Metamodeling for Variable Annuity Valuation: What works and what does not

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October 13th, 2021



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- Variable Annuities (VAs) are separate account life insurance contracts linked to a list of financial instruments with tax benefits
- 2 trillion USD in net assets (25% of US insurance industry's total assets)
- Payoff contingent on market performance, policyholder mortality, and withdrawal behavior
- VAs are difficult to evaluate
 - No direct replicating securities in the market
 - Path dependency
 - Uncertain policyholder behavior
 - ⇒ Complex financial derivatives

Page 2 Background and Literature

Key Actuarial Problems for Variable Annuities

Valuation and Hedging

- Background: Black & Scholes (1973), Harrison & Kreps (1979),...
- Numerical valuation: Bauer et al. (2008), Chen & Forsyth (2008),...
- Analytical valuation: Milevsky & Salisbury (2006), Feng & Volkmer (2012),...
- Hedging: Coleman et al. (2007, 2008),...

Policyholder Behavior

- American option pricing: Milevsky & Salisbury (2006), Dai et al. (2008), Shah & Bertsimas (2008),...
- Utility optimization: Horneff et al. (2010, 2015), Steinorth & Mitchell (2015), Moenig (2021),...

Portfolio Valuation

Gan (2015), Gan & Lin (2017), Wu et al. (2018), Quan et al. (2021),...

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• **Research Question:** How well do metamodeling approaches work on real-world VA contracts?

- All papers so far rely on synthetic datasets
- Gan and Valdez (2017): "...extremely difficult, if not impossible, for researchers to obtain real datasets..."

Extract contract features and build a data set of VAs with GMABs

Implement a flexible MC simulation process for VA valuation

Test Metamodeling with different sampling and learning components

- Larger sample size
- Sophisticated learners
- Sampling methods ×

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Page 4 VA Features and Data Construction

VAs with Guaranteed Minimum Accumulation Benefits (GMABs)

- Investment in portfolio of stocks and bonds
 - Policyholder's choice (subject to restrictions)
- If surviving to maturity, policyholder is guaranteed the benefit base
 - For a fee, regardless of investment performance
- One of the simplest riders, yet contain high heterogeneity:
 - Maturity: fixed, multiple fixed, step-up reset, etc.
 - Benefit base: combinations of return-of-premium, roll-up, ratchet, etc.
 - Fees: base, rate, specialities (such as being tied to market volatility)
 - Free withdrawals, surrender charge schedule, impact of withdrawals, etc.

...

 VA prospectus: Typically several hundred-page long documents with detailed description

- Our source: Morningstar Annuity Intelligence
 - 2,346 VA + GMAB combinations (22,623 in total for all benefit types)
 - Starting from 1994
 - Numerical values on fees and benefits
 - Textual description on features and conditions

VA Features and Data Construction

Morningstar Report Example

| to ans Platinum III B | Co | | Surrender Schedul | e | Expenses and Fees | | |
|--|---|---|---|--|---|------|--|
| innuity Pedile Bepart | | | Duration (Marca) | - | | 4.00 | |
| | | | Duration (Years) | / | Mortality and Expense Risk (M&E) | 1.30 | |
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| ante des 1997-1991 | Contraling Inc. Busine | Allowing Descer Leave | Surrender Charge | 8, 7, 6, 5, 4, 3, 2** | Administrative Charge | 0.00 | |
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| officiale. Media | | Mark! | obligatio (re) | | Distribution Charge | 0.00 | |
| led vetre & seet 96262/2/ 8 | Volte Tessaria Volte | Theorem Service Junit | Free Withdrawals | 10% of adjusted purchase | - | | |
| 200 white 2010/2012/101 | V06 Hated | Dates With Detections | TIDE WILIIGIAWAIS | | Total Annual Expense | 1.30 | |
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Page 6

Page 7 VA Features and Data Construction Morningstar Report Example

- M&E Fee: The M&E is based on the Return of Premium Death Benefit. <u>The optional Standard Death Benefit is available for 0.15%</u> less. Annual Policy Fee: The annual policy fee is \$30 for contracts issued in NM. Prior to 7/18/2011, the annual policy fee is \$35 (\$30 for contracts issued in ND) and is waived if the anniversary contract value is at least \$50,000. Prior to 7/18/2011, the surrender schedule is 7,6,6,5,4,3,2 and the free withdrawal amount is the greater of 10% of adjusted purchase payments (must be systematic for first year) or all earnings.
- Keywords: optional
- When: Standard Death Benefit
- What: M&E Fee
- How: 0.15% less

Page 8 VA Features and Data Construction

Example of Feature Extraction

| Feature Description | Original Text | Extracted Variable | Variable Value | | |
|---------------------------|--|--------------------|-------------------------|--|--|
| Depafit Charge | 0.750% assessed annually and calculated | Fee_B_Base | AV (Account Value) | | |
| Benefit Charge | against the account value | Fee_B_Rate | 0.0075 | | |
| | | SC | 8, 7, 6, 5, 4, 3, 2 | | |
| | | Length | 7 | | |
| Surrender Charge Schedule | 8, 7, 6, 5, 4, 3, 2 | Slope | -1 | | |
| | | Max | 8 | | |
| | | Min | 2 | | |
| Impact of Withdrawal | Proportionate | IW | Proportionate | | |
| | The M&E is based on the Beturn of Premium | | If BenefitOption == | | |
| Specialty 1 | | Specialty 1 | Standard Death Benefit: | | |
| Specially 1 | Death Benefit. The optional Standard Death Benefit is available for 0.15% less. | Specially 1 | Fee_VA_Rate == | | |
| | Benefit is available for 0.15% less. | | Fee_VA_Rate - 0.0015 | | |
| | | | If ID <11/1/2010: | | |
| | (Depatit) Fee perceptore is 0 FE9(prior to | | fee_B_Rate == 0.0055; | | |
| Specialty 2 | (Benefit) Fee percentage is 0.55% prior to | Specialty 2 | If ID <3/2/2009: | | |
| Specially 2 | 11/1/2010, 0.40% prior to 3/2/2009 and | Specially 2 | fee_B_Rate == 0.0040; | | |
| | 0.25% prior to 5/1/2008. | | If ID <5/1/2008: | | |
| | | | fee_B_Rate == 0.0025; | | |

53 contract features for valuation and learning

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Scenario Generation

- Investment performance (stock market participation and volatility)
- Policyholder characteristics (age and sex)

Policyholder Behavior

- Withdrawal (none, free only, deep out-of-money only, random shock)
- Step-up (never, always before a certain age, large benefit increase)

Financial Model

- Standard Black-Scholes model
- Constant volatility and risk-neutral valuation
- 3 × 3 × 3 × 2 × 4 × 3 = 648 scenarios per contract (1.5 million in total)
 - Evaluation on UW's High Throughput Computing (HTC) Resources

Page 10 Metamodeling

Sample Selection and Statistical Learning

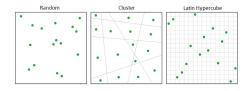
Select ⇒ Calculate ⇒ Learn (Train and Tune) ⇒ Predict

Representative Sample Selection

- Random Sampling (benchmark)
- Clustering (k-means)
- Latin Hypercube Sampling

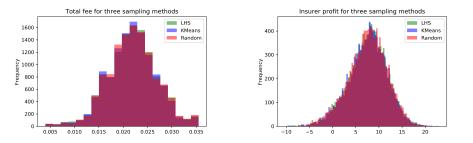
Statistical Learning

- Simple GLM (benchmark)
- Tree-Based Models
- Neural Network



Page 11 Results Representative Observations from the Sampling Component

Similar distributions on explanatory and response variables



| | Ran | Random Sample | | | Hypercube | Cluster Sampling | | |
|------------------|-----|---------------|-----|------|-----------|------------------|-------|--|
| Sample Size | 1% | 5% | 20% | 1% | 5% | 1% | 5% | |
| Sampling Time (h |) - | | | 3.42 | 20.51 | 3.54 | 24.74 | |

Page 12 Results

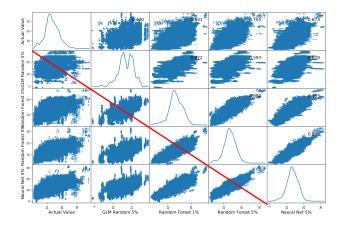
Important Features from the Learning Component

- GLM picks up dummies for feature categories
- Boosted Trees emphasize on fees

| GLM | | Boosted Trees | | | | |
|--------------------------|-------------|-----------------------|------------|--|--|--|
| Feature Name | Coefficient | Feature Name | Importance | | | |
| IW Speciality | 10.0609 | BenefitFee | 11.07% | | | |
| StepUp Next | 8.6718 | SubAccountFee L | 8.69% | | | |
| StepUp Initial | -8.3399 | VA Fee | 6.98% | | | |
| IW (min) | -5.0211 | M&E Fee | 6.16% | | | |
| IW (dollar) | -3.4742 | SubAccountFee U | 6.06% | | | |
| FreeWithdrawal Base (AV) | -3.182 | WithdrawalStrategy 1 | 5.12% | | | |
| FreeWithdrawal Base (BB) | -2.8338 | SurranderCharge Slope | 4.65% | | | |
| BenefitFee Speciality | 2.6186 | Age | 3.97% | | | |
| WithdrawalStrategy 3 | -2.4723 | AnnuitizationAge | 3.87% | | | |
| BenefitFee Base (max) | 2.4433 | WithdrawalStrategy 3 | 3.75% | | | |

Page 13 Results

Correlation of "Actual Values" and Predictions

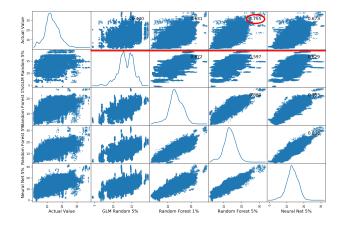


Histograms of the "actuarial values" and predictions

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Page 13 Results

Correlation of "Actual Values" and Predictions



Scatter plots for correlation

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Page 14 Results

Accuracy and Runtime of Metamodeling

| | | Random Sample | | | Latin Hypercube | | Cluster Sampling | |
|----------------|-----------------|---------------|-------|--------|-----------------|-------|------------------|-------|
| | Sample Size | 1% | 5% | 20% | 1% | 5% | 1% | 5% |
| GLM | Tuning Time (h) | - | - | - | - | - | - | - |
| | OOS RMSE | 4.29 | 4.29 | 4.27 | 4.29 | 4.29 | 4.30 | 4.30 |
| | | | | | | | | |
| Boosted Trees | Tuning Time (h) | 1.79 | 22.17 | 192.29 | 2.52 | 21.43 | 2.15 | 21.15 |
| | OOS RMSE | 3.77 | 3.15 | 3.04 | 3.78 | 3.20 | 3.77 | 3.21 |
| | | | | | | | | |
| Random Forest | Tuning Time (h) | 0.08 | 0.43 | 2.70 | 0.08 | 0.44 | 0.13 | 0.56 |
| | OOS RMSE | 3.69 | 3.13 | 2.65 | 3.70 | 3.16 | 3.72 | 3.14 |
| | | | | | | | | |
| Neural Network | Tuning Time (h) | 7.40 | 34.15 | 193.56 | 7.80 | 30.50 | 6.26 | 25.68 |
| | OOS RMSE | 4.00 | 3.53 | 3.45 | 4.02 | 3.53 | 4.13 | 3.55 |
| | | | | | | | | |

• GLM isn't improving with more samples

Page 14 Results

Accuracy and Runtime of Metamodeling

| ample Size | 1% | | | | | Cluster Sampling | |
|----------------|---|--|--|--|--|--|--|
| | | 5% | 20% | 1% | 5% | 1% | 5% |
| uning Time (h) | - | - | - | - | - | - | - |
| OS RMSE | 4.29 | 4.29 | 4.27 | 4.29 | 4.29 | 4.30 | 4.30 |
| | | | | | | | |
| uning Time (h) | 1.79 | 22.17 | 192.29 | 2.52 | 21.43 | 2.15 | 21.15 |
| OS RMSE | 3.77 | 3.15 | 3.04 | 3.78 | 3.20 | 3.77 | 3.21 |
| | | | | | | | |
| uning Time (h) | 0.08 | 0.43 | 2.70 | 0.08 | 0.44 | 0.13 | 0.56 |
| OS RMSE | 3.69 | 3.13 | 2.65 | 3.70 | 3.16 | 3.72 | 3.14 |
| | | | | | | | |
| uning Time (h) | 7.40 | 34.15 | 193.56 | 7.80 | 30.50 | 6.26 | 25.68 |
| OOS RMSE | 4.00 | 3.53 | 3.45 | 4.02 | 3.53 | 4.13 | 3.55 |
| | OS RMSE uning Time (h) OS RMSE uning Time (h) OS RMSE uning Time (h) | OS RMSE 4.29 uning Time (h) 1.79 OS RMSE 3.77 uning Time (h) 0.08 OS RMSE 3.69 uning Time (h) 7.40 | OS RMSE 4.29 4.29 uning Time (h) 1.79 22.17 OS RMSE 3.77 3.15 uning Time (h) 0.08 0.43 OS RMSE 3.69 3.13 uning Time (h) 7.40 34.15 | OS RMSE 4.29 4.29 4.27 uning Time (h) 1.79 22.17 192.29 OS RMSE 3.77 3.15 3.04 uning Time (h) 0.08 0.43 2.70 OS RMSE 3.69 3.13 2.65 uning Time (h) 7.40 34.15 193.56 | OS RMSE 4.29 4.29 4.27 4.29 uning Time (h) 1.79 22.17 192.29 2.52 OS RMSE 3.77 3.15 3.04 3.78 uning Time (h) 0.08 0.43 2.70 0.08 OS RMSE 3.69 3.13 2.65 3.70 uning Time (h) 7.40 34.15 193.56 7.80 | OS RMSE 4.29 4.29 4.27 4.29 4.29 uning Time (h) 1.79 22.17 192.29 2.52 21.43 OS RMSE 3.77 3.15 3.04 3.78 3.20 uning Time (h) 0.08 0.43 2.70 0.08 0.44 OS RMSE 3.69 3.13 2.65 3.70 3.16 uning Time (h) 7.40 34.15 193.56 7.80 30.50 | OS RMSE 4.29 4.29 4.27 4.29 4.29 4.30 uning Time (h) 1.79 22.17 192.29 2.52 21.43 2.15 OS RMSE 3.77 3.15 3.04 3.78 3.20 3.77 uning Time (h) 0.08 0.43 2.70 0.08 0.44 0.13 OS RMSE 3.69 3.13 2.65 3.70 3.16 3.72 uning Time (h) 7.40 34.15 193.56 7.80 30.50 6.26 |

- Tuning time scales heavily with sample sizes
- RMSE decreases for about 20% with a 20× increase in sample size

Page 14 Results

Accuracy and Runtime of Metamodeling

| | | Random Sample | | Latin | Latin Hypercube | | Cluster Sampling | |
|----------------|-----------------|---------------|-------|--------|-----------------|-------|------------------|-------|
| | Sample Size | 1% | 5% | 20% | 1% | 5% | 1% | 5% |
| GLM | Tuning Time (h) | - | - | - | - | - | - | - |
| | OOS RMSE | 4.29 | 4.29 | 4.27 | 4.29 | 4.29 | 4.30 | 4.30 |
| Boosted Trees | Tuning Time (h) | 1.79 | 22.17 | 192.29 | 2.52 | 21.43 | 2.15 | 21.15 |
| | OOS RMSE | 3.77 | 3.15 | 3.04 | 3.78 | 3.20 | 3.77 | 3.21 |
| Random Forest | Tuning Time (h) | 0.08 | 0.43 | 2.70 | 0.08 | 0.44 | 0.13 | 0.56 |
| | OOS RMSE | 3.69 | 3.13 | 2.65 | 3.70 | 3.16 | 3.72 | 3.14 |
| Neural Network | Tuning Time (h) | 7.40 | 34.15 | 193.56 | 7.80 | 30.50 | 6.26 | 25.68 |
| | OOS RMSE | 4.00 | 3.53 | 3.45 | 4.02 | 3.53 | 4.13 | 3.55 |

- RMSE of \$2-\$3, with mean of the actual value around \$13 (s.d. of \$5)
- MAPE of 20%, PE on the portfolio level < 0.1%

Accuracy and Runtime of Metamodeling

Metamodeling to predict insurer's profit loading

Similar results for GLM

Results

Page 15

- Advanced sampling approaches
 - \star No effect at 1% sample size, moderate effect at 5% sample size
- Advanced learning methods
 - \star Improving prediction at the cost of longer runtime

- Metamodeling to predict VAs from a single insurer
 - Significantly reduced runtime and moderately improved accuracy
 - Similar patterns in metamodeling components

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Metamodeling to predict insurer's profit loading

Similar results for GLM

Results

Page 15

- Advanced sampling approaches
 - \star No effect at 1% sample size, moderate effect at 5% sample size
- Advanced learning methods
 - \star Improving prediction at the cost of longer runtime

- Metamodeling to predict VAs from a single insurer
 - Significantly reduced runtime and moderately improved accuracy
 - Similar patterns in metamodeling components

- Constructed a data set of real-world VAs with GMAB riders
 - Extract and formalize payoff related contract features from textual description

- Implemented a flexible simulation based valuation process
 - Accommodate the high number of features and complex structures

- Tested metamodeling with different sampling and learning components
 - Sophisticated learners are better at picking up meaningful relations
 - Larger sample size increases prediction accuracy with longer runtime
 - Sampling methods do not significantly affect the performance

Thank you!

Metamodeling for Variable Annuity Valuation

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