Asymmetric Information and Longevity Risk Transfer

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Abstract

This paper proposes a principal-agent framework to study the optimal transfer of longevity risk between a reinsurer and a hedger under information asymmetry. Most hedgers, such as a defined-benefit pension plan or a life insurer, in the real world have rather small portfolios, the liabilities of which are hard to be accurately estimated by the reinsurer. Using indemnity longevity swaps as an example of reinsurance products, we derive the analytical solution to the optimal risk premiums and incentive-compatible hedge demands in a separating equilibrium and examine the conditions for the existence of the separating equilibrium. The theoretical results are evaluated using real-world mortality data in extensive empirical analyses. We find that the expected profit of the reinsurer can be substantially increased if the adverse selection issue in longevity risk transfer is appropriately addressed.

Keywords: Longevity Risk; Adverse selection; Longevity Swap; Economic Pricing; Principal-Agent-Model

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1 Introduction

In the past few decades, unprecedented rise in human life expectancy has been witnessed. While it is per se a very pleasant development, the increasing life expectancy has led to a sequence of societal problems related to longevity risk, i.e. the risk that people live longer than expected. In particular, the increasing uncertainty surrounding future longevity forecasts has made the liability risk management for life insurers and pension funds all around the world more challenging. The rapid increase in longevity risk has also a huge impact on the global longevity risk transfer market over the past 15 years. As the most important sector of the longevity risk transfer market, the reinsurance industry has insured £300 billion of liabilities from defined benefit (DB) pension schemes in UK alone since the pension risk transfer market took off in 2007, and expects to insure an additional £700 billion of liabilities by the end of 2031, resulting in £1 trillion of DB pension scheme liabilities, which is around 50% of the total (Blake & Cairns, 2021). The majority of hedgers (such as a defined-benefit pension plan or a life insurer) around the world have a small portfolio, in which asymmetric information between the reinsurer and hedgers is likely to arise as the hedgers have more precise information about their own liability processes. This paper proposes a principal-agent framework to study the optimal longevity risk reinsurance decision between a reinsurer and a hedger taking account of the information asymmetry.

As one of the most studied classical economic problems, information asymmetry and the resultant adverse selection of the agents have been widely discussed in the insurance market between insurers and individuals. For example, in their seminal work, Rothschild & Stiglitz (1978) lay the theoretical foundation of the market equilibrium between an insurer and individuals under information asymmetry. Specifically, they characterize a self-selecting, separating equilibrium between the insurer and two types (high-risk and low-risk) of individuals, in which both the prices and the quantities of insurance are specified for both types. The conditions under which the separating equilibrium exists are also discussed. The adverse selection issue has later been empirically evaluated in different fields of insurance, including general insurance (Cohen & Siegelman, 2010), automobile insurance (Puelz & Snow, 1994; Dionne *et al.*, 2001), long-term care (Sloan & Norton, 1997), health insurance (Cutler & Zeckhauser, 1998; Cutler & Reber, 1998), and agriculture insurance (Makki & Somwaru, 2001), just to name a few.

When it comes to the annuity market, abundant literature has discussed the issue of adverse selection at policyholder level, i.e., healthier people are more likely to purchase annuities (see, among many others, Davies & Kuhn, 1992; Finkelstein & Poterba, 2004; McCarthy & Mitchell, 2010; Fong *et al.*, 2011; Hosseini, 2015; Heijdra *et al.*, 2019). However, adverse selection related to longevity risk has not yet been touched upon in the context of reinsurance, i.e., the risk transfer between insurance and reinsurance companies, by the existing literature. Although there have been plenty of studies on longevity risk transfer at institutional level (see, for example, Wills & Sherris, 2010; Coughlan *et al.*, 2011; Cairns, 2013; Blake *et al.*, 2014; Li *et al.*, 2017; Blake *et al.*, 2019), it is typically assumed that all parties have symmetric information about the longevity risk of the hedgers (the pension plans or the life insurers).[1] While the perfect information assumption might be somewhat justified for very large portfolios, the mortality experiences of which are more stable and predictable, it is likely to be improper for hedgers with small or new portfolios. Specifically, as the historical mortality data of such portfolios are shorter and/or noisier, it is rather difficult for the reinsurer to select the "correct" mortality model that captures the real underlying mortality patterns. In such cases, the hedger may have better knowledge of the future mortality of its policyholders, which results in information asymmetry between the hedger and the reinsurer. The potential consequences of adverse selection in the longevity risk reinsurance market is not negligible, as so far the majority of hedgers all around the world have a small portfolio. Taking the US for example, in 2019, there were in total 46,370 private DB pension plans in the US, and 39,586 of them had fewer than 100 participants (United States Department of

[1] There are a few studies, such as Cairns (2013) and Li *et al.* (2017), which assume that the true mortality law of the hedgers is not known, but their focus is on model risk rather than information asymmetry.

Labor, 2021). Therefore, adverse selection may harm the profitability of the longevity risk reinsurance business, or even result in financial losses for the reinsurer, and consequently hinder the development of the global longevity reinsurance market.

In this paper, we utilize the principal-agent framework proposed by Rothschild & Stiglitz (1978) to consider longevity risk reinsurance between a reinsurer (principal) and a hedger (agent) under information asymmetry. The hedger can be either a low- or a high-risk type with exogenous probabilities. The reinsurer offers two types of longevity risk reinsurance products tailor-made for each risk type, and determines the optimal prices and quantities of both products that maximize its expected profit. A set of participation and incentive constraints is imposed to ensure that both risk types will participate in the reinsurance market and reveal their real types. In this paper, we use the *indemnity longevity swaps* as an example of the reinsurance products. As an innovative longevity risk hedging instrument, indemnity longevity swaps involve periodic exchanges of cash flows between the hedger and the reinsurer. The cash flows are linked to the mortality experience of the hedger's portfolio, and could mitigate or eliminate the hedger's longevity risk. In contrast to traditional reinsurance products, such as pension buy-in (also known as the bulk annuity) or buy-out,[2] which cover all risks of the hedger's portfolio (including investment risks and operation risks, etc), indemnity longevity swaps only focus on the transfer of longevity risk, and thus could be much cheaper for the hedger. Due to its cost efficiency, indemnity longevity swap has gained substantial popularity in the global longevity risk transfer market in the past decade (Blake & Cairns, 2021). Using the longevity swaps, analytical solution in the separating equilibrium, as well as the conditions under which the equilibrium exists, are derived. Consistent with the findings in Rothschild & Stiglitz (1978), the high-risk type hedger always entails a complete hedge while the low-risk type opts for a partial hedge. Moreover, the equilibrium price of the swap for the high-risk type is lower than the price under perfect information, while the opposite is true for the low-risk type. This indicates that the high-risk type is subsidized by the low-risk type in

[2]For more discussions of buy-in and buy-out, we refer to De Ferrars (2009) and Cox *et al.* (2018).

the presence of information asymmetry.

The theoretical results of the separating equilibrium are supplemented with extensive empirical analyses using real-world mortality data and a sophisticated mortality model. In particular, the mortality data of England from 1956 to 2016 are fitted to the Age-Period-Cohort-Improvement mortality model, which is used by the CMI[3] mortality projections committee to generate life tables for UK life insurers and pension funds. Using the simulated future survival probabilities from the fitted model, we evaluate the optimal prices and quantities of the longevity swaps under two representative scenarios, under which the outstanding liability of the high-risk type is higher in expectation and/or more volatile than that of the low-risk type. Further, we compare the separating equilibrium contracts with two extreme situations: the first-best situation in which the reinsurer has perfect information about the hedger's type, and a Stackelberg game in which the reinsurer does not address the adverse selection issue and simply offers a single longevity swap to both types. We find that, while the separating equilibrium contracts lead to only slightly lower expected profit for the reinsurer than the first-best solution, they result in a distribution of profit substantially higher in mean and all percentiles (e.g., the 0.5% percentile)[4] than the case in which adverse selection is not addressed. All in all, our analyses suggest that taking the potential information asymmetry and adverse selection into account could substantially improve the profitability and meanwhile reduce the uncertainty of the longevity risk reinsurance business for the reinsurer.

The contribution of this paper is three-fold. First, this paper fills the gap between the literature of adverse selection and longevity risk reinsurance. The analytical equilibrium solution provides a convenient way to evaluate the impact of information asymmetry on the optimal prices and quantities of the longevity swaps. Furthermore, the equilibrium

[3]CMI (Continuous Mortality Investigation) is an organization supported by the Institute and Faculty of Actuaries in UK. Source: `https://www.actuaries.org.uk/learn-and-develop/continuous-mortality-investigation/about-cmi`.

[4]The 0.5% percentile is a key statistics in (re)insurance solvency capital management under the new international solvency and accounting regulations, such as Solvency II and IFRS 17 (England *et al.*, 2019).

pricing formulas shed light on the longevity risk pricing problem, which is itself a hot topic in the literature (see, for example, Denuit *et al.*, 2007; Wills & Sherris, 2010; Leung *et al.*, 2018; Xu *et al.*, 2020, and the references therein). The equilibrium pricing formulas also allow for straightforward analysis of the impact of different characteristics of the hedger's portfolio on the price of the swaps under information asymmetry. Second, our analysis provides conditions under which the longevity swap market exists under information asymmetry. In particular, we find that the market will break down when the high-risk type's liability is substantially higher in expectation and/or substantially more volatile than that of the low-risk type. This finding could be valuable for real-world reinsurers operating in or planning to enter the longevity transfer market. It also indicates the importance of mitigating information asymmetry to the further development of the longevity risk transfer market. Finally, in contrast to most of the existing insurance studies of adverse selection, which are either purely theoretical or empirical, this paper combines a theoretical analysis of optimal longevity risk transfer problem with extensive empirical analyses using real-world mortality data and a realistic mortality model.

Finally, we remark that although the indemnity longevity swap is chosen as an illustration, the proposed framework can be applied in a straightforward manner to a variety of longevity hedging instruments, including the widely-used pension buy-out and bulk annuity, as well as the index-based longevity-linked securities, which is a rather hot topic in the recent literature of longevity risk management.[5]

The remainder of this paper is organized as follows. Section 2 introduces the preliminaries. Section 3 describes the principal-agent framework and discusses the equilibrium contracts in their general forms and under two representative scenarios. Section 4 introduces the mortality data and model, and discusses the numerical analyses. Finally,

[5]The index-based longevity-linked securities provide payments linked to public indices rather than the mortality experience of individual portfolios. Theoretically, they could be treated as a class of standardized products that can be traded in the secondary market. This class of securities is rather new and exists mostly in theory so far. For more discussions on such securities, see, among many others, Blake *et al.* (2006); Coughlan *et al.* (2007); Dawson *et al.* (2010); Cairns *et al.* (2014); Li (2018); Li *et al.* (2019); Li & Luo (2012).

Section 5 concludes. Proofs and graphical illustrations of the mortality data and model are summarized in the appendix.

2 Preliminaries

In this section, we introduce the preliminaries and consider how a contract on an indemnity longevity swap can be arranged between a reinsurer (swap seller) and a hedger (swap buyer). For ease of exposition, we assume that the hedger is a life insurer selling only life annuities with a stream of in-arrears fixed unitary payments to l_x policyholders. All policyholders belong to the same cohort with age x in year 0, with homogeneous mortality experience, i.e., their future survival probabilities are governed by the same mortality law.[6] Due to longevity risks prevailing in the annuities products, the hedger is interested in purchasing longevity swaps to shift a part of the risk to the reinsurer. Let us first introduce the notations used throughout this paper:

- l_x: the initial number (in year 0) of policyholders in the hedger's portfolio;
- l_{x+t}: the random number of remaining policyholders in the hedger's portfolio in year t, $t \in \{1, ..., \omega_x\}$, where ω_x is the maximal remaining life time for the policyholders aged x;
- $\hat{l}_{x+t}$: the time-0 expected number of remaining policyholders in the hedger's portfolio in year t, $t \in \{1, ..., \omega_x\}$;
- ${}_tp_x = \frac{l_{x+t}}{l_x}$: the (random) t-year survival probability of the hedger's portfolio;
- ${}_t\hat{p}_x = \frac{\hat{l}_{x+t}}{l_x}$: the time-0 expected t-year survival probability of the hedger's portfolio.

A more detailed table summarizing the notations used in this paper is provided in Appendix A. The time-0 random present value of the hedger's annuity portfolio is then given

[6] We have chosen the simplest setting to keep the paper stay focused on the main topic. The annuity business of a real life insurer could be more complex, for example, it may include multiple cohorts from different risk classes. Such generalizations can be incorporated by our analysis in a straightforward manner.

by:

$$\left(A - \sum_{t=1}^{\omega_x} e^{-rt} l_{x+t}\right),$$

where A is the exogenous, initial premium collected from the policyholders. Further, we assume for simplicity a constant annual interest rate r for all t. In real world applications, A would be determined using the hedger's pricing assumptions, which could be different from the (reserving) mortality assumptions used in the hedging procedure discussed in the sequel. Furthermore, because the numbers of future remaining policyholders, l_{x+t}'s, are uncertain, and it deals with annuity products, the hedgers are exposed to longevity risk.

We assume there are two types of hedgers, a low (L)- and a high (H)-risk type. The future mortality rates of the hedger's portfolio depend on its risk type. Specifically, we use ${}_tp_x^L$ and ${}_tp_x^H$, $t = 1, \cdots, \omega_x$ to denote the conditional survival probability of the low- and high-risk hedger, which are random variables underlying different probability distributions. The underlying mortality models will be discussed in Section 4.1. Further, we assume that the reinsurer knows the probability distribution of both sets of survival probabilities, but does not know which type the hedger is. More specifically, the reinsurer knows that the hedger is a low-risk type with a probability of ϵ, which is strictly between 0 and 1, and a high-risk type with a probability $1 - \epsilon$. This assumption is equivalent to the case where there are multiple hedgers with the same initial portfolio size, where ϵ is the proportion of hedgers of the low-risk type.

We consider indemnity longevity swaps between the hedger and the reinsurer, with the aim to mitigate the hedger's longevity risk exposure. Specifically, the reinsurer, which acts as the hedge provider, offers protection against the uncertainty of the portfolio of the hedger over its lifespan. Formally, in each year t, the reinsurer provides two swaps, one designed for the low-risk type and the other for the high-risk type. By acquiring the swap, the hedger of risk type i pays a floating payment of $M^i \cdot FLT_t^i$ in exchange of a fixed payment of $(1+\pi_t^i)\cdot M^i \cdot FIX_t^i$, with M^i being the notional amount, and FIX_t^i and FLT_t^i the fixed and the floating rate of the swap respectively for $i = L, H$. In general, the risk premium loading at each time t can be negotiated separately between the two counterparties. However, this will lead to substantial complexity to our economic model, as well as computational burden to the numerical analysis. Therefore, in this paper we follow Chen *et al.* (2021) and assume the following functional form for the risk loading:

$$\pi_t^L = \alpha_L^A\, \tilde{\pi}_t^L,$$
$$\pi_t^H = \alpha_H^A\, \tilde{\pi}_t^H.$$

That is, the risk loading π_t^i can be presented as the product of a constant and time-varying component, where $\alpha_L^A \geq 0$ and $\alpha_H^A \geq 0$ are the constant components which will become two choice variables in our optimal contracting, and $\tilde{\pi}_t^L$ and $\tilde{\pi}_t^H$ the time-varying parts determined at time 0 for the low- and the high-risk type, respectively. In general, the payments of the reinsurer, both the fixed and floating leg, could be any cash flows linked to the mortality experience of the hedger's portfolio. We follow a popular choice in the existing literature and let FLT_t^i be the t-year realized survival probability, ${}_tp_x^i$, $i = L, H$, and FIX_t^i be the corresponding best estimate, i.e., ${}_t\hat{p}_x^L$ for the low-risk type and ${}_t\hat{p}_x^H$ for the high-risk type (Dowd *et al.*, 2006; Dawson *et al.*, 2010).

In order to focus on the annuity portfolios considered in our model, we assume that the reinsurer has no pre-existing liability, and its profit solely depends on the aggregate net payments from trading the longevity swaps. Further, when both types of swaps are available to the hedger, we consider for the moment that the hedger will purchase the contract design for its risk type, i.e., it will not have the incentive to misrepresent and take the contract designed for the other type.[7] Formally, let $M^i = z_i^A\, l_x$, where z_i^A is the

[7] In principle, the hedger of risk type i can deviate to the contract designed for the other type. We ignore this possible choice for now, and will later work out the conditions which justify the negligence.

hedge rate for type i, the expected profit of the reinsurer is given by:

$$\begin{aligned}
&\mathbb{E}_P[PR_R^A] \\
&= \mathbb{E}_P\left[z_L^A\, \mathbb{1}_{\{L\}}\, l_x \sum_{t=1}^{\omega_x} e^{-rt}\big[(1+\pi_t^L)_t\hat{p}_x^L - {}_tp_x^L\big] + z_H^A\, \mathbb{1}_{\{H\}} l_x \sum_{t=1}^{\omega_x} e^{-rt}\big[(1+\pi_t^H)_t\hat{p}_x^H - {}_tp_x^H\big] \right] \\
&= \epsilon\, z_L^A\, \alpha_L^A\, l_x \sum_{t=1}^{\omega_x} e^{-rt}\tilde{\pi}_t^L\, {}_t\hat{p}_x^L + (1-\epsilon)\, z_H^A\, \alpha_H^A\, l_x \sum_{t=1}^{\omega_x} e^{-rt}\tilde{\pi}_t^H\, {}_t\hat{p}_x^H \qquad (2.1) \\
&= \epsilon\, z_L^A\, \alpha_L^A\, \mathcal{B}^L + (1-\epsilon)\, z_H^A\, \alpha_H^A\, \mathcal{B}^H,
\end{aligned}$$

with $\mathcal{B}^i = \sum_{t=1}^{\omega_x} e^{-rt}\tilde{\pi}_t^i\, \hat{l}_{x+t}^i$ for $i = L, H$, and $\mathbb{1}_{\{L\}}$ (resp. $\mathbb{1}_{\{H\}}$) the indicator function which equals to 1 if the hedger is the low- (resp. high-) risk type and 0 otherwise. This assumption is to prevent the model from being degenerate. The reinsurer will maximize the expected profit in (2.1) by choosing the optimal risk premiums (α_i^A) and hedge rates (z_i^A), $i = L, H$. This optimization will be discussed in the next section, together with a set of participation and incentive constraints which eliminate the hedger's incentive to misrepresent and find incentive-compatible contracts.

3 The Principal-Agent Problem

In this paper, we follow typical principal-agent optimization problems (see, for example, Rothschild & Stiglitz, 1978; McAfee & McMillan, 1986), and assume that the swap transactions will be executed in the following order.

1. The reinsurer announces the two types of swaps, with the specified risk premiums and hedge rates (α_i^A, z_i^A), $i = L, H$, and commits to them.

2. The hedger announces its type.

3. The hedger will enter the swap transaction if doing so could improve its utility level, and it will buy the swap which gives it the higher utility level.

The optimal contract parameters will be determined by the reinsurer under a set of participation and incentive constraints, which ensure that the hedger is willing to participate in the swap transaction and will not misrepresent its type. In order to come to these constraints, let us first write down the random net profit function of both types as a function of the hedge rate and the risk premium. Assuming that the hedger can only purchase swaps of one type, the random profit function of the type-i hedger purchasing the type-j swap with risk premium α_j^A and hedge rate z_j^A is given by:

$$PR^i(\alpha_j^A, z_j^A) = \left(A - \sum_{t=1}^{\omega_x} e^{-rt} l_{x+t}^i\right) + z_j^A \, l_x \sum_{t=1}^{\omega_x} e^{-rt} \Big[{}_t p_x^i - (1 + \alpha_j^A \, \tilde{\pi}_t^j) {}_t \hat{p}_x^j \Big]. \tag{3.1}$$

The superscript i in PR^i means the hedger is of type i, and (α_j^A, z_j^A) means the hedger purchases the type-j swap (j could be the same as i).
Furthermore, we assume that the hedger has the following mean-variance preference:

$$U\big(PR^i(\alpha_j^A, z_j^A)\big) = \mathbb{E}_{\mathbb{P}}\big[PR^i(\alpha_j^A, z_j^A)\big] - \frac{1}{2}\gamma Var_{\mathbb{P}}\big[PR^i(\alpha_j^A, z_j^A)\big], \; i, j = L, H,$$

where γ describes the degree of risk aversion. In this section, we assume identical risk aversion for both types. Mean-variance preference is a widely used preference in decision theory and portfolio choice literature (see e.g. Ormiston & Schlee, 2001). In particular, it is shown to be fully compatible with the expected utility theory under certain conditions (Sinn, 1990). Further, it can approximate decisions of a wide variety of (concave) utility functions (Kroll *et al.*, 1984; Markowitz, 2014).

Next, we will introduce the constraints for the reinsurer's optimization problem. First, given a risk type (high/low), the hedger owns different investment possibilities. Hence, in general, it is not necessarily the case that a hedger will honestly announce its type, and purchase the "right" swap. Instead, the hedger will not misrepresent its type only if the following incentive constraints are satisfied:

$$U\big(PR^i(\alpha_i^A, z_i^A)\big) \geq U\big(PR^i(\alpha_j^A, z_j^A)\big), \quad \text{with } i, j = L, H \text{ and } i \neq j,$$

that is, the hedger could receive a higher utility by choosing the swap designed for its type. Second, both risk types have a reservation utility level, denoted by $\bar{U}^L$ and $\bar{U}^H$, respectively, representing the corresponding type's utility level without trading any swap. The hedger will participate in the swap transaction only if doing so leads to a utility improvement, namely, if the following participation constraints are satisfied:

$$U\big(PR^i(\alpha_i^A, z_i^A)\big) \geq \bar{U}^i, \quad \text{with } i, j = L, H.$$

Under these constraints, the optimization problem of the reinsurer is given by:

$$\max_{\alpha_L^A, z_L^A, \alpha_H^A, z_H^A} \mathbb{E}_P[PR_R^A] \quad \text{subject to} \tag{3.2}$$

$$U\big(PR^L(\alpha_L^A, z_L^A)\big) \geq \bar{U}^L, \tag{PC1}$$

$$U\big(PR^H(\alpha_H^A, z_H^A)\big) \geq \bar{U}^H, \tag{PC2}$$

$$U\big(PR^L(\alpha_L^A, z_L^A)\big) \geq U\big(PR^L(\alpha_H^A, z_H^A)\big), \tag{IC1}$$

$$U\big(PR^H(\alpha_H^A, z_H^A)\big) \geq U\big(PR^H(\alpha_L^A, z_L^A)\big). \tag{IC2}$$

Typically, (PC1) and (IC2) will be binding at the optimal solution, if it exists. This means that the low-risk type will be indifferent between participating or not, and the high-risk type will have no incentive to deviate to the contract offered to the low-risk type at the optimal solution. Classic literature on adverse selection (Rothschild & Stiglitz, 1978) shows that, if the above conditions are not satisfied, the hedge provider could always increase its expected profit by changing the prices of the two products. As for the conditions (PC2) and (IC1), they are satisfied with inequality and do not need to be taken into consideration. After the optimal contracts are derived, we show that these two conditions are indeed satisfied as well.

We proceed with the optimal contracting problem as follows: we first assume that the two conditions (PC1) and (IC2) are binding, based on which the first order and complementary slackness conditions are derived. In other words, we assume the existence of the separating equilibrium, where the hedgers will choose the swaps tailor-made to their type. We then examine the existence of the solution which satisfies conditions (PC2) and (IC1). With the two aforementioned constraints binding, we get the following Lagrangian function:

$$\mathcal{L} = \epsilon\, z_L^A\, \alpha_L^A\, \mathcal{B}^L + (1-\epsilon)\, z_H^A\, \alpha_H^A\, \mathcal{B}^H + \lambda_1^A \Big(U\big(PR^L(\alpha_L^A, z_L^A)\big) - \bar{U}^L \Big) \\ + \lambda_2^A \Big(U\big(PR^H(\alpha_H^A, z_H^A)\big) - U\big(PR^H(\alpha_L^A, z_L^A)\big) \Big) \tag{3.3}$$

with both λ_k^A's, $k = 1, 2$ larger than or equal to 0. Then, based on the Lagrangian function, the solution to the optimization problem (3.2) is stated in the following Proposition.

Proposition 1 (Optimal contracting with adverse selection)**.** *The solution to the optimization problem* (3.2) *is given by:*

- $z_L^{(A,*)} = 1 + \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\big((1-\epsilon)\mathcal{V}^H - \mathcal{V}^L\big)}$, $\alpha_L^{(A,*)} = \frac{\gamma \mathcal{V}^L}{2\mathcal{B}^L}\Big(1 - \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\big((1-\epsilon)\mathcal{V}^H - \mathcal{V}^L\big)}\Big)$,
- $z_H^{(A,*)} = 1$, $\alpha_H^{(A,*)} = \frac{\gamma \mathcal{V}^H}{2\mathcal{B}^H} + \frac{z_L^{(A,*)}}{\mathcal{B}^H}(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \frac{\gamma}{2\mathcal{B}^H}\big((z_L^{(A,*)})^2 - 2 z_L^{(A,*)}\big)(\mathcal{V}^H - \mathcal{V}^L)$,
- $\lambda_1^{(A,*)} = 1$ *and* $\lambda_2^{(A,*)} = 1 - \epsilon$,

with $\hat{\mathcal{D}}^i = \sum_{t=1}^{\omega_x} e^{-rt} \hat{l}_{x+t}^i$, $\mathcal{B}^i = \sum_{t=1}^{\omega_x} e^{-rt} \tilde{\pi}_t^i\, \hat{l}_{x+t}^i$, *and* $\mathcal{V}^i = l_x^2\ Var(\sum_{t=1}^{\omega_x} e^{-rt}\,{}_t p_x^i)$ *for* $i = L, H$.

Proof. The proof can be found in Appendix C.1. □

Due to the complexity of the longevity swap contracts in which many parameters are involved, no simple interpretations can be provided to the optimal solution in Proposition 1. The optimal solution depends on how specifically the low- and high-risk type are defined. Per se, there are multiple ways to define the low- and high-risk type of the hedger. In the next subsection, we will consider two representative definitions of the risk types, and discuss the optimal contracts in each scenario.

3.1 Scenario Analysis

In this paper, we consider the following two scenarios of the hedger's portfolio:

a) the high-risk type hedger has a higher expectation of its outstanding liability than the low-risk type, and the variances of both risk types' liability are identical, i.e., $\hat{\mathcal{D}}^H > \hat{\mathcal{D}}^L$ and $\mathcal{V}^H = \mathcal{V}^L = \mathcal{V}$.

b) the high-risk type hedger's outstanding liability is higher in both the expectation and variance, i.e., $\hat{\mathcal{D}}^H > \hat{\mathcal{D}}^L$ and $\mathcal{V}^H > \mathcal{V}^L$.

In the following analysis, we only consider optimal hedge rates that are between 0 and 1, i.e., $z_L^{(A,*)} \in [0,1]$ and $z_H^{(A,*)} \in [0,1]$. This assumption resembles the real-world situations in which the reinsurer offers partial or full longevity hedge to the life insurers or pension plans, and does not offer short-sell or over-hedge opportunities of the indemnity swap (or other reinsurance products). This assumption is also typical in the reinsurance literature, such as Højgaard & Taksar (1998), Schmidli (2001), and Gu *et al.* (2010). We then derive conditions of the parameter values (e.g., $\mathcal{B}^{(i)}$, $\mathcal{D}^{(i)}$, and $\mathcal{V}^{(i)}$) such that the optimal hedge rates fall within the desired range.

Before proceeding to the scenario analysis of the optimal contracting, the optimal solution *in the absence* of information asymmetry, i.e., when the reinsurer knows exactly the risk type of the hedger, is presented. With perfect information, Chen *et al.* (2021) show that it is optimal for the reinsurer to negotiate with each type of hedger separately, and sell them the swap designed for their own type. The resulting optimal hedge rates and the risk premiums, which we refer to as the first-best solution, are summarized in the following Proposition.

Proposition 2 (Optimal contracting with perfect information)**.** *The optimization problem under perfect information (P) for each hedger $i \in \{L, H\}$ is given by:*

$$\max_{\alpha_i^P, z_i^P} \; \mathbb{E}_P[PR_R^P] \quad \textit{subject to}$$

$$U\big(PR_i^P(\alpha_i^P, z_i^P)\big) \geq \bar{U}^i,$$

where

$$PR_R^P = z_i^P\, l_x^i \sum_{t=1}^{\omega_x} e^{-rt} \Big[\big(1 + \alpha_i^P\, \tilde{\pi}_t^i\big)\, {}_t\hat{p}_x^i - {}_t p_x^i \Big],$$

$$PR_i^P(\alpha_i^P, z_i^P) = \left(A - \sum_{t=1}^{\omega_x} e^{-rt} l_{x+t}^i \right) + z_i^P\, l_x^i \sum_{t=1}^{\omega_x} e^{-rt} \Big[{}_t p_x^i - \big(1 + \alpha_i^P\, \tilde{\pi}_t^i\big)\, {}_t\hat{p}_x^i \Big], \text{ and}$$

$\tilde{\pi}_t^i$ *is the hedger specific risk loading. The optimal solution is given by*

$$z_i^{(P,*)} = 1,\; \alpha_i^{(P,*)} = \frac{\gamma \mathcal{V}^i}{2\mathcal{B}^i},$$

where γ *is the risk aversion parameter of the hedger, and* $\mathcal{B}^i = \sum_{t=1}^{\omega_x} e^{-rt} \tilde{\pi}_t^i \hat{l}_{x+t}^i$.

Proof. The proof can be found in Proposition 2 in Chen *et al.* (2021). □

The subscript P in the proposition above refers to *perfection information.* With perfect information, we see that both risk types opt for a full indemnity hedge ($z_i^{(P,*)} = 1$, $i = L, H$). Moreover, the risk premiums depend only on the hedger's own portfolio, since each contract is negotiated individually. Next, we proceed to the scenario analysis under asymmetric information. First, the optimal solution under Scenario a) is summarized in Lemma 1.

Lemma 1 (Optimal contracting under Scenario a): $\hat{\mathcal{D}}^H > \hat{\mathcal{D}}^L$ and $\mathcal{V}^H = \mathcal{V}^L = \mathcal{V}$)**.** The optimal solution is given by:

$$\begin{aligned}
z_L^{(A,*)} &= 1 - \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma \epsilon \mathcal{V}}, \\
z_H^{(A,*)} &= 1, \\
\alpha_L^{(A,*)} &= \frac{\gamma \mathcal{V}}{2\mathcal{B}^L} \left(1 + \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma \epsilon \mathcal{V}} \right), \\
\alpha_H^{(A,*)} &= \frac{1}{2} \gamma \frac{\mathcal{V}}{\mathcal{B}^H} + \frac{z_L^{(A,*)}}{\mathcal{B}^H} (\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H).
\end{aligned} \tag{3.4}$$

We see that the optimal risk premium $\alpha_H^{(A,*)}$ of the high-risk type is lower than the first-best case, which means the high-risk type hedger enjoys a cheaper swap due to information asymmetry. Consequently, the high-risk type hedger will opt for a full hedge, as in the first-best case. On the other hand, the optimal hedge rate (resp. risk premium) of the low-risk type hedger is reduced (resp. increased) by the same percentage adjustment factor $\frac{(1-\epsilon)(\hat{\mathcal{D}}^H-\hat{\mathcal{D}}^L)}{\gamma\epsilon\mathcal{V}}$. With the assumption of $\hat{\mathcal{D}}^H > \hat{\mathcal{D}}^L$, the adjustment factor is positive, making the premium rate higher than the first-best level, which consequently makes a partial hedge optimal for the low-risk type. In other words, in order to prevent the high-risk type from deviating, the reinsurer needs to charge a higher risk premium for the low-risk type swap, which causes the low-risk type hedger to choose a lower hedge rate. Further, the adjustment factor increases when the difference between the expected liability of the two types, $\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L$, is larger, making $\alpha_L^{(A,*)}$ and $z_L^{(A,*)}$ deviate further from the first-best level. Finally, $\alpha_L^{(A,*)}$ is a decreasing function of ϵ. That is, when the probability of the hedger being the high-risk type is larger, the separating equilibrium requires a higher $\alpha_L^{(A,*)}$ for the incentive constraints to hold.

From Solution (3.4), we see that the adjustment factor $\frac{(1-\epsilon)(\hat{\mathcal{D}}^H-\hat{\mathcal{D}}^L)}{\gamma\epsilon\mathcal{V}}$ needs to be bounded above by 1 in order to ensure $z_L^{(A,*)} > 0$. This condition translates to:

$$\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L < \frac{\gamma\epsilon\mathcal{V}}{1-\epsilon}. \tag{3.5}$$

In other words, the difference between the expected liability of the two risk types cannot exceed a threshold dependent on $\mathcal{V}$, ϵ, and γ. If the expected liabilities are too different, then $\alpha_L^{(A,*)}$ needs to be so high that the low-risk type hedger finds the longevity swap not attractive as a hedging instrument. Moreover, the threshold is increasing in all the three parameters, which means that (combinations of) a more volatile liability, a higher probability of the hedger being the low-risk type, and a higher risk aversion parameter would lead to a greater tolerance of the difference between the two expected liabilities. Next, the optimal solution under Scenario b) is shown in Lemma 2.

Lemma 2 (Optimal contracting in Scenario b): $\hat{\mathcal{D}}^H > \hat{\mathcal{D}}^L$ and $\mathcal{V}^H > \mathcal{V}^L$)**.** The optimal solution has the general form of:

$$
\begin{aligned}
z_L^{(A,*)} &= 1 - \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\big(\mathcal{V}^L - (1-\epsilon)\mathcal{V}^H\big)}, \\
z_H^{(A,*)} &= 1, \\
\alpha_L^{(A,*)} &= \frac{\gamma \mathcal{V}^L}{2\mathcal{B}^L}\left(1 - \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\big(\mathcal{V}^L - (1-\epsilon)\mathcal{V}^H\big)}\right), \\
\alpha_H^{(A,*)} &= \frac{\gamma \mathcal{V}^H}{2\mathcal{B}^H} + \frac{z_L^{(A,*)}}{\mathcal{B}^H}(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \frac{\gamma}{2\mathcal{B}^H}\big((z_L^{(A,*)})^2 - 2z_L^{(A,*)}\big)(\mathcal{V}^H - \mathcal{V}^L). \qquad (3.6)
\end{aligned}
$$

Similar to Scenario a), we see that the high-risk type hedger still opts for a full hedge. Also, $\alpha_L^{(A,*)}$ and $z_L^{(A,*)}$ are increased and reduced by a percentage adjustment factor, $\frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\left(\mathcal{V}^L - (1-\epsilon)\mathcal{V}^H\right)}$, respectively. When this adjustment factor is bounded by 0 and 1, it holds that $z_L^{(A,*)} \in [0,1]$ and $\alpha_H^{(A,*)}$ lower than the first-best value. To keep the adjustment factor within the desired range, the following conditions need to hold:

$$
\mathcal{V}^H < \frac{\mathcal{V}^L}{(1-\epsilon)}, \quad \text{and} \qquad (3.7)
$$

$$
\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L \leq \frac{\gamma\big(\mathcal{V}^L - (1-\epsilon)\mathcal{V}^H\big)}{1-\epsilon}. \qquad (3.8)
$$

Several observations can be made. First, from Constraint (3.7), we see that $\mathcal{V}^H$ cannot be substantially larger than $\mathcal{V}^L$ for a fixed ϵ. Second, the difference in $\mathcal{V}^H$ and $\mathcal{V}^L$ can be larger if ϵ is large, i.e., the probability of the hedger being the high-risk type is small. Third, Constraint (3.8) indicates that, similar to Scenario a), the difference between $\hat{\mathcal{D}}^H$ and $\hat{\mathcal{D}}^L$ cannot exceed a threshold dependent on $\mathcal{V}^H$, $\mathcal{V}^L$, ϵ, and γ. That is, when the expected liability of the two risk types are too different, $\alpha_L^{(A,*)}$ has to be so high that the low-risk type finds the longevity swap not as attractive as a hedging instrument. In fact, when Constraint (3.8) is violated, $\alpha_H^{(A,*)}$ would be larger than its first-best value, and the participation constraint of the high-risk type hedger would be violated:

$$
\begin{aligned}
U(PR^H(\alpha_H^{(A,*)}, z_H^{(A,*)})) - \bar{U}^H &= -\alpha_H^{(A,*)}\, z_H^{(A,*)}\, B^H - \frac{1}{2}\gamma((z_H^{(A,*)})^2 - 2z_H^{(A,*)})\mathcal{V}^H \\
&= -\alpha_H^{(A,*)}\, B^H + \frac{\gamma \mathcal{V}^H}{2}
\end{aligned}
$$

$$= -\left(\frac{z^L}{\mathcal{B}^H}(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \frac{\gamma}{2\mathcal{B}^H}\big((z_L^{(A,*)})^2 - 2z_L^{(A,*)}\big)(\mathcal{V}^H - \mathcal{V}^L)\right) < 0$$

and thus no solution exists to the principal-agent problem. In other words, when the expected liability of the high-risk type hedger is too large, the reinsurer would have to charge higher risk premiums for both types to ensure that they do not deviate. However, due to the overly high risk premium, even the high-risk type hedger would not be willing to purchase any swap at all, and thus the market collapses.

All in all, the optimal hedge rate of the low-risk type will fall outside the $[0, 1]$ interval when the two risk types are "too" different, i.e., when $\mathcal{D}^H$ and/or $\mathcal{V}^H$ are substantially larger than $\mathcal{D}^L$ and/or $\mathcal{V}^L$. Since the two risk types are determined only by information not observed by the reinsurer, they are not likely to be substantially different from each other after all observable information is taken into account. Indeed, in the subsequent numerical analyses, we will see that moderate differences of the mortality law between the two risk types will lead to reasonable values of $z_L^{(A,*)}$.

4 Numerical Analysis

In this section, illustrative numerical analyses are performed to evaluate the optimal longevity swap contracts, as well as their impact on the reinsurer and the hedger under information asymmetry. Section 4.1 introduces the mortality model used in this paper to simulate the outstanding liabilities of the hedger, Section 4.2 discusses the benchmark numerical analysis. Finally, Section 4.3 compares the optimal contracts under the benchmark analysis with two extreme cases: the first-best case in which the reinsurer has perfect information, and the Stackelberg game case in which the reinsurer does not address adverse selection in the presence of asymmetric information.

4.1 The mortality model

In this paper, we use the Age-Period-Cohort-Improvement (APCI) model to simulate future survival probabilities. The APCI model is used by the CMI Mortality Projections Committee[8] to generate life tables for UK life insurers and pension funds. Specifically, denote by $m_{x,t}$ the annual death rates at age x and in year t, the APCI model describes the logarithm of the death rate as:

$$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)}(t - \bar{t}) + \kappa_t + \gamma_c, \tag{4.1}$$

where $\beta_x^{(1)}$ is the age effect representing the average mortality level at age x, $\beta_x^{(2)}$ is the period effect representing the age-specific exposure to the linear time trend, $\bar{t}$ is a normalization parameter set to the average year in the sample, κ_t is a time-varying parameter representing the residual mortality improvements not captured by the linear trend, and finally, γ_c is the cohort-effect, i.e., the unique mortality pattern shared by the group of people born in the same year.

The parameters of Model (4.1) are estimated by the least squares approach. Specifically, let X be the number of ages and T the number of years in the data, there are $3X+2T-1$ parameters to be fitted. The parameters are estimated using the unisex mortality data of England from 1956 to 2016 and ages from 20 to 100. We do not consider ages above 100 in the estimation, as the number of death is scarce and volatile for these ages, and thus the mortality rates could introduce large estimation errors when included. In the numerical analysis, we assume that the maximally attainable age is 120, which is typically the maximum age of life tables used by insurance companies in practice. In order to generate the mortality rates from 101 to 120, we need to extrapolate the age and the period effects. Specifically, we follow Dowd & Blake (2019) and Dowd *et al.* (2019), and extrapolate $\beta_x^{(1)}$ using a linear regression fitted to the estimated parameters for ages $x \geq 70$, and $\beta_x^{(2)}$ using the estimated parameters for age 100, i.e. $\beta_x^{(2)} = \beta_{100}^{(2)}$ for $x > 100$. In other words,

[8] Source: CMI Mortality Projections Committee *et al.* (2016).

we assume that the log-mortality level increases linearly for ages above 100, while their period effects are identical to that of age 100. We refer to Dowd & Blake (2019) and Dowd *et al.* (2019) for more detailed discussions on old age mortality extrapolation. The estimated parameters and the extrapolated age and period effects are shown in Appendix B.

In order to forecast future survival probabilities, the mortality improvement factor κ_t needs to be projected. To this end, we fit a set of candidate time-series models, ARIMA(p, d, q) with $p, q = 1, 2, 3$, and $d = 1, 2$ to the estimated κ_t process. Based on the Bayesian Information Criterion (BIC), ARIMA(0,1,1) is selected as the optimal model specification:

$$\kappa_t = \kappa_{t-1} + \theta\sigma_\kappa\epsilon_{t-1} + \sigma_\kappa\varepsilon_t,$$

where θ is the moving-average parameter, σ_κ is the standard deviation of the error terms, and ε_t are i.i.d. standard normal distributed error-terms. We obtain the estimation result: $\hat{\theta} = -0.3008$ and $\hat{\sigma}_\kappa = 0.0194$. Finally, if only the existing cohorts are considered, as is the case in our analysis, the cohort effect γ_c does not need to be projected.

4.2 Benchmark Analysis

In this subsection, we evaluate the reinsurer's expected profit and the hedger's utility improvement under the two scenarios described in Section 3. First, the future survival probabilities of the two risk types are simulated. To this end, we assume that the survival probabilities of the low-risk type follow the same mortality law under both scenarios, and are generated by the estimated APCI model discussed above, whereas the survival probabilities of the high-risk type are simulated by the estimated APCI model with the following two modifications:

$$\hat{\beta}_x^{(2,H)} = B\hat{\beta}_x^{(2)},$$
$$\hat{\sigma}_\kappa^H = K\hat{\sigma}_\kappa,$$

i.e., the period effects and the standard deviation of the mortality improvement effect of the high-risk type hedger are adjusted by the factor B and K, respectively. By selecting appropriate values of B and K, the two scenarios can be constructed.

The parameter setup of the benchmark numerical analysis is summarized in Table 1. In particular, we consider a single cohort with age 65 at time 0, and the hedger's initial portfolio size is 1,000. The probability of the hedger being the low-risk type is set to be $\epsilon = 0.5$, that is, both risk types are equally probable given the observable information of the hedger. The value of γ is in line with the values used in the existing literature (see, for example, Zhang *et al.*, 2009; Li *et al.*, 2017). For each Scenario, $S = 1,000$ future paths of survival probabilities are simulated for both risk types. Based on this setup, the simulated life expectancy is 21.68 years for the low-risk type hedger, and 22.75 for the high-risk type hedger under both scenarios.

Risk-free rate	Initial age	Pool size	Risk aversion
$r = 2\%$	$x_0 = 65$	$l_{x_0} = 1,000$	$\gamma = 0.05$
Scenario	K	B	Prob. of low-risk type
a)	1.01	1.2	$\epsilon = 0.5$
b)	1.3	1.2	

Table 1: Base case parameter setup.

When calculating the payoff of the swaps, the time-varying risk loading $\tilde{\pi}_t^i$ is set as the standard-deviation premium principle, $\tilde{\pi}_t^i = \sigma\left(\frac{l_{x+t}^i}{\hat{l}_{x+t}^i}\right)$, $i = L, H$. The resulting key quantities of the outstanding liabilities for both scenarios are summarized in Table 2. We see that the factors $B = 1.2$ and $K = 1.3$ lead to a 4% increase in the high-risk type's expected liability $\hat{D}^H$ under both scenarios, and a 37% increase in the variance of the liability's present value $\mathcal{V}^H$ under Scenario b). Further, $\mathcal{B}^H = \sum_{t=1}^{\omega_x} e^{-rt}\tilde{\pi}_t^H \hat{l}_{x+t}^H$ is higher under Scenario b) because of higher time-varying risk loadings $\tilde{\pi}_t^H = \sigma\left(\frac{l_{x+t}^H}{\hat{l}_{x+t}^H}\right)$.

Scenario	Risk type	$\mathcal{B}^i$	$\hat{\mathcal{D}}^i$	$\mathcal{V}^i$
a)	L	396	16,820	93,091
	H	397	17,493	93,091
b)	L	396	16,820	93,091
	H	430	17,497	127,808

Table 2: Key quantities of the outstanding liabilities under both scenarios.

The optimal solution under both scenarios is shown in Table 3. For the sake of comparison, we also include the optimal contracts in the case of perfect information. First, we see $\alpha_L^{(A,*)} > \alpha_L^{(P,*)}$ and $\alpha_H^{(A,*)} < \alpha_H^{(P,*)}$ under both scenarios. This is consistent with the theoretical results derived in Section 3, i.e., the reinsurer has to increase $\alpha_L^{(A,*)}$ and reduce $\alpha_H^{(A,*)}$ in order to prevent the high-risk type from deviating in the presence of asymmetric information. As a result, the high-risk type hedger benefits from the cheaper swap and enjoys an improvement in expected utility, whereas the low-risk type hedger would opt for a lower hedge rate due to the higher price. Second, $\alpha_H^{(A,*)}$ is lower under Scenario b), where the liability of the high-risk type hedger is more volatile. This is at odd with the optimal contracts under perfect information, where the optimal risk premium increases with the liability's volatility. In fact, although the survival probabilities of the low-risk type hedger are unchanged, $\alpha_L^{(A,*)}$ has increased when the high-risk type hedger becomes riskier. This observation implies that, due to the incentive constraints, the low-risk type hedger will subsidize the high-risk type hedger more when the two risk types are more different. Finally, the optimal hedge rate of the low-risk type hedger is between 0 and 1 under both scenarios, meaning that both conditions (3.7) and (3.8) are satisfied with the chosen parameter values.

The effect of the probability ϵ

In the benchmark analysis, we assumed $\epsilon = 0.5$, i.e., the probability of the hedger being the low-risk type is 50% given the observable information. In this subsection, we analyse the effect of this probability on the optimal solution. An ϵ larger (smaller) than 0.5 rep-

Asymmetric Information				
Scenario	$z_L^{(A,*)}$	$\alpha_L^{(A,*)}$	$z_H^{(A,*)}$	$\alpha_H^{(A,*)}$
a)	0.856	6.711	1	4.414
b)	0.769	7.218	1	4.132
Perfect Information				
Scenario	$z_L^{(P,*)}$	$\alpha_L^{(P,*)}$	$z_H^{(P,*)}$	$\alpha_H^{(P,*)}$
a)	1	5.86	1	5.84
b)	1	5.86	1	7.11

Table 3: The optimal results for both scenarios and $\epsilon = 0.5$ and $\gamma = 0.05$.

resents an optimistic (pessimistic) scenario regarding the hedger's type.

First, we compute the minimally acceptable probability of the low-risk type, i.e. ϵ, such that the low-risk type hedger will not prefer short-selling the swap. This number is 12.6% under Scenario a) and 34.1% under Scenario b). That is, when the difference between the two risk types is larger (under Scenario b), in which the high-risk type hedger is also riskier), the reinsurer needs to set a higher $\alpha_L^{(A,*)}$ to prevent the high-risk type hedger from deviating. Consequently, the set of ϵ for the longevity swap transaction to happen is narrower (c.f. Lemma 2). Furthermore, Table 4 reports the optimal contracts with different values of ϵ. We see that the risk premium of the high-risk type swap, $\alpha_H^{(A,*)}$, decreases with ϵ under both scenarios. Consequently, the minimal value of $\alpha_L^{(A,*)}$ required to prevent the high-risk type hedger from deviating is also decreasing in ϵ, and thus the optimal hedge rate of the low-risk type hedger, $z_L^{(A,*)}$, is increasing in ϵ. Finally, when ϵ approaches 1, both $\alpha_L^{(A,*)}$ and $z_L^{(A,*)}$ converge to the first-best solution (c.f. Lemmas 1 and 2), as the impact of the high-risk type hedger on the expected profit of the reinsurer diminishes.

	Scenario a)					Scenario b)				
ϵ	0.13	0.30	0.50	0.75	0.95	0.35	0.45	0.6	0.75	0.95
$z_L^{(A,*)}$	0.03	0.66	0.86	0.95	0.99	0.13	0.66	0.87	0.94	0.99
$z_H^{(A,*)}$	1	1	1	1	1	1	1	1	1	1
$\alpha_L^{(A,*)}$	11.53	7.84	6.71	6.15	5.91	10.99	7.77	6.61	6.19	5.91
$\alpha_H^{(A,*)}$	5.80	4.74	4.41	4.25	4.18	6.47	4.37	3.91	3.77	3.69

Table 4: The optimal contract parameters with different values of ϵ (probability of the low-risk type) under Scenario a) (left panel) and b) (right panel).

4.3 Further analysis of the adverse selection effect

In this subsection, we further evaluate the impact of the information asymmetry on the reinsurer's profit. To this end, we compare the benchmark analysis discussed in Section 4.2 with the following two cases:

1. The perfect information case, in which the reinsurer knows the type of the hedger and offers the corresponding indemnity longevity swap. The optimal contracts in this case are given in Proposition 2.

2. An imperfect information case, in which the reinsurer does not know the type of the hedger, neither does it wish to address the adverse selection issue. Instead, it only offers a single longevity swap based on a Stackelberg game (S) and with risk loading α^S.

In the second case, we assume that the reinsurer and hedger will play a Stackelberg game, a strategic game widely applied in economics (see, among many others, Maharjan *et al.*, 2013; Sinha *et al.*, 2013; Li & Sethi, 2017), where the reinsurer is the Stackelberg leader, and the hedger is the Stackelberg follower. The resulting reinsurance contract will be a subgame perfect Nash equilibrium, which serves both the reinsurer and hedger best. The Stackelberg game proceeds as follows.

1. The reinsurer stipulates the premium for the longevity swap, α^S.

2. Based on the given α^S, the hedger chooses the hedge rate z_i^S based on its type. The chosen z_i^S will maximize its utility $U\big(PR^i(z_i^S, \alpha^S)\big)$.

3. The reinsurer maximizes its expected profit by choosing the optimal risk premium, taking into account the optimal responses of the hedger, i.e. the hedge rate as a function of α^S.

4.3.1 The Stackelberg game

We now derive the optimal contracts with the Stackelberg game. In this case, since the reinsurer only offers a common longevity swap, we assume that this common swap is written on a synthetic portfolio, $l_{x+t} = \epsilon\, l^L_{x+t} + (1-\epsilon) l^H_{x+t}$, $t = 1, 2, ..., \omega_x$. In other words, at each time t, the number of policyholders alive in the synthetic portfolio is the average of the low-risk and the high-risk type portfolio, weighted by their respective probabilities. The time-0 best estimated survival probabilities of this synthetic portfolio is defined as ${}_t\hat{p}_x = \frac{\hat{l}_{x+t}}{l_x}$, $t = 1, 2, ..., \omega_x$.

In the first step of the Stackelberg game, the hedger chooses a hedge rate z_i^S based on its type to maximize its expected utility given an exogenous α^S. The optimization problem has the following form:

$$\max_{z_i^S} \; U\big(PR_i^S(z_i^S, \alpha^S)\big), \text{ where} \tag{4.2}$$
$$PR_i^S = \left(A - \sum_{t=1}^{\omega_x} e^{-rt} l^i_{x+t}\right) + z_i^S l_x \sum_{t=1}^{\omega_x} e^{-rt} \left[{}_tp^i_x - \big(1 + \alpha^S \tilde{\pi}^S_t\big){}_t\hat{p}_x\right].$$

Based on the optimal hedge rate of the hedger, the reinsurer then maximizes its expected profit by choosing the optimal α^S.

At this stage, the reinsurer has two options: it could offer a lower risk premium so that the hedger would purchase the swap regardless of its risk type, or it could set a higher risk premium at which only the high-risk type hedger will be interested in buying the swap. With the former option, the reinsurer enjoys a larger expected transaction volume in exchange of a lower price, whereas the opposite holds for the latter option. A priori, it is unclear which option is better. Hence, we will discuss both of them. The optimization problem of the first option is given by:

$$\begin{aligned}
&\max_{\alpha^S} \ \mathbb{E}_P[PR_R^S], \ \text{where} \\
&PR_R^S = z_L^S \mathbb{1}_{\{L\}} l_x \sum_{t=1}^{\omega_x} e^{-rt} \Big[\big(1+\alpha^S \tilde{\pi}_t^S\big)_t\hat{p}_x - {}_tp_x^L \Big] \\
&\qquad + z_H^S \mathbb{1}_{\{H\}} l_x \sum_{t=1}^{\omega_x} e^{-rt} \Big[\big(1+\alpha^S \tilde{\pi}_t^S\big)_t\hat{p}_x - {}_tp_x^H \Big].
\end{aligned} \tag{4.3}$$

With the second option, the reinsurer sets an α^S so high that the low-risk type hedger will find it too expensive to participate. Formally, denote by $\tilde{\alpha}$ the threshold risk premium that makes the low-risk type hedger indifferent between participating or not (i.e., $z_L^{(S,*)} = 0$). If $\alpha^{(S,*)} > \tilde{\alpha}$, the low-risk type hedger will not enter the market. The optimization problem of the reinsurer in this case is given by:

$$\begin{aligned}
&\max_{\alpha^S} \ \mathbb{E}_P[PR_R^S], \ \text{subject to} \\
&\qquad \alpha^S \geq \tilde{\alpha}, \text{where} \\
&PR_R^S = z_H^S \mathbb{1}_{\{H\}} l_x \sum_{t=1}^{\omega_x} e^{-rt} \Big[\big(1+\alpha^S \tilde{\pi}_t^S\big)_t\hat{p}_x - {}_tp_x^H \Big].
\end{aligned} \tag{4.4}$$

After solving the two optimization problems above, the reinsurer will then compare the resultant expected profits, and track down the final optimal risk loading α^S. The optimal solution of the optimization problems above is summarized in the following proposition.

Proposition 3 (Optimal contracting of the Stackelberg game)**.** *The optimal solution of the maximization problem* (4.2)-(4.3) *is given by:*

$$\alpha^{(S,*)} = \frac{\frac{1}{2}\gamma \mathcal{V}^H \mathcal{V}^L + \epsilon(\hat{\mathcal{D}}^L - \hat{\mathcal{D}})\mathcal{V}^H + (1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}})\mathcal{V}^L}{\mathcal{B}^S\big(\epsilon \mathcal{V}^H + (1-\epsilon)\mathcal{V}^L\big)} \quad \textit{and} \tag{4.5}$$

$$z_i^{(S,*)} = 1 + \frac{\hat{\mathcal{D}}^i - \hat{\mathcal{D}} - \alpha^{(S,*)}\mathcal{B}^S}{\gamma \mathcal{V}^i}, \ \textit{for } i = L, H, \tag{4.6}$$

where $\hat{\mathcal{D}} = \sum_{t=1}^{\omega_x} e^{-rt}\hat{l}_{x+t}$*, and* $\mathcal{B}^S = l_x \sum_{t=1}^{\omega_x} e^{-rt}\tilde{\pi}_t^S {}_t\hat{p}_x$*. The threshold risk premium is*

$\tilde{\alpha} = \frac{\hat{\mathcal{D}}^L - \hat{\mathcal{D}} + \gamma \mathcal{V}^L}{\mathcal{B}^S}$.

The optimal solution of the maximization problem (4.2) *and* (4.4) *exists only if* $\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L \geq \gamma \mathcal{V}^L - \frac{1}{2}\gamma \mathcal{V}^H$, *in which case is given by:*

$$\alpha^{(S,*)} = \frac{\frac{1}{2}\gamma \mathcal{V}^H + (\hat{\mathcal{D}}^H - \hat{\mathcal{D}})}{\mathcal{B}^S}$$
$$z_L^{(S,*)} = 0 \text{ and } z_H^{(S,*)} = 1 + \frac{\hat{\mathcal{D}}^H - \hat{\mathcal{D}} - \alpha^S \mathcal{B}^S}{\gamma \mathcal{V}^H} = \frac{1}{2}.$$

Proof. The derivations are collected in Appendix C.2. □

Hence, when $\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L < \gamma \mathcal{V}^L - \frac{1}{2}\gamma \mathcal{V}^H$, i.e., the difference between the expected liability of the two risk types is not too large compared to the difference in the volatility, it is optimal for the reinsurer to set a lower premium and attract both risk types. In fact, the constraint $\alpha^S \geq \tilde{\alpha}$ will be violated when $\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L < \gamma \mathcal{V}^L - \frac{1}{2}\gamma \mathcal{V}^H$ and hence the optimization problem (4.4) has no solution. On the other hand, when $\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L \geq \gamma \mathcal{V}^L - \frac{1}{2}\gamma \mathcal{V}^H$, the reinsurer will compare the two options and select the $\alpha^{(S,*)}$ that leads to the higher expected profit.

4.3.2 The numerical comparison

In this subsection, we numerically compare the expected profit of the reinsurer resulting from the three cases discussed above. In the latter two cases, the similar standard deviation premium principle is used for the time-varying loadings: $\tilde{\pi}_t^{(P,i)} = \sigma\left(\frac{l_{x+t}^i}{\hat{l}_{x+t}^i}\right)$, $i = L, H$ and $\tilde{\pi}_t^S = \sigma\left(\frac{l_{x+t}}{\hat{l}_{x+t}}\right)$, $t = 1, 2, ..., \omega_x$. Similar to the benchmark analysis, the comparison is done under the two scenarios described in Section 4.2. Under both scenarios, it holds that $\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L < \gamma \mathcal{V}^L - \frac{1}{2}\gamma \mathcal{V}^H$ and thus the solution of the Stackelberg game is given by (4.5) and (4.6), i.e. both risk types will purchase the longevity swap.

Let us define the utility improvement of the hedger by

$$\hat{U}_i^{\Diamond} = U_i(PR_i^{\Diamond}(\alpha_i^{(\Diamond,*)}, z_i^{(\Diamond,*)})) - \bar{U}_i$$

for $i = L, H$ and $\Diamond \in \{A, P, S\}$, i.e. the utility gain achieved from trading the swap. For the three cases, they are given by:

- Benchmark analysis: $\hat{U}_i^A = -\alpha_i^{(A,*)} z_i^{(A,*)} B^i - \frac{1}{2}\gamma\big((z_i^{(A,*)})^2 - 2z_i^{(A,*)}\big)\mathcal{V}^i$.
- Perfect information: $\hat{U}_i^P = -\alpha_i^{(P,*)} z_i^{(P,*)} B^i - \frac{1}{2}\gamma\big((z_i^{(P,*)})^2 - 2z_i^{(P,*)}\big)\mathcal{V}^i$.
- Stakelberg game: $\hat{U}_i^F = z_i^{(S,*)}\hat{\mathcal{D}}^i - z_i^{(S,*)}(\hat{\mathcal{D}} + \alpha^{(S,*)}\mathcal{B}^S) - \frac{1}{2}\gamma\big((z_i^{(S,*)})^2 - 2z_i^{(S,*)}\big)\mathcal{V}^i$.

Finally, we remark that in the case of perfect information, ϵ would be either 0 or 1 because the reinsurer knows exactly the hedger's type. However, in order to facilitate the comparison with the other two cases, we still consider the following expected profit formula in the perfect information case:

$$\begin{aligned}\mathbb{E}_P[PR_R^P] = \mathbb{E}_P\Bigg[& z_L^P\, \mathbb{1}_{\{L\}}\, l_x \sum_{t=1}^{\omega_x} e^{-rt}\big[(1+\alpha_L^P\tilde{\pi}_t^L)_t\hat{p}_x^L - {}_tp_x^L\big] \\ &+ z_H^P\, \mathbb{1}_{\{H\}} l_x \sum_{t=1}^{\omega_x} e^{-rt}\big[(1+\alpha_H^P\tilde{\pi}_t^H)_t\hat{p}_x^H - {}_tp_x^H\big]\Bigg],\end{aligned}$$

with $\mathbb{E}_P[\mathbb{1}_{\{L\}}] = \epsilon$ and $\mathbb{E}_P[\mathbb{1}_{\{H\}}] = 1-\epsilon$.

The optimal contracts, the expected profit of the reinsurer, as well as the utility improvement of both risk types under Scenario a) are shown in Table 5. The total present value of the fixed leg, $\Pi_i^{(\Diamond)} = \alpha_i^{(\Diamond,*)} \sum_{t=1}^{\omega_x} e^{-rt}\tilde{\pi}_t^{(\Diamond)}\, \hat{l}_{x+t}^{(\Diamond)}$, is also shown for each risk type for each case. First, we see that the reinsurer obtains the highest expected profit in the case of perfect information. This is not surprising, as the reinsurer can design contracts that exploit all utility improvement from both risk types. Second, and more importantly, taking the adverse selection into account and offering type-dependent swaps leads to much higher expected profit of the reinsurer under information asymmetry. This is because the benchmark principal-agent problem utilizes type-dependent swaps, and can thus extract more profit from both risk types. This is not possible in the Stackelberg game, where only one swap is offered. In particular, we see that Π_L^A is much higher than Π_H^A in the benchmark case, which indicates the value of type-dependent pricing. Another reason that the reinsurer's expected profit is lower in the Stackelberg game is that the hedger is able to select a hedge rate that maximizes its utility level, which is not the case in both the benchmark analysis and the perfect information case. Hence, both risk types end up with larger utility improvement in the Stackelberg game.

Optimal Contracts					
Benchmark Analysis		Perfect Information		Stackelberg Game	
$z_L^{(A,*)} = 0.86$	$\alpha_L^{(A,*)} = 6.71$	$z_L^{(P,*)} = 1$	$\alpha_L^{(P,*)} = 5.86$	$z_L^{(S,*)} = 0.43$	$\alpha^{(S,*)} = 5.85$
$z_H^{(A,*)} = 1$	$\alpha_H^{(A,*)} = 4.41$	$z_H^{(P,*)} = 1$	$\alpha_H^{(P,*)} = 5.86$	$z_H^{(S,*)} = 0.57$	
Present Value of the Fixed Leg					
$\Pi_L^A = 2663.2$	$\Pi_H^A = 1752.3$	$\Pi_L^P = 2327.3$	$\Pi_H^P = 2327.2$	$\Pi_L^S = \Pi_H^S = 2327.2$	
Expected Profit					
$\mathbb{E}[PR_R^A] = 2015.6$		$\mathbb{E}[PR_R^P] = 2327.1$		$\mathbb{E}[PR_R^S] = 1139.3$	
Utility improvement of the hedger					
$\hat{U}_L^A = 0$	$\hat{U}_H^A = 575$	$\hat{U}_L^P = 0$	$\hat{U}_H^P = 0$	$\hat{U}_L^S = 426$	$\hat{U}_H^S = 762$

Table 5: Optimal contract parameters of the three swaps (top panel), the expected profits of the reinsurer (mid panel) and the utility improvement of the hedger (bottom panel) under Scenario a).

The distribution of the reinsurer's profit in each case under Scenario a) are displayed in Figure 1 and the 0.5%, 5%, 95%, and 99.5% quantiles are shown in Table 6. First, we see that the distribution is most concentrated in the Stackelberg game. This is because the fixed payments of the swap in this case are the same regardless of the hedger's type. Further, the profit distribution is wider in the benchmark case than the perfect information case. The reason is that the optimal hedge rate of the low-risk type hedger is lower in the benchmark case (0.86 vs. 1). Therefore, the profit of the reinsurer resulting from the two risk types are more different than in the benchmark case. We can also see from Table 6 that the benchmark analysis leads to substantially higher quantiles than the Stackelberg game.

The results under Scenario b) can be found in Table 7, from which we observe similar

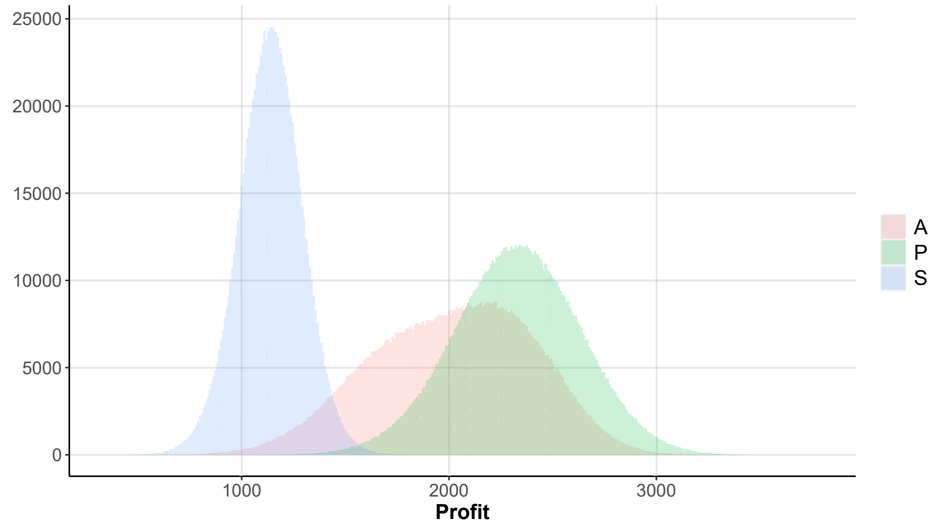


Figure 1: The histogram of the distribution of the profit of the reinsurer for Scenario a) and for each case, where A denotes the benchmark case, P the perfect information case and S the Stackelberg game.

Quantile	0.5%	5%	95%	99.5%
A	1024.8	1360.3	2614.1	2879.2
P	1520.9	1819.4	2824.6	3102.4
S	717.1	883.7	1390.2	1546.8

Table 6: Selected quantiles of the profit distributions of the reinsurer resulting from the benchmark analysis (A), the perfect information case (P), and the Stackelberg game (S) under Scenario a).

patterns as Scenario a). Specifically, offering type-dependent swaps in the case of adverse selection significantly increases the expected profit of the reinsurer. However, compared to Scenario a), the lack of information leads to a greater decrease in the expected profit of the reinsurer (compared to the profit in the perfect information case). This is because the reinsurer needs to set a higher $\Pi_L^{(A,*)}$ and lower $\Pi_H^{(A,*)}$ (compared to $\Pi_L^{(P,*)}$ and $\Pi_H^{(P,*)}$ in the perfect information case) to prevent the now riskier high-risk type hedger from deviating. These risk premiums lead to a lower $z_L^{(A,*)}$ and a higher utility improvement of the high-risk type hedger. The high-risk type hedger enjoys a higher utility improvement in the Stackelberg game. Hence, under information asymmetry, the more the two risk types differ, the more the high-risk type hedger benefits. Finally, in the Stackelberg game, the optimal hedge rate of the two risk types differ more than under Scenario a). Also, the utility improvement has reduced for the low-risk type while increased for the high-risk type, which indicates that the low-risk type subsidizes the high-risk type more when the two risk types are more different.

Optimal Contracts					
Benchmark Analysis		Perfect Information		Stackelberg Game	
$z_L^{(A,*)} = 0.77$	$\alpha_L^{(A,*)} = 7.22$	$z_L^{(P,*)} = 1$	$\alpha_L^{(P,*)} = 5.86$	$z_L^{(S,*)} = 0.36$	$\alpha^{(S,*)} = 6.25$
$z_H^{(A,*)} = 1$	$\alpha_H^{(A,*)} = 4.13$	$z_H^{(P,*)} = 1$	$\alpha_H^{(P,*)} = 7.11$	$z_H^{(S,*)} = 0.64$	
Present Value of the Fixed Leg					
$\Pi_L^A = 2864.0$	$\Pi_H^A = 1855.5$	$\Pi_L^P = 2327.3$	$\Pi_H^P = 3195.2$	$\Pi_L^S = \Pi_H^S = 2640.1$	
Expected Profit					
$\mathbb{E}[PR_R^A] = 2029.5$		$\mathbb{E}[PR_R^P] = 2761.2$		$\mathbb{E}[PR_R^S] = 1273.1$	
Utility improvement of the hedger					
$\hat{U}_L^A = 0$	$\hat{U}_H^A = 1340$	$\hat{U}_L^P = 0$	$\hat{U}_H^P = 0$	$\hat{U}_L^S = 302$	$\hat{U}_H^S = 1307$

Table 7: Optimal contract parameters of the three swaps (top panel), the expected profits of the reinsurer (mid panel) and the utility improvement of the hedger (bottom panel) under Scenario b).

Quantile	0.5%	5%	95%	99.5%
A	990.4	1395.8	2540.3	2784.8
P	1598.6	1935.4	3648.6	4015.0
S	799.0	924.9	1763.1	1997.5

Table 8: Selected quantiles of the profit distributions of the reinsurer resulting from the benchmark analysis (A), the perfect information case (P), and the Stackelberg game (S) under Scenario b)

The profit distributions under Scenario b) are displayed in Figure 2 and the corresponding quantiles are shown in Table 8. We see that the profit distributions become bi-modal for the Stackelberg game and the perfect information case. The reason is that the difference between the risk premium (of the indemnity swaps) and the hedge rate (of the single swap) of the two risk types becomes substantially larger under Scenario b). Consequently, the two risk types result in rather varied profits for the reinsurer. Also, for both distributions, the left peak is higher and more concentrated due to the lower and more certain profit

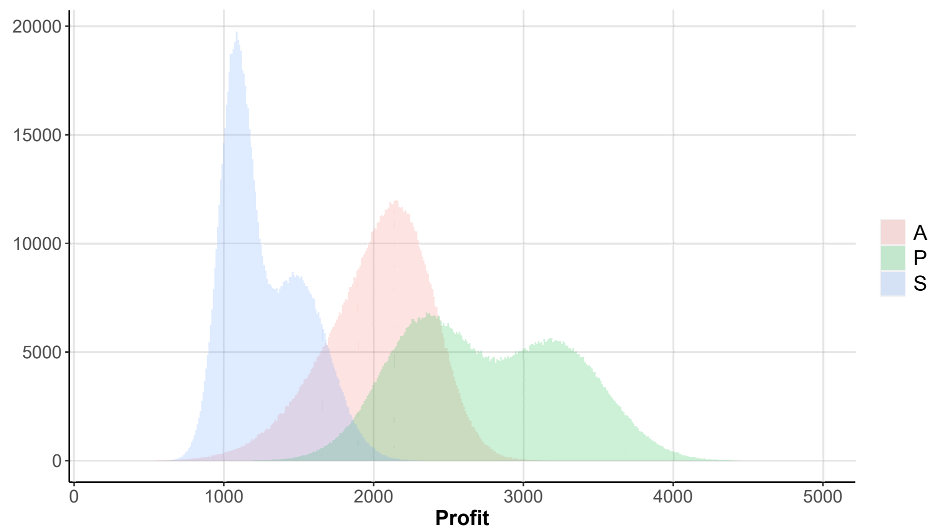


Figure 2: The histogram of the distribution of the profit of the reinsurer for Scenario b) and for each case, where A denotes the benchmark case, P the perfect information case and S the Stackelberg game.

resulting from the low-risk type. Interestingly, the bi-modal phenomenon is not observed in the benchmark case. This is because, while $z_L^{(A,*)}$ becomes lower than $z_H^{(A,*)}$, $\alpha_L^{(A,*)}$ becomes much higher than $\alpha_H^{(A,*)}$. These two changes make the profit resulting from the low-risk type even closer to that from the high-risk type, compared to Scenario a). Finally, the benchmark analysis leads to substantially higher quantiles than the Stackelberg game under Scenario b) as well. Hence, we see that properly addressing adverse selection in longevity swap transaction can lead to not only a higher expected profit, but also much lower downside risk for the reinsurer.

5 Conclusion

In this work, we study the optimal longevity swap deal between a reinsurer and a hedger (e.g., pension funds and life insurers) in a principal-agent model in the presence of adverse selection. We derive analytical solution to the optimal risk premium of the swap contracts and the incentive-compatible hedge demands in a separating equilibrium, and discuss the conditions under which the optimal solution fits to the classic literature in this field. In particular, the high-risk type's incentive constraint is binding and obtains full coverage as optimal insurance. On the other hand, the low-risk type has a binding participation constraint with partial insurance, because the equilibrium price of the longevity swap offered to the low-risk type is higher than the price it would have been offered under perfect information, at which the low-risk type would opt for full insurance. Furthermore, we show that the equilibrium will break down if the high-risk type is substantially riskier than the low-risk type, in which case purchasing the longevity swap would in fact lower the low-risk type's utility level.

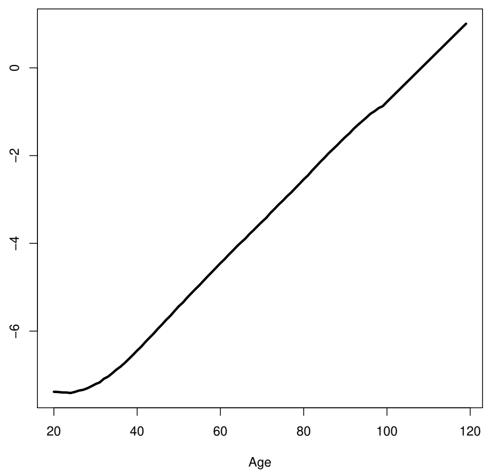
To further study the impact of adverse selection, we determine the optimal hedge demands both under perfect information and in a Stackelberg game where the reinsurer is the Stackelberg leader, and compare these results with our setting under adverse selection. Using real-world mortality data and a sophisticated mortality model, we find that potentially severe losses could be generated for the reinsurer if adverse selection is not taken into consideration in the contracting.

Finally, although the proposed principal-agent framework is illustrated with a longevity swap, it could be applied in other contexts, including traditional longevity reinsurance and the innovative products discussed in the literature, such as tail longevity risk reinsurance, longevity swaptions, and index-based longevity-linked derivatives. Furthermore, our analysis assumes the same degree of risk aversion of both risk types. It might be the case that the high-risk type is more risk averse than the low-risk type. Exploration of these interesting topics are left for future research.

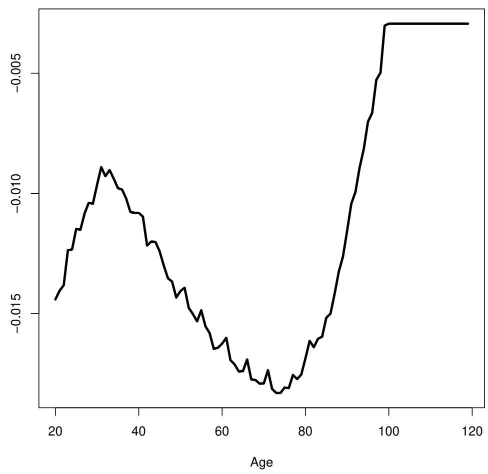
A Key Variable definitions

Variable	Definition
l_{x+t} $(\hat{l}_{x+t})$	the random (expected) number of policyholders alive in year t
ω_x	maximal remaining life time for the policyholders with x at time 0
${}_tp_x$ $({}_t\hat{p}_x)$	the random (expected) t-year survival probability of the policyholders
r	constant annual interest rate
$\tilde{\pi}_t^i$	the time-varying risk loading at time t of the swap designed for type $i = L, H$
α_i	the constant risk premium of the swap designed for type $i = L, H$
z_i	the hedge rate of the hedger of risk type $i = L, H$
ϵ	probability that the hedger is the low-risk type
$\mathcal{B}^i$	the expected present value of the initial risk loading for risk type i
$U(x)$	the mean-variance utility function
$\bar{U}^i$	reservation utility level of risk type i
$\hat{\mathcal{D}}^i$	the expected present value of the type-i hedger's liability
$\mathcal{V}^i$	the variance of the present value of the type-i hedger's liability
Π_i^J	total present value of the fixed leg of risk type i with swap J

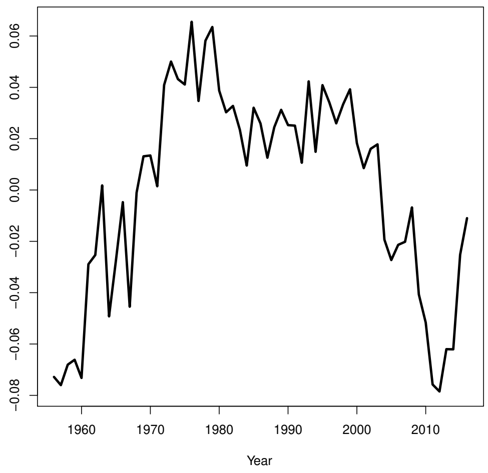
B Estimation and extrapolation results



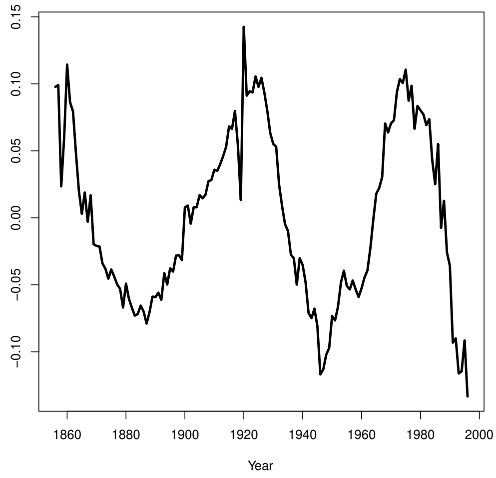
(a) The estimated and extrapolated values of $\beta_x^{(1)}$.



(b) The estimated and extrapolated values of $\beta_x^{(2)}$.



(c) The estimated values of κ_t.



(d) The estimated values of γ_c.

Note that a linear extrapolation of $\beta_x^{(2)}$ would result in positive values for very old ages and consequently leads to increasing mortality rates which is unrealistic. Therefore, we set $\beta_x^{(2)}$ equal to $\beta_{100}^{(2)}$ for very old ages.

C The solution of the Lagrangian function

C.1 Indemnity swap with adverse selection

Based on (3.2), we have the following participation and incentive constraints:

$$
\begin{aligned}
(PC1): \quad & U(PR^L(\alpha_L^A, z_L^A)) - \bar{U}^L = -\alpha_L^A\, z_L^A\, \mathcal{B}^L - \frac{1}{2}\gamma((z_L^A)^2 - 2z_L^A)\mathcal{V}^L \\
(IC2): \quad & U(PR^H(\alpha_H^A, z_H^A)) - U(PR^H(\alpha_L^A, z_L^A)) \\
& = \alpha_L^A\, z_L^A\, \mathcal{B}^L - \alpha_H^A\, z_H^A\, \mathcal{B}^H + z_L^A(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) - \frac{1}{2}\gamma\Big((z_H^A)^2 - 2z_H^A - (z_L^A)^2 + 2z_L^A \Big)\mathcal{V}^H.
\end{aligned}
$$

The optimal solution shall satisfy the following first order conditions and complementary slackness conditions of the Lagrangian function described in (3.3):

$$
\begin{aligned}
0 = \frac{\partial \mathcal{L}}{\partial z_L^A} &= \epsilon \alpha_L^A \mathcal{B}^L - \lambda_1^A(\alpha_L^A \mathcal{B}^L + \gamma \mathcal{V}^L(z_L^A - 1)) \\
&+ \lambda_2^A\Big(\alpha_L^A \mathcal{B}^L + \hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H + \gamma\big(z_L^A - 1\big)\mathcal{V}^H \Big)
\end{aligned} \tag{C.1}
$$

$$0 = \frac{\partial \mathcal{L}}{\partial z_H^A} = (1-\epsilon)\alpha_H^A\, \mathcal{B}^H - \lambda_2^A(\alpha_H^A \mathcal{B}^H + \gamma \mathcal{V}^H (z_H^A - 1)) \tag{C.2}$$

$$0 = \frac{\partial \mathcal{L}}{\partial \alpha_L^A} = \epsilon z_L^A \mathcal{B}^L - \lambda_1^A\, z_L^A\, \mathcal{B}^L + \lambda_2^A\, z_L^A\, \mathcal{B}^L = z_L^A\, (\epsilon \mathcal{B}^L - \lambda_1^A \mathcal{B}^L + \lambda_2^A \mathcal{B}^L) \tag{C.3}$$

$$0 = \frac{\partial \mathcal{L}}{\partial \alpha_H^A} = (1-\epsilon) z_H^A\, \mathcal{B}^H - \lambda_2^A\, z_H^A\, \mathcal{B}^H \Rightarrow \lambda_2^A = 1-\epsilon \tag{C.4}$$

$$0 = -\alpha_L^A\, z_L^A\, \mathcal{B}^L - \frac{1}{2}\gamma \mathcal{V}^L z_L^A (z_L^A - 2) \tag{C.5}$$

$$0 = \alpha_L^A\, z_L^A\, \mathcal{B}^L - \alpha_H^A\, z_H^A\, \mathcal{B}^H + z_L^A(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) - \frac{1}{2}\gamma\Big((z_H^A)^2 - 2z_H^A - (z_L^A)^2 + 2z_L^A \Big)\mathcal{V}^H \tag{C.6}$$

With $\lambda_2^A = 1-\epsilon$ and (C.2) we get

$$\begin{aligned} 0 &= (1-\epsilon)\alpha_H^A\, \mathcal{B}^H - (1-\epsilon)(\alpha_H^A\, \mathcal{B}^H + \gamma \mathcal{V}^H (z_H^A - 1)) \\ &= (1-\epsilon)\gamma \mathcal{V}^H (1 - z_H^A) \Rightarrow z_H^A = 1 \end{aligned} \tag{C.7}$$

If we assume that $z_L^A > 0$ we get from (C.3)

$$\lambda_1^A = \epsilon + 1 - \epsilon = 1.$$

Then we plug this into (C.1)

$$\begin{aligned} 0 &= (\epsilon-1)\alpha_L^A \mathcal{B}_L^A - \gamma \mathcal{V}^L (z_L^A - 1) + (1-\epsilon)\Big(\alpha_L^A\, \mathcal{B}^L + \hat{D}^L - \hat{D}^H + \gamma\big(z_L^A - 1\big)\mathcal{V}^H\Big)\Big) \\ &= -\gamma \mathcal{V}^L (z_L^A - 1) + (1-\epsilon)(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + (1-\epsilon)\big(\gamma\big(z_L^A - 1\big)\mathcal{V}^H\big) \\ \Rightarrow z_L^A &= 1 + \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\big((1-\epsilon)\mathcal{V}^H - \mathcal{V}^L\big)}. \end{aligned} \tag{C.8}$$

Then from Equation (C.5) we have

$$\begin{aligned} 0 &= -\alpha_L^A\, \mathcal{B}^L - \frac{1}{2}\gamma \mathcal{V}^L (z_L^A - 2) = -\alpha_L^A\, \mathcal{B}^L + \frac{1}{2}\gamma \mathcal{V}^L (2 - z_L^A) \\ \Rightarrow \alpha_L^A &= \frac{\gamma \mathcal{V}^L}{2\mathcal{B}^L}\left(1 - \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\big((1-\epsilon)\mathcal{V}^H - \mathcal{V}^L\big)}\right) \end{aligned} \tag{C.9}$$

Finally, from Equations (C.5) to (C.6), we have

$$\begin{aligned} 0 &= -\alpha_H^A\, z_H^A\, \mathcal{B}^H + z_L^A(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \frac{1}{2}\gamma\mathcal{V}^H + \frac{1}{2}\gamma\big((z_L^A)^2 - 2z_L^A\big)(\mathcal{V}^H - \mathcal{V}^L) \\ \Rightarrow \alpha^H &= \frac{\gamma\mathcal{V}^H}{2\mathcal{B}^H} + \frac{z_L^A}{\mathcal{B}^H}(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \frac{\gamma}{2\mathcal{B}^H}\big((z_L^A)^2 - 2z_L^A\big)(\mathcal{V}^H - \mathcal{V}^L). \end{aligned} \tag{C.10}$$

Next, we check if the inequalities are satisfied. Therefore, we define $\tilde{\Omega} \equiv \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L)}{\gamma\Big((1-\epsilon)\mathcal{V}^H - \mathcal{V}^L\Big)}$. For the incentive constraint of the first hedger (IC1), we have

$$\begin{aligned} (IC1):\quad & U(PR^L(\alpha_L^A, z_L^A)) - U(PR^L(\alpha_H^A, z_H^A)) \\ &= \alpha_H^A\, z_H^A\, \mathcal{B}^H - \alpha_L^A\, z_L^A\, \mathcal{B}^L + z_H^A(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L) - \frac{1}{2}\gamma\Bigg((z_L^A)^2 - 2z_L^A - (z_H^A)^2 + 2z_H^A\Bigg)\mathcal{V}^L \\ &= \frac{\gamma\mathcal{V}^H}{2} + (1+\tilde{\Omega})(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \frac{\gamma}{2}(\tilde{\Omega}^2 - 1)(\mathcal{V}^H - \mathcal{V}^L) - (\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) \\ &\quad - \frac{\gamma\mathcal{V}^L}{2}(1 - \tilde{\Omega}^2) - \frac{\gamma\mathcal{V}^L}{2}(1 - 2\tilde{\Omega} + \tilde{\Omega}^2 - 2 + 2\tilde{\Omega} + 1) \\ &= \tilde{\Omega}(\hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H) + \tilde{\Omega}^2\frac{\gamma}{2}(\mathcal{V}^H - \mathcal{V}^L) \\ &> 0, \end{aligned}$$

if $\tilde{\Omega} < 0$. That is, the low-risk type hedger would prefer the swap tailor-made for its type. The participation constraint for the high-risk type hedger is:

$$\begin{aligned} (PC2):\quad & U(PR^H(\alpha_H^A, z_H^A)) - \bar{U}^H = -\alpha_H^A\, z_H^A\, \mathcal{B}^H - \frac{1}{2}\gamma\big((z_H^A)^2 - 2z_H^A\big)\mathcal{V}^H \\ &= -\alpha_H^A\, \mathcal{B}^H + \frac{\gamma\mathcal{V}^H}{2} \\ &= (1+\tilde{\Omega})(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}^L) + \frac{\gamma}{2}(1 - \tilde{\Omega}^2)(\mathcal{V}^H - \mathcal{V}^L) \\ &> 0, \end{aligned}$$

if $\tilde{\Omega} > -1$. Note that $\tilde{\Omega} \in [-1, 0]$ if $z_L^A \in [0, 1]$.

C.2 Stackelberg game

Optimization Problem (4.2)**.** The expected utility of hedger of risk type i is:

$$U(PR_i^S(z_i^S,\alpha^S)) = A - (1-z_i^S)\hat{\mathcal{D}}^i - z_i^S(\hat{\mathcal{D}} + \alpha^S\mathcal{B}^S) - \frac{1}{2}\gamma\mathcal{V}^i(1-z_i^S)^2,$$

and the first order condition is given by:

$$0 = \frac{\partial\mathcal{L}}{\partial z_i^S} = \hat{\mathcal{D}}^i - \hat{\mathcal{D}} - \alpha^S\mathcal{B}^S + \gamma\mathcal{V}^i(1-z_i^S).$$

Optimization Problem (4.3)**.** The first order conditions are given by:

$$\begin{aligned}
0 = \frac{\partial\mathcal{L}}{\partial\alpha^S} &= \frac{\epsilon\mathcal{B}^S(\hat{\mathcal{D}}^L - \hat{\mathcal{D}})}{\gamma\mathcal{V}^L} + \frac{(1-\epsilon)\mathcal{B}^S(\hat{\mathcal{D}}^H - \hat{\mathcal{D}})}{\gamma\mathcal{V}^H} \\
&+ \mathcal{B}^S + \epsilon\frac{(\hat{\mathcal{D}}^L - \hat{\mathcal{D}})\mathcal{B}^S - 2\alpha^S(\mathcal{B}^S)^2}{\gamma\mathcal{V}^L} + (1-\epsilon)\frac{(\hat{\mathcal{D}}^H - \hat{\mathcal{D}})\mathcal{B}^S - 2\alpha^S(\mathcal{B}^S)^2}{\gamma\mathcal{V}^H} \\
&= \frac{\epsilon(\hat{\mathcal{D}}^L - \hat{\mathcal{D}})}{\gamma\mathcal{V}^L} + \frac{(1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}})}{\gamma\mathcal{V}^H} + 1 + \epsilon\frac{\hat{\mathcal{D}}^L - \hat{\mathcal{D}} - 2\alpha^S\mathcal{B}^S}{\gamma\mathcal{V}^L} \\
&+ (1-\epsilon)\frac{\hat{\mathcal{D}}^H - \hat{\mathcal{D}} - 2\alpha^S\mathcal{B}^S}{\gamma\mathcal{V}^H} \\
\Leftrightarrow \frac{2\alpha^S\mathcal{B}^S}{\gamma}&\left(\frac{\epsilon}{\mathcal{V}^L} + \frac{1-\epsilon}{\mathcal{V}^H}\right) = 1 + \epsilon\frac{2(\hat{\mathcal{D}}^L - \hat{\mathcal{D}})}{\gamma\mathcal{V}^L} + (1-\epsilon)\frac{2(\hat{\mathcal{D}}^H - \hat{\mathcal{D}})}{\gamma\mathcal{V}^H} \\
\Leftrightarrow \alpha^S\mathcal{B}^S&\big(\epsilon\mathcal{V}^H + (1-\epsilon)\mathcal{V}^L\big) = \frac{1}{2}\gamma\mathcal{V}^H\mathcal{V}^L + \epsilon(\hat{\mathcal{D}}^L - \hat{\mathcal{D}})\mathcal{V}^H + (1-\epsilon)(\hat{\mathcal{D}}^H - \hat{\mathcal{D}})\mathcal{V}^L.
\end{aligned}$$

Optimization Problem (4.4)**.** The first order conditions are given by:

$$\begin{aligned}
0 &= \frac{\partial\mathcal{L}}{\partial\alpha^S} = (1-\epsilon)(\mathcal{B}^S + 2(\frac{\mathcal{B}^S(\hat{\mathcal{D}}^H - \hat{\mathcal{D}}) - \alpha^S(\mathcal{B}^S)^2}{\gamma\mathcal{V}^H}) + \lambda \\
0 &= \lambda(\alpha^S - \tilde{\alpha}).
\end{aligned}$$

Suppose $\alpha^S > \tilde{\alpha}$, then $\lambda = 0$, and the optimal solution is $\alpha^{(S,*)} = \frac{\frac{1}{2}\gamma\mathcal{V}^H + (\hat{\mathcal{D}}^H - \hat{\mathcal{D}})}{\mathcal{B}^S}$. Otherwise, if $\alpha^S = \tilde{\alpha}$, then the solution is simply $\tilde{\alpha}$. Denote by $\mathbb{E}_{\mathbb{P}}\big[PR_R^S(\alpha^{(S,*)})\big]$ and $\mathbb{E}_{\mathbb{P}}\big[PR_R^S(\tilde{\alpha})\big]$ the expected profit of the reinsurer given $\alpha^{(S,*)}$ and $\tilde{\alpha}$, respectively, we have:

$$\mathbb{E}_{\mathbb{P}}\big[PR_R^S(\alpha^{(S,*)})\big] - \mathbb{E}_{\mathbb{P}}\big[PR_R^S(\tilde{\alpha})\big] = \big(\frac{1}{2}\gamma\mathcal{V}^H - (\gamma\mathcal{V}^L + \hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H)\big)^2.$$

Hence, $\alpha^{(S,*)}$ is preferred to $\tilde{\alpha}$ whenever $\frac{1}{2}\gamma\mathcal{V}^H \neq \gamma\mathcal{V}^L + \hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H$. However, when $\frac{1}{2}\gamma\mathcal{V}^H < \gamma\mathcal{V}^L + \hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H$, it holds that $\alpha^{(S,*)} < \tilde{\alpha}$, and thus the low-risk type hedger will enter the market as well. In this case, the optimization problem (4.4) no longer reflects the reinsurer's expected profit. Therefore, optimal solution exists only when $\frac{1}{2}\gamma\mathcal{V}^H \geq \gamma\mathcal{V}^L + \hat{\mathcal{D}}^L - \hat{\mathcal{D}}^H$.

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