Asymmetric Information and Longevity Risk Transfer

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Motivation

- Increasing life expectancy all over the world $\Rightarrow$ longevity risk
- Challenging liability risk management for life insurers and pension funds

$\Rightarrow$ Transfer longevity risk to reinsurer

- UK: Reinsurer insured £300 billion liabilities from defined benefit pension plans since 2007

$\Rightarrow$ Expected to increase up to £1 trillion until 2031 (Blake & Cairns, 2021)

- There is a large amount of literature about longevity risk transfer, e.g., Coughlan *et al.* (2011) or Blake *et al.* (2019)
- Assumption: Perfect information
- Reality: Most DB pension plans are small (United States Department of Labor, 2021) $\Rightarrow$ Information asymmetry

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Objective

- Analyse longevity swap in a principal-agent framework under information asymmetry
- Compute optimal contract parameters in a separating equilibrium
- Derive conditions such that the market exists
- Illustrate the results in a numerical analysis
- Compare the separating equilibrium contracts with two cases
    1. Perfect information
    2. Single longevity swap

Structure

Notation

- l_x: the initial number (in year 0) of policyholders in the hedger's portfolio
- l_{x+t}: the random number of remaining policyholders in the hedger's portfolio in year t, $t \in \{1, ..., \omega_x\}$
- ω_x: the maximal remaining life time of a policyholder aged x
- $\hat{l}_{x+t}$: the time-0 expected number of remaining policyholders in the hedger's portfolio in year t, $t \in \{1, ..., \omega_x\}$
- ${}_tp_x$: the (random) t-year survival probability of the hedger's portfolio
- ${}_t\hat{p}_x$: the time-0 expected t-year survival probability of the hedger's portfolio

The Indemnity Longevity Swap

- Contract between a hedger and a reinsurer
- The hedger pays periodic fixed payments of $(1 + \pi_t) \cdot M \cdot {}_t\hat{p}_x$
- The reinsurer pays periodic floating payments of $M \cdot {}_t p_x$
- M denotes the notional amount
- π_t denotes time varying risk loading
- Common setup, cf. Dowd *et al.* (2006) or Dawson *et al.* (2010)

Information Asymmetry

Assumption:

- Two types of hedger, a low (L) and high (H) risk type

$\Rightarrow$ Type dependent survival probabilities, i.e., ${}_tp_x^i$ for $i = L, H$

- The probability that the hedger is a low risk type is given by $\epsilon \in (0,1)$
- $M^i = z_i\, l_x^i$, where z_i denotes the type dependent hedge rate
- $\pi_t = \alpha_i \tilde{\pi}_t$, where $\alpha_i \geq 0$ and $\tilde{\pi}_t > 0$ is fixed by the reinsurer

$\Rightarrow$ Choice variables of the contract: z_i and α_i for $i = L, H$

The Principal-Agent Problem

- Principal-agent setup (cf., e.g., Rothschild & Stiglitz (1978) or McAfee & McMillan (1986)), where in this case the principal is the reinsurer and the hedger is the agent

We assume that the swap transaction has the following order

1. The reinsurer offers two swaps with specified contract parameters (α_i^A, z_i^A), $i = L, H$
2. The hedger announces its type
3. The hedger will buy the swap which gives it the largest utility gain

$\Rightarrow$ The contract parameters have to satisfy a set of participation and incentive constraints

The Reinsurer

Expected profit of the reinsurer:

$$\begin{aligned}\mathbb{E}_P[PR_R^A] &= \mathbb{E}_P\Bigg[z_L^A \, \mathbb{1}_{\{L\}} \, l_x \sum_{t=1}^{\omega_x} e^{-rt} \big[(1+\pi_t^L)\, {}_t\hat{p}_x^L - {}_t p_x^L\big] \\ &\quad + z_H^A \, \mathbb{1}_{\{H\}} \, l_x \sum_{t=1}^{\omega_x} e^{-rt} \big[(1+\pi_t^H)\, {}_t\hat{p}_x^H - {}_t p_x^H\big]\Bigg] \\ &= \epsilon \, z_L^A \, \alpha_L^A \, \mathcal{B}^L + (1-\epsilon) \, z_H^A \, \alpha_H^A \, \mathcal{B}^H,\end{aligned}$$

with $\mathcal{B}^i = \sum_{t=1}^{\omega_x} e^{-rt} \tilde{\pi}_t^i \, \hat{l}_{x+t}^i$ for $i = L, H$ and $\mathbb{1}$ denotes the indicator function

The Hedger

The profit function of the hedger $PR^i(\alpha_j^A, z_j^A)$:

$$= \left(A - \sum_{t=1}^{\omega_x} e^{-rt} l^i_{x+t} \right) + z_j^A \, l_x \sum_{t=1}^{\omega_x} e^{-rt} \left[{}_t p_x^i - (1 + \alpha_j^A \, \tilde{\pi}^j_t) \, {}_t \hat{p}_x^j \right], \; i, j = L, H$$

- A denotes the initial wealth of the hedger

The hedger has a mean-variance utility with risk aversion $\gamma > 0$:

$$U(PR^i(\alpha_j^A, z_j^A)) = \mathbb{E}[PR^i(\alpha_j^A, z_j^A)] - \frac{1}{2}\gamma Var[PR^i(\alpha_j^A, z_j^A)], \; i, j = L, H$$

The reservation utility is denoted by $\bar{U}^i$, $i = L, H$

The Optimization Problem

The principal agent optimization problem and the participation and incentive constraints:

$$\max_{\alpha_L^A, z_L^A, \alpha_H^A, z_H^A} \mathbb{E}_P[PR_R^A] \quad \text{subject to} \tag{1}$$

$$U\big(PR^L(\alpha_L^A, z_L^A)\big) \geq \bar{U}^L, \tag{PC1}$$

$$U\big(PR^H(\alpha_H^A, z_H^A)\big) \geq \bar{U}^H, \tag{PC2}$$

$$U\big(PR^L(\alpha_L^A, z_L^A)\big) \geq U\big(PR^L(\alpha_H^A, z_H^A)\big), \tag{IC1}$$

$$U\big(PR^H(\alpha_H^A, z_H^A)\big) \geq U\big(PR^H(\alpha_L^A, z_L^A)\big). \tag{IC2}$$

Typically, (PC1) and (IC2) will be binding at the optimal solution

Solution of the Optimization Problem

Proposition (Optimal contracting with adverse selection)

The solution to the optimization problem (1) *is given by:*

- $z_L^{(A,*)} = 1 - \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\left(\mathcal{V}^L - (1-\epsilon)\mathcal{V}^H\right)}$, $\alpha_L^{(A,*)} = \frac{\gamma \mathcal{V}^L}{2\mathcal{B}^L}\left(1 + \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\left(\mathcal{V}^L - (1-\epsilon)\mathcal{V}^H\right)}\right)$,
- $z_H^{(A,*)} = 1$,
  $\alpha_H^{(A,*)} = \frac{\gamma \mathcal{V}^H}{2\mathcal{B}^H} + \frac{z_L^{(A,*)}}{\mathcal{B}^H}(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \frac{\gamma}{2\mathcal{B}^H}\left((z_L^{(A,*)})^2 - 2z_L^{(A,*)}\right)(\mathcal{V}^H - \mathcal{V}^L)$,
- $\lambda_1^{(A,*)} = 1$ *and* $\lambda_2^{(A,*)} = 1 - \epsilon$,

with $\hat{\mathcal{D}}^i = \sum_{t=1}^{\omega_x} e^{-rt}\hat{l}^i_{x+t}$, $\mathcal{B}^i = \sum_{t=1}^{\omega_x} e^{-rt}\tilde{\pi}^i_t\,\hat{l}^i_{x+t}$, *and*
$\mathcal{V}^i = l_x^2\, Var(\sum_{t=1}^{\omega_x} e^{-rt}\,{}_tp^i_x)$ *for* $i = L, H$.

Scenario Analysis

Derive conditions such that the reinsurer offers a partial or full longevity hedge. Therefore, we consider two scenarios

a) The high-risk type has higher expected liabilities and the variances of the liabilities of both risk types are identical, i.e., $\hat{\mathcal{D}}^H > \hat{\mathcal{D}}^L$ and $\mathcal{V}^H = \mathcal{V}^L = \mathcal{V}$

b) The high-risk type has higher expected liabilities and a higher variance, i.e., $\hat{\mathcal{D}}^H > \hat{\mathcal{D}}^L$ and $\mathcal{V}^H > \mathcal{V}^L$

Under a) $z_L^{(A,*)} = 1 - \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\epsilon\mathcal{V}}$ and $\alpha_L^{(A,*)} = \frac{\gamma\mathcal{V}}{2\mathcal{B}^L}\left(1 + \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\epsilon\mathcal{V}}\right)$

$\Rightarrow$ We observe that the expected liabilities of the risk types cannot be too different

Numerical Analysis

- The mortality model
- Base case analysis
- Adverse selection effect

Mortality Model

- Age-Period-Cohort-Improvement (APCI) model
- Calibrated on the unisex UK data from 1956 to 2016 and ages from 20 to 100
- Extrapolate higher ages

$\Rightarrow$ Life expectancy of 21.68 years for an 65- year old person

- Modify the survival probabilities of the high risk type
  - Adjust the period effect and standard deviation of the mortality improvement effect by factor ($B > 1$ and $K > 1$)

Base Case Analysis

Risk-free rate	Initial age	Pool size	Risk aversion
$r = 2\%$	$x_0 = 65$	$l_{x_0} = 1,000$	$\gamma = 0.05$
Scenario	K	B	
a)	1.01	1.2	
b)	1.3	1.2	

Table: Base case parameter setup

Scenario	Risk type	$\mathcal{B}^i$	$\hat{\mathcal{D}}^i$	$\mathcal{V}^i$
a)	L	396	16,820	93,091
	H	397	17,493	93,091
b)	L	396	16,820	93,091
	H	430	17,497	127,808

Table: Key quantities of the outstanding liabilities under both scenarios.

Base Case Analysis

- Vary the probability of being of low risk type (ϵ)
- Compute minimum acceptable ϵ

	Scenario a)					Scenario b)				
ϵ	0.13	0.30	0.50	0.75	0.95	0.35	0.50	0.60	0.75	0.95
$z_L^{(A,*)}$	0.03	0.66	0.86	0.95	0.99	0.13	0.77	0.87	0.94	0.99
$z_H^{(A,*)}$	1	1	1	1	1	1	1	1	1	1
$\alpha_L^{(A,*)}$	11.53	7.84	6.71	6.15	5.91	10.99	7.22	6.61	6.19	5.91
$\alpha_H^{(A,*)}$	5.80	4.74	4.41	4.25	4.18	6.47	4.13	3.91	3.77	3.69

Table: The optimal contract parameters with different values of ϵ (probability of the low-risk type) under Scenario a) (left panel) and b) (right panel).

Adverse Selection Effect

Compare the previous results with two different cases:

1. The perfect information case (I), i.e., the reinsurer knows the type of the hedger
2. An imperfect information case, where the reinsurer offers a single longevity swap based on a Stackelberg game (S) with a risk loading α^S

The Stackelberg game proceeds as follow

1. The reinsurer stipulates the premium for the longevity swap (α^S)
2. Based on α^S, the hedger chooses z_i^S which maximizes its utility
3. The reinsurer maximizes its expected profit by choosing the optimal risk premium, taking into account the optimal responses of the hedger, i.e. the hedge rate as a function of α^S.

Adverse Selection Effect - Results

Optimal Contracts					
Benchmark Analysis		Perfect Information		Stackelberg Game	
$z_L^{(A,*)} = 0.77$	$\alpha_L^{(A,*)} = 7.22$	$z_L^{(P,*)} = 1$	$\alpha_L^{(P,*)} = 5.86$	$z_L^{(S,*)} = 0.36$	$\alpha^{(S,*)} = 6.25$
$z_H^{(A,*)} = 1$	$\alpha_H^{(A,*)} = 4.13$	$z_H^{(P,*)} = 1$	$\alpha_H^{(P,*)} = 7.11$	$z_H^{(S,*)} = 0.64$	
Present Value of the Fixed Leg					
$\Pi_L^A = 2864.0$	$\Pi_H^A = 1855.5$	$\Pi_L^P = 2327.3$	$\Pi_H^P = 3195.2$	$\Pi_L^S = \Pi_H^S = 2640.1$	
Expected Profit					
$\mathbb{E}[PR_R^A] = 2029.5$		$\mathbb{E}[PR_R^P] = 2761.2$		$\mathbb{E}[PR_R^S] = 1273.1$	
Utility improvement of the hedger					
$\hat{U}_L^A = 0$	$\hat{U}_H^A = 1340$	$\hat{U}_L^P = 0$	$\hat{U}_H^P = 0$	$\hat{U}_L^S = 302$	$\hat{U}_H^S = 1307$

Table: Optimal contract parameters of the three swaps (top panel), the present value of the fixed leg (2nd panel), the expected profits of the reinsurer (3rd panel) and the utility improvement of the hedger (bottom panel) under Scenario b).

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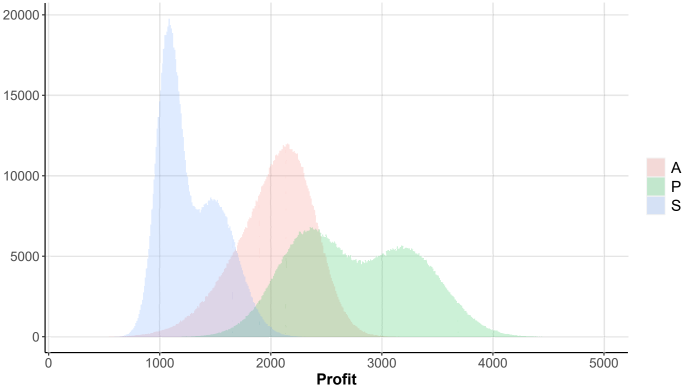

Figure: The histogram of the distribution of the profit of the reinsurer for Scenario b) and for each case, where A denotes the benchmark case, P the perfect information case and S the Stackelberg game.

Conclusion

- Study optimal longevity swap contracting under information asymmetry
- Derive analytical solutions which satisfy the participation and incentive constraints in a principal-agent model

⇒ The high risk type obtains full coverage

⇒ The low risk type obtains partial insurance and subsidizes the high risk type

⇒ Exists only if the high risk type is not substantially riskier

- If not addressed adverse selection can cause severe losses for the reinsurer

The principal-agent model can be applied to other longevity reinsurance products

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The principal-agent model can be applied to other longevity reinsurance products

Thank you for your attention!

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Adverse selection effect - Results

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$z_H^{(A,*)} = 1$	$\alpha_H^{(A,*)} = 4.41$	$z_H^{(P,*)} = 1$	$\alpha_H^{(P,*)} = 5.86$	$z_H^{(S,*)} = 0.57$	
Present Value of the Fixed Leg					
$\Pi_L^A = 2663.2$	$\Pi_H^A = 1752.3$	$\Pi_L^P = 2327.3$	$\Pi_H^P = 2327.2$	$\Pi_L^S = \Pi_H^S = 2327.2$	
Expected Profit					
$\mathbb{E}[PR_R^A] = 2015.6$		$\mathbb{E}[PR_R^P] = 2327.1$		$\mathbb{E}[PR_R^S] = 1139.3$	
Utility improvement of the hedger					
$\hat{U}_L^A = 0$	$\hat{U}_H^A = 575$	$\hat{U}_L^P = 0$	$\hat{U}_H^P = 0$	$\hat{U}_L^S = 426$	$\hat{U}_H^S = 762$

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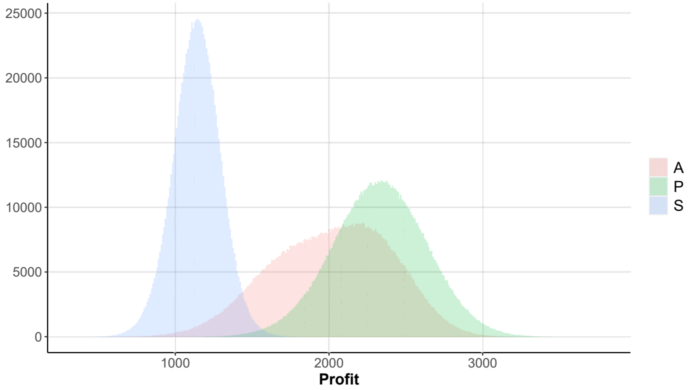

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