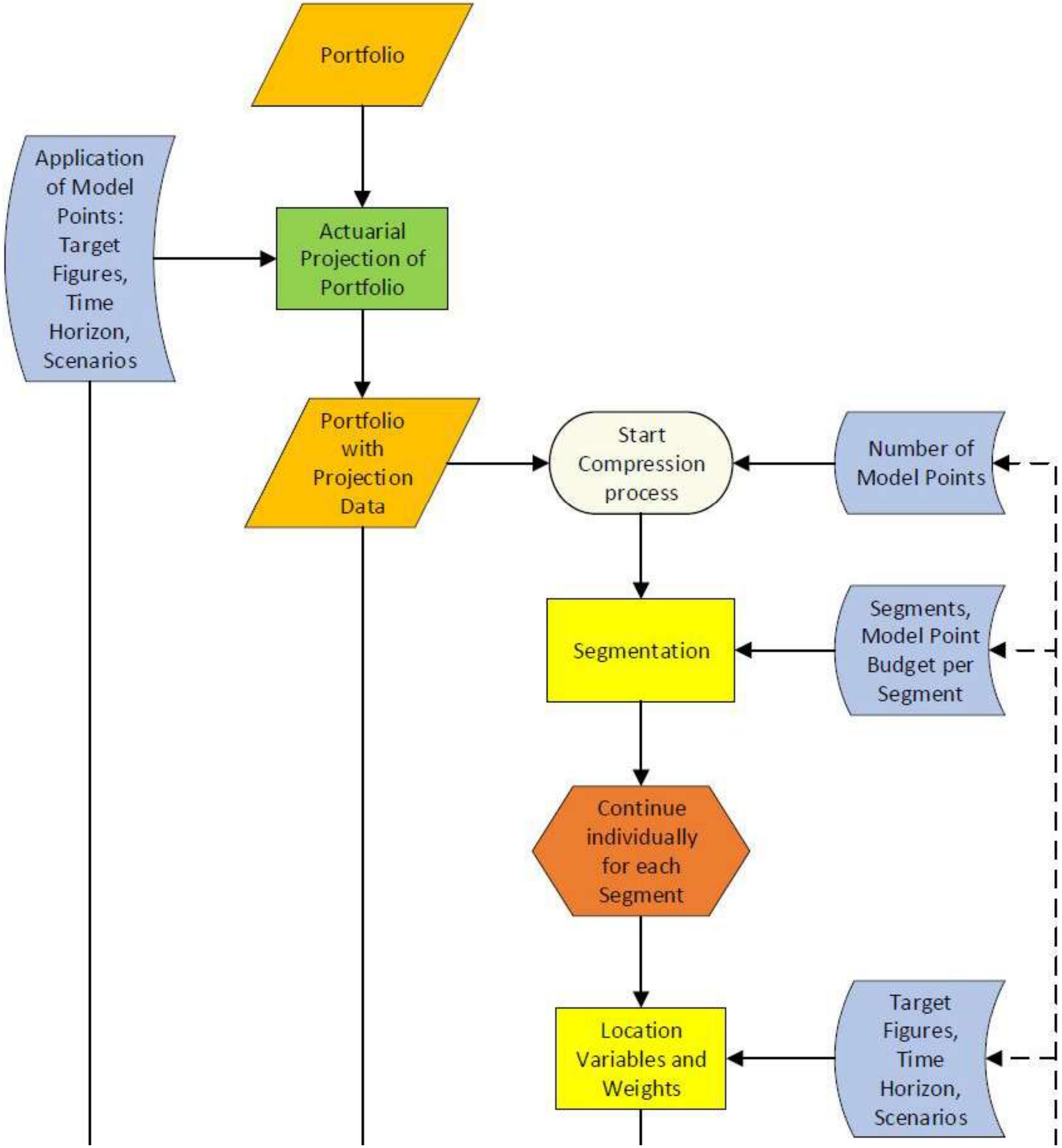
- include significantly less policies than the original portfolio P: $|P^{MP}| \leq m$
- accurately represent the portfolio P in actuarial projections: $d_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P, P^{MP}) \to \min$

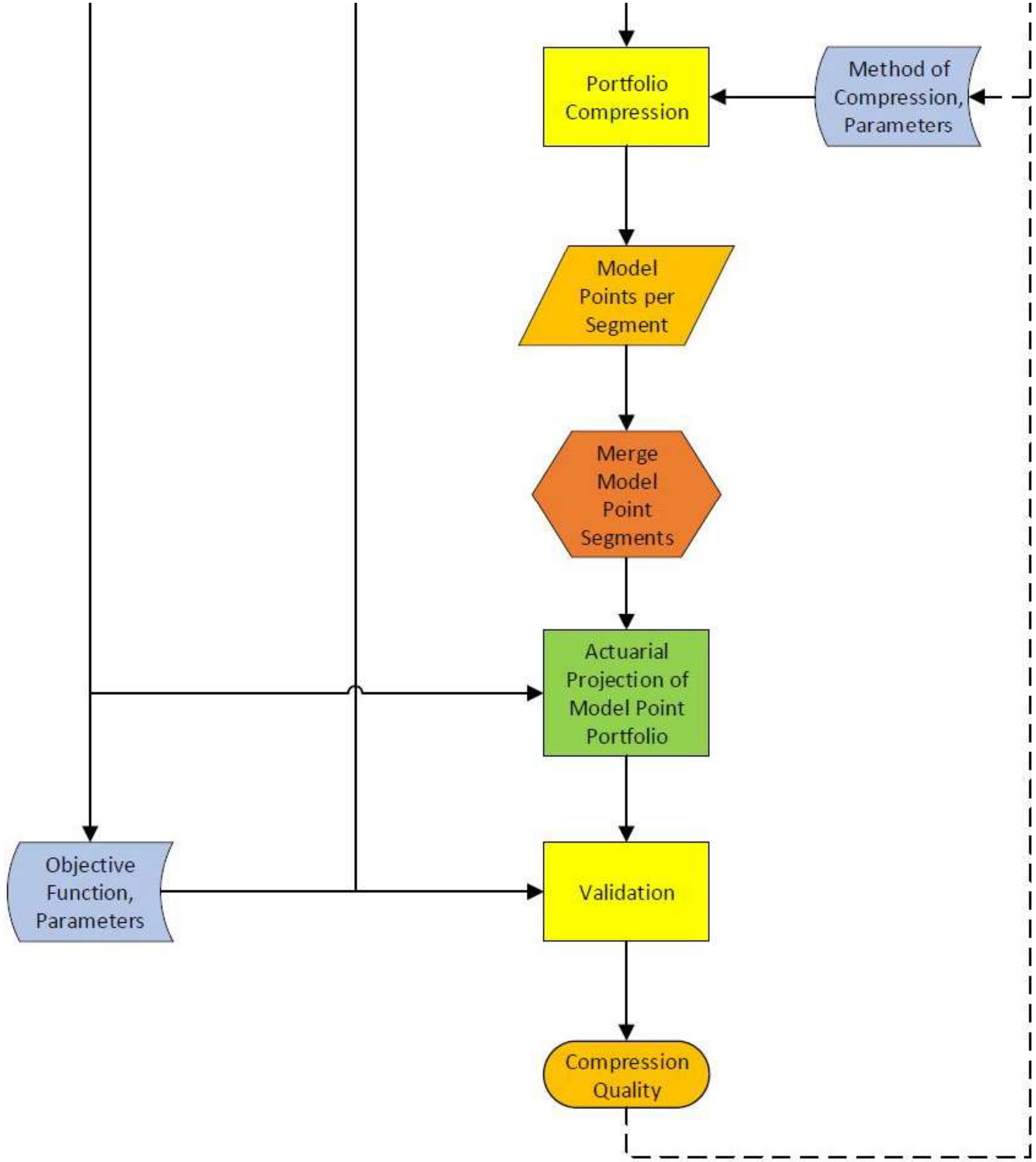
Optimization problem:

$$(OP) \begin{cases} d_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P, P^{MP}) \to \min, \\ e_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P, P^{MP}) \leq \mathbf{0}, \\ |P^{MP}| \leq m, \end{cases}$$

$\Rightarrow$ How to set up $d_{\mathcal{Q},\mathcal{T},\mathcal{S}}$?

Process model





Portfolio Compression
Method of Compression, Parameters
Model Points per Segment
Merge Model Point Segments
Actuarial Projection of Model Point Portfolio
Objective Function, Parameters
Validation
Compression Quality

Standard actuarial grouping methods

- split portfolio manually into segments based on contract features
- represent each segment by a single model point by
- aggregating contract features within each segment, e.g. by summation, averaging

Shortcomings:

- insufficient compression ratios [3]
- number of model points cannot be set in advance
- expert knowledge required $\Rightarrow$ problematic for automation, new products
- artificial policies $\Rightarrow$ inconsistencies
- no projection data taken into account

Monte Carlo methods

- consider location variables $y_j^{(i)}$, $j = 1, \ldots, M$, $i = 1, \ldots, m$ of the model points and $y_1^{(P)}, \ldots, y_M^{(P)}$ of the whole portfolio
- draw a sample of m policies and optimize their weights $x_1, \ldots, x_m \geq 0$ to solve

$$||A \cdot x - b|| \to \min,$$

$$A = \begin{pmatrix} y_1^{(1)} & y_1^{(2)} & \cdots & y_1^{(m)} \\ y_2^{(1)} & y_2^{(2)} & \cdots & y_2^{(m)} \\ \vdots & \vdots & & \vdots \\ y_M^{(1)} & y_M^{(2)} & \cdots & y_M^{(m)} \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}, \quad b = \begin{pmatrix} y_1^{(P)} \\ y_2^{(P)} \\ \vdots \\ y_M^{(P)} \end{pmatrix}$$

Clustering

- model each policy C_i as a data point in $\mathbb{R}^M$ w.r.t. its location variables $y_1^{(i)}, \dots, y_M^{(i)}$
- identify m disjoint homogeneous clusters of policies w.r.t a distance measure, e.g.

$$d(C_i, C_k) = \sqrt{\sum_{j=1}^{M} \left(y_j^{(i)} - y_j^{(k)}\right)^2}$$

Applied clustering methods in the context of compression:

- hierarchical clustering, e.g. Ward's method
- partitional clustering, e.g. k-means
- model-based-clustering, e.g. EM-algorithm

$\Rightarrow$ choice of one representative model point per cluster

Validation: general aspects

- Underlying projection data of validation

$$\left\{ \{y_{jts}^{(P)}\}, \{y_{jts}^{(P_1^{MP})}\}, \ldots, \{y_{jts}^{(P_R^{MP})}\}, j \in \mathcal{Q}, t \in \mathcal{T}, s \in \mathcal{S} \right\}.$$

- compression quality depends on homogeneity of portfolio
- check compression ratio and structure of compressed portfolio [1]
- static validation: compare portfolio key figures, cash flows and balance sheet items at the valuation date
- dynamic validation: [6]
  - forward validation: compare future projection values
  - backward validation: projection for previous period and comparison with portfolio at valuation date
- validate within-sample or out-of-sample

General requirements towards a validation function

- **Completeness**: incorporate all aspects being part of the main goals of the compression
- **Expediency**: quantify the compression quality by using adequate methods w.r.t the goals of the compression
- **Non-redundancy**: similar aspects should not be considered simultaneously in order not to overstate their impact on the validation
- **Balance**: not only multiple aspects combined should not dominate the validation function, but also single aspects should not become too influential in comparison to others

Validation function

Assume $y_{jts}^{(P)} \cdot y_{jts}^{(P^{MP})} > 0$, $j \in \mathcal{Q} = \mathcal{Q}_1 \cup \mathcal{Q}_2, t \in \mathcal{T}, s \in \mathcal{S}$:

$$d_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P, P^{MP}) =$$

$$\left(\sum_{j\in\mathcal{Q}_1} \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} w_j g_t^j f_s \left| \log \left(\frac{y_{jts}^{(P^{MP})}}{y_{jts}^{(P)}} \right) \right|^p \right)^{\frac{1}{p}} + \alpha \cdot g_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P, P^{MP})$$

where $p \geq 1$, $\alpha \geq 0$, weights $w_j \geq 0$, $g_t^j \geq 0$, $f_s \geq 0$.
Violation of constraints $h_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P, P^{MP})_k \leq 0$ is penalized

$$g_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P, P^{MP}) =$$

$$\left(\sum_{k\in\mathcal{Q}_2} \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} w_k g_t^k f_s \left| \log \left(\frac{y_{kts}^{(P^{MP})}}{y_{kts}^{(P)}} \right) \right|^p \cdot \mathbb{1}_{\left\{ h_{\mathcal{Q},\mathcal{T},\mathcal{S}}(P,P^{MP})_k > 0 \right\}} \right)^{\frac{1}{p}}.$$

Properties of validation function

- function constitutes a metric on $\mathcal{P}$
- in line with the theory of multiple criteria decision making (MCDM) [2,4,5,9]

Advantages of logarithm:

$$\log\left(\frac{y_{jts}^{(P^{MP})}}{y_{jts}^{(P)}}\right) \approx \frac{y_{jts}^{(P^{MP})} - y_{jts}^{(P)}}{y_{jts}^{(P^{MP})}} \text{ for } \left(\frac{y_{jts}^{(P^{MP})}}{y_{jts}^{(P)}}\right) \to 1$$

- general form of relative deviations with useful properties [8]
- limits the problem that relative deviations w.r.t. small values can become very large

Calibration of parameters: idea

- calibrate parameters to underlying validation data in order to ensure non-redundancy and balance of validation function
- no calibration necessary for p
- set g_t^j, $t \in \mathcal{T}$, $j \in \mathcal{Q}$, and f_s, $s \in \mathcal{S}$, according to relevance
- compose w_j, $j \in \mathcal{Q}$, from factors representing its relevance, correlation with other figures (non-redundancy) and the magnitude of its deviations (balance)
- determine α such that the two parts of the validation function represent a reasonable ratio (balance)

References

[1] Ausschuss Enterprise Risk Management der Deutschen Aktuarvereinigung (DAV). ”Validierung der versicherungstechnischen Rückstellungen unter Solvabilität II”. Nov. 16, 2018.
[2] Shu-Jen Chen and Ching-Lai Hwang. ”Fuzzy Multiple Attribute Decision Making, Methods and Applications. Lecture Notes in Economics and Mathematical Systems”. Vol. 375. Springer, 1992.
[3] Avi Freedman and Craig Reynolds. ”Cluster Analysis: A Spatial Approach to Actuarial Modeling”. Miliman Research Report. Aug. 2008.

[4] Ching-Lai Hwang and Kwangsun Yoon. ”Multiple Attribute Decision Making, Methods and Applications. Lecture Notes in Economics and Mathematical Systems”. Vol. 186. Springer, 1981.

[5] Ralph L Keeney and Howard Raiffa. ”Decisions with Multiple Objectives: Preferences and Value Trade-offs”. Cambridge University Press, 1993.

[6] Richard D. Olswang. ”Validating a Liability Model”. SOA. 1999.

[7] Milanthi K Sarukkali. “Replicated Stratified Sampling for Sensitivity Analysis”. PhD thesis. University of Connecticut, 2013.

[8] Leo Törnqvist, Pentti Vartia, and Yrjö O Vartia. “How Should Relative Changes Be Measured?” In: The American Statistician 39.1 (1985), pp. 43–46.
[9] Evangelos Triantaphyllou. ”Multi-Criteria Decision Making Methods: A Comparative Study”. Springer, 2000.
[10] Peter Uetermeier. ”Verdichtung in der Lebensversicherung”. Master’s thesis. Ulm University. 2019